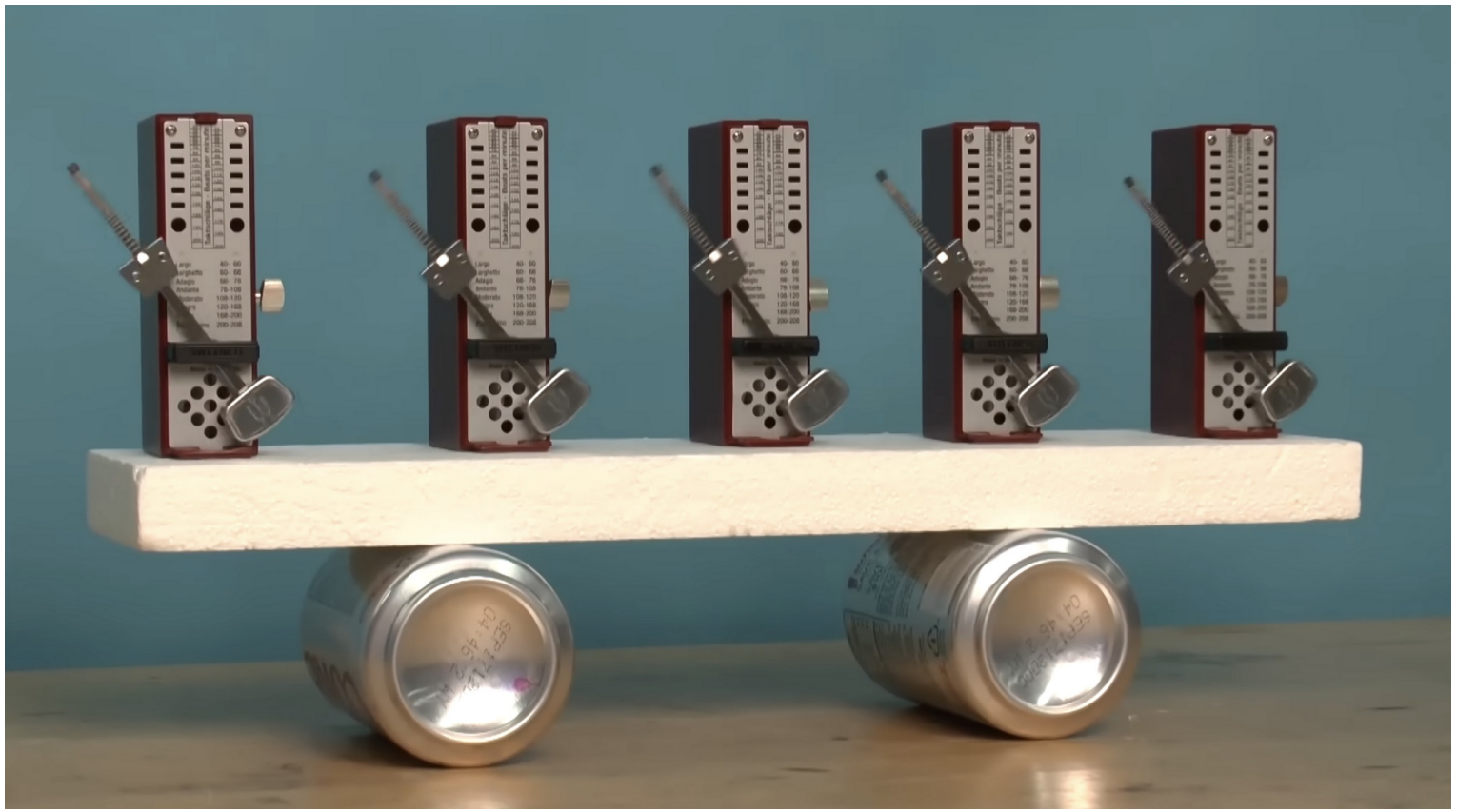
Transient Stability

ELEN90060 Power System Analysis

Keith Moffat



https://www.youtube.com/watch?v=T58lGKREubo&t=1s

“Stable operation” means keeping enormous rotating machines in step.



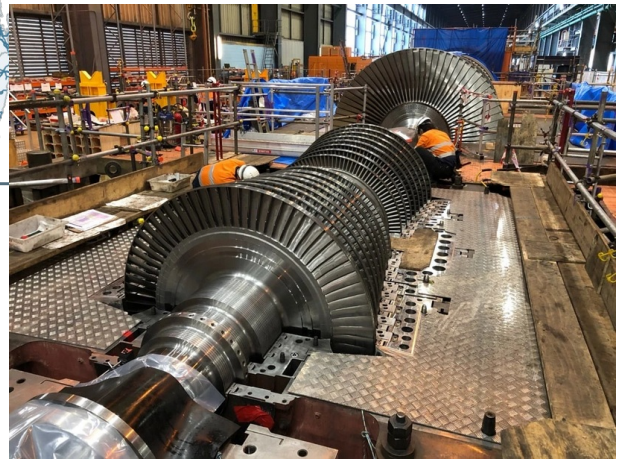
https://theenergy.co



https://www.csenergy.com.au

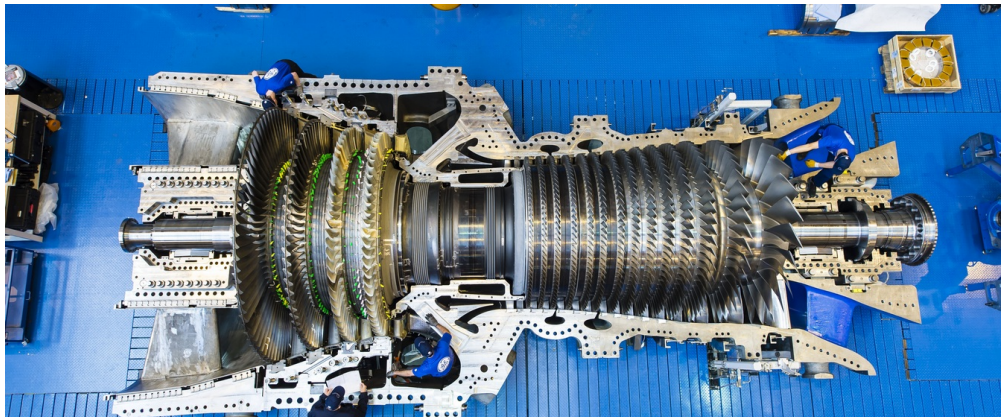


https://portal.engineersaustralia.org.au



https://tic.toshiba.com.au

All generators connected to the grid must synchronize and stay synchronized at all times.



https://www.ge.com/news/reports/its-only-natural-new-ohio-power-plant-to-use-ges-record-setting-natural-gas-turbines

Large gas turbine + generator

Combined rotor inertia:

$J = 30{,}000$ kg m²

Rotation: 3,000 rpm

$\omega_m = 2\pi \times 3{,}000 / 60 \approx 314$ rad/s

$W = \frac{1}{2} J \omega_m^2$

$= \frac{1}{2} \times 30{,}000 \times 314^2$

≈ 1.5 GJ



https://commons.wikimedia.org/wiki/File:Series-E5-U2_Hayabusa-30.jpg

High-speed train

Mass: 400 tonnes

$m = 400{,}000$ kg

Speed: 300 km/h = 83.3 m/s

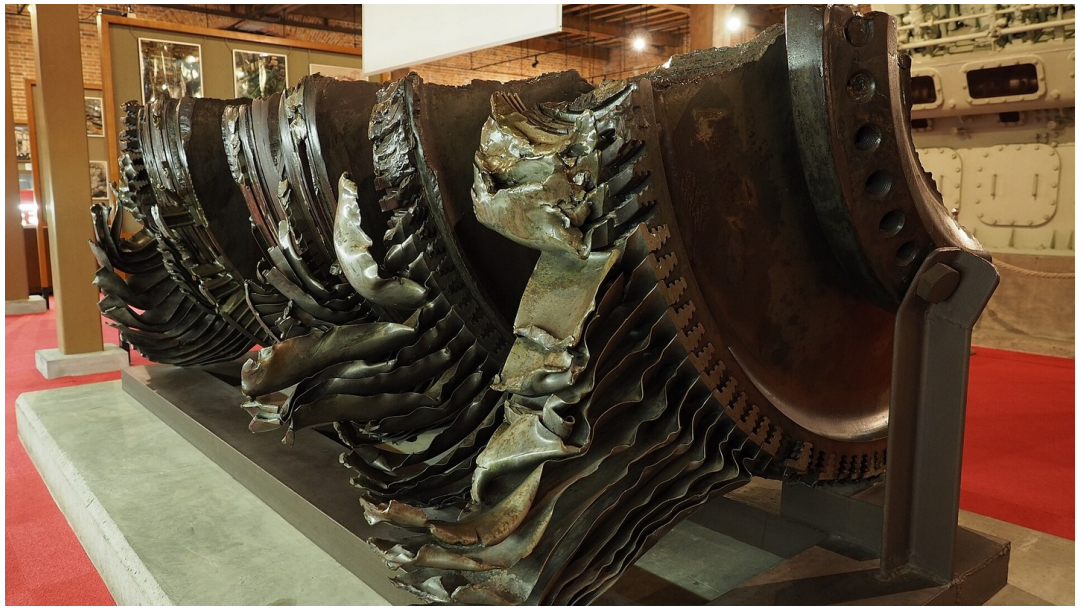
$W = \frac{1}{2} m v^2$

$= \frac{1}{2} \times 400{,}000 \times 83.3^2$

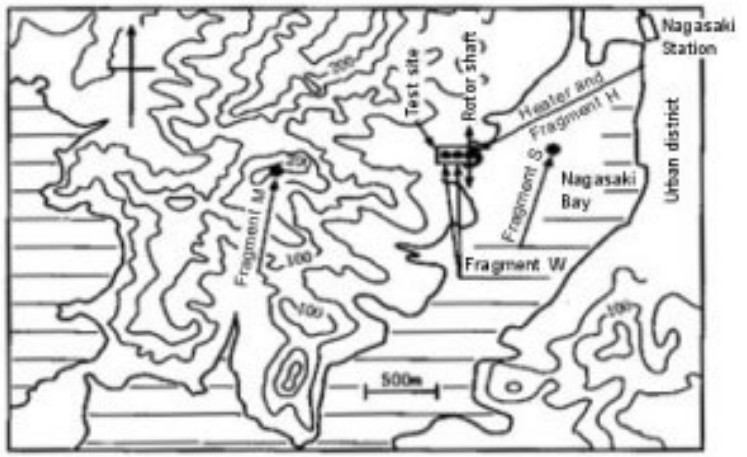
≈ 1.4 GJ

The rotating turbine and generator store about as much energy as the whole train.

Nagasaki Turbine failure, 1970



https://commons.wikimedia.org/wiki/File:Steam_turbine_rotor_1970.jpg



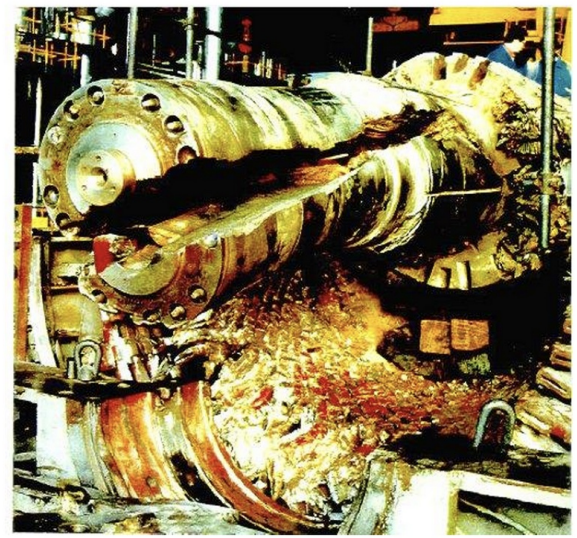
In 1970, a steam turbine fractured during an overspeed test.

This 9 ton weight fragment was found in Nagasaki Bay 880 m away from the accident site.

Generator manufacturers put a lot of time and effort into avoiding such catastrophes

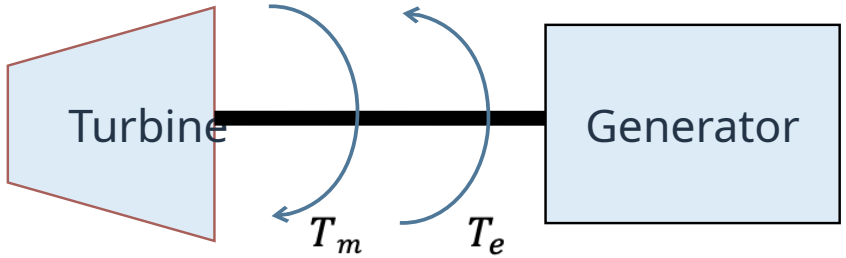
Pole slip / out of step protection (78)

- Asynchronous running of a synchronous machine with the rest of the system but **with excitation intact (opposed to loss of Field)**
- Characterized by power (P & Q) oscillation
- Manifests as impedance movement in R & X plane
- destructive mechanical impact on turbine and shaft
- Pole Slip typically caused by:
  - Long fault clearance time (especially close by 3Ph faults are critical)
  - Inadvertent tripping of a transmission line (increase of transmission impedance between generator and load)
  - Loss of large generator unit



© ABB Group

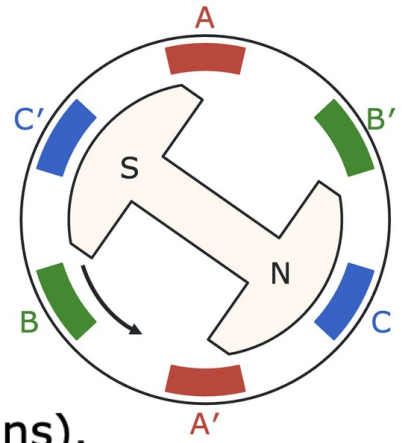
https://library.e.abb.com/public/38405c7967bf4222a36befd9877d0b87/2015%20Generator%20Protection%20Fundamentals_JC.pdf#page=64



The turbine and generator share a shaft.

Loss of synchronism causes large current and torque oscillations that can cause damage.

Protection disconnects the generator if a pole "slips."



A pole slip is when the generator's rotor and stator phase angles pass $180°$ (π radians).

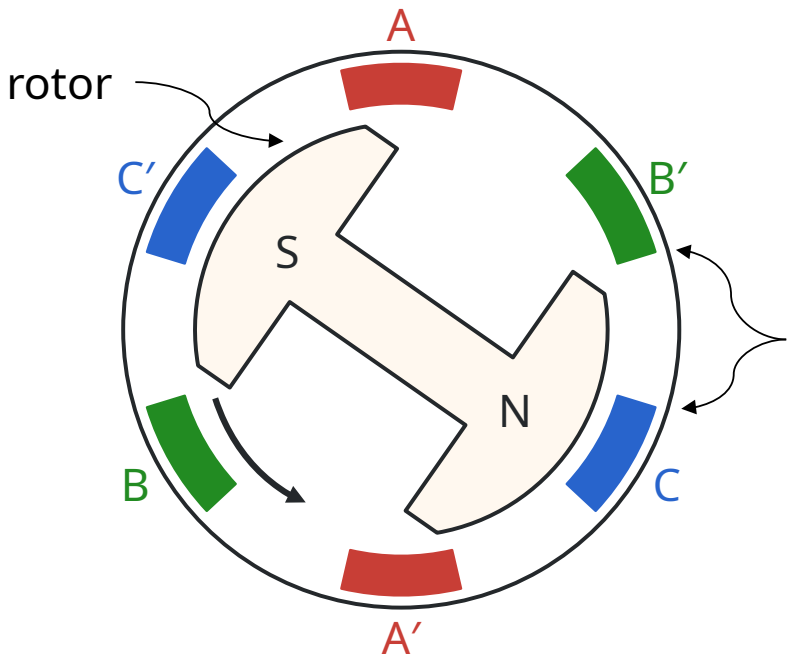
The term comes from the generator rotor's magnetic poles "slipping" past the stator's poles.
(the stator poles come from the grid's 3-phase AC signal)

A pole slip can cause large current and torque stresses.

180° is when pole slip occurs but it is not the stability limit!
→ a disturbance can make loss of synchronism inevitable before that angle is reached.

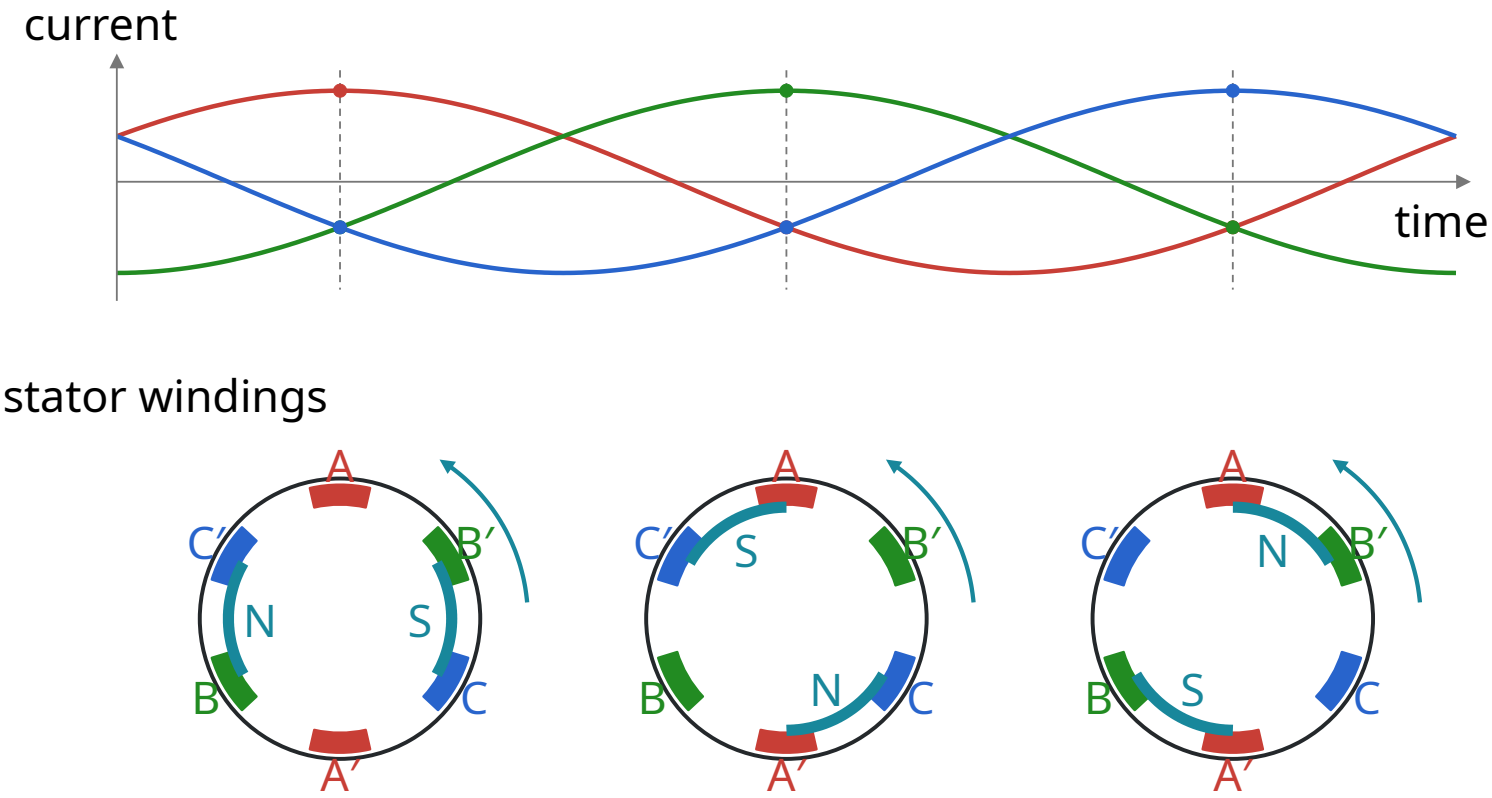
Synchronous machine: the turbine drives the rotor magnetic pole, pulling the stator magnetic pole.

Rotor Pole



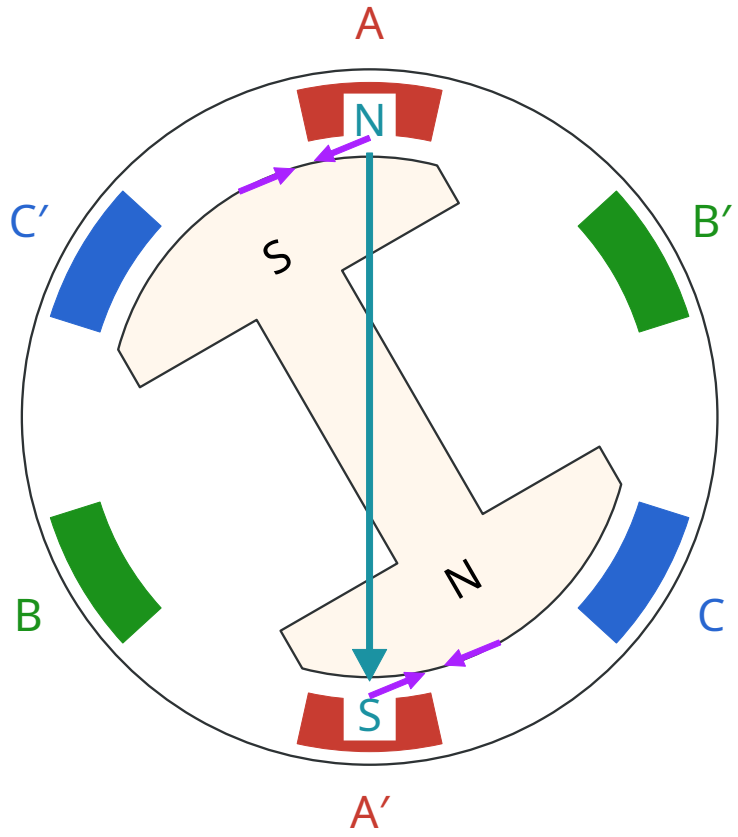
The rotor pole position is determined by the position of the rotor in space.

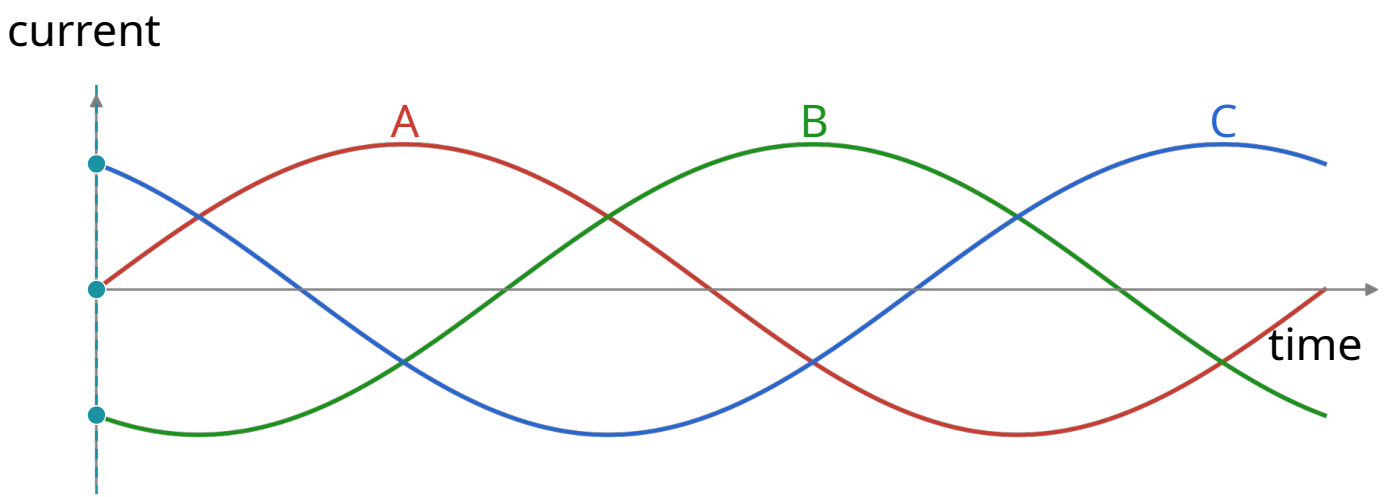
Stator Pole



The stator pole position is determined by the (change in) current through the stator windings.

Synchronous machine: the turbine drives the rotor magnetic pole, pulling the stator magnetic pole.

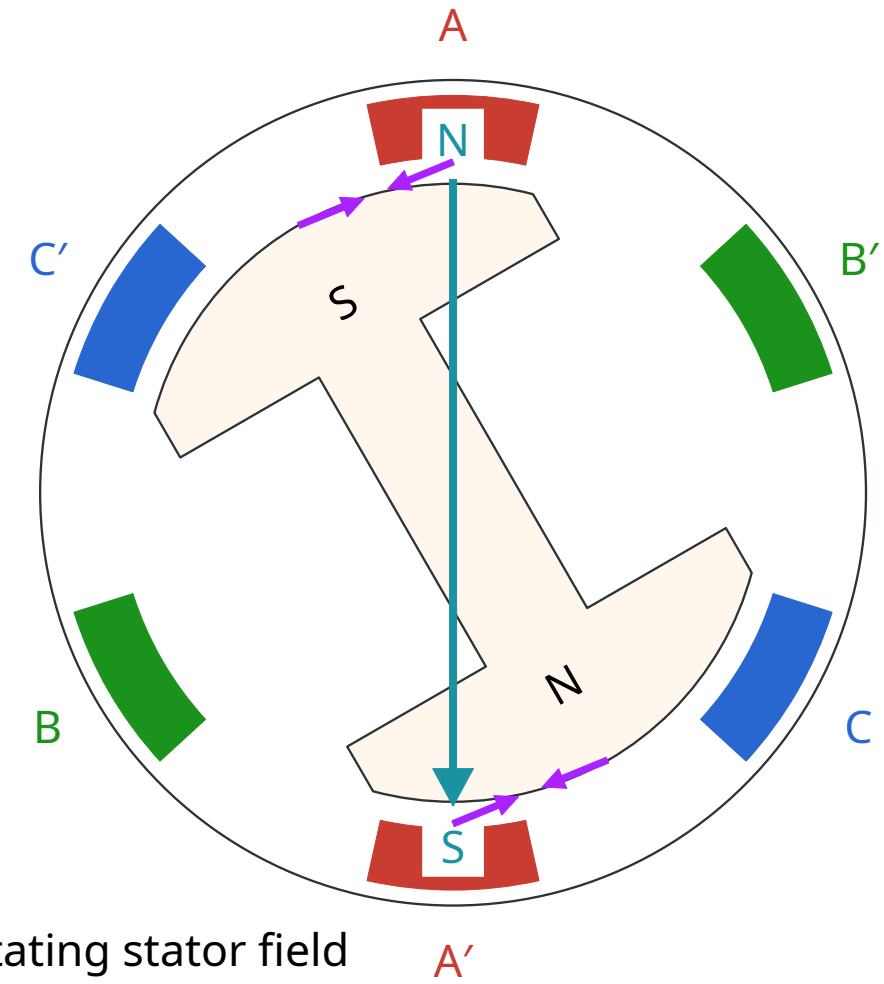


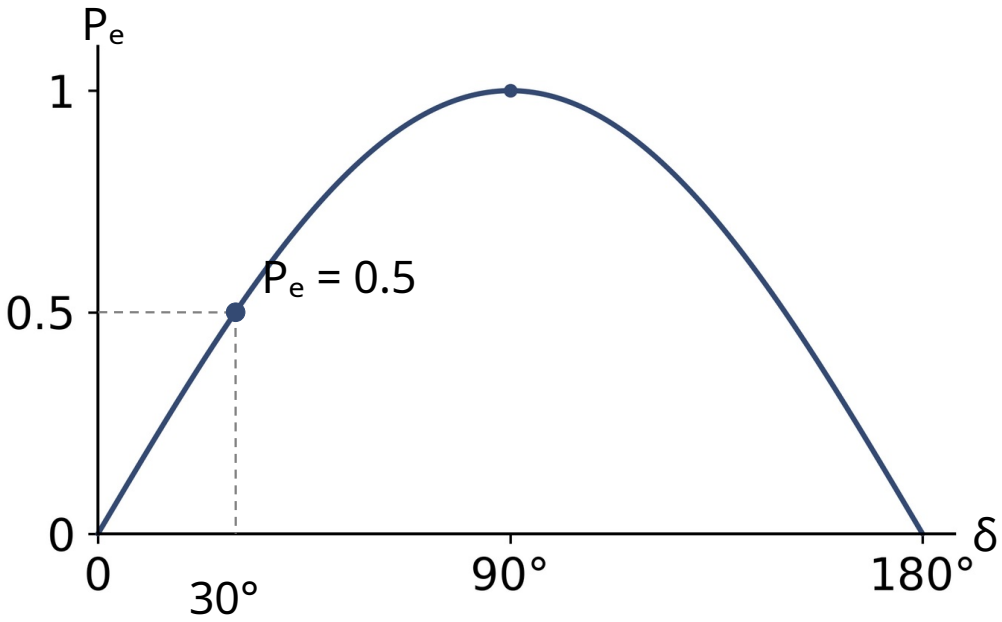


→ Rotating stator field

→ Magnetic attraction

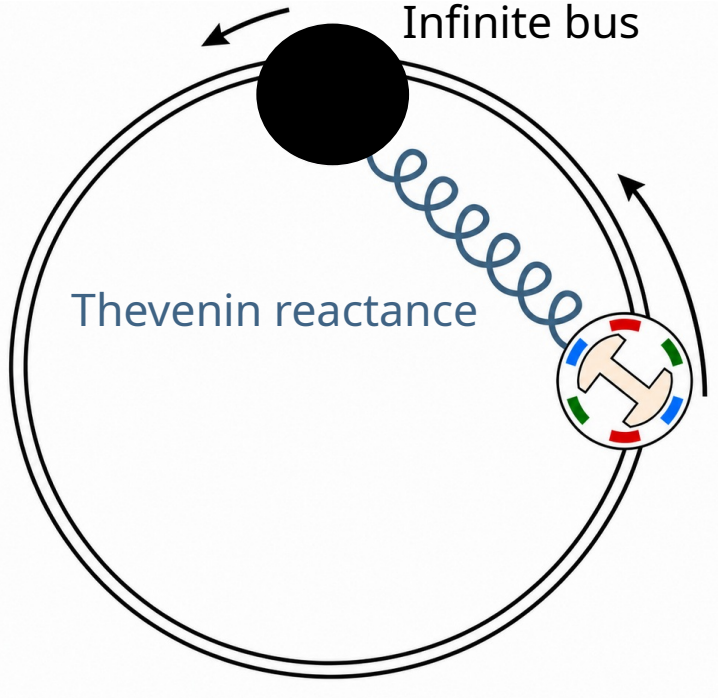
The angle between the rotor and the grid determines the electrical power flowing onto the grid.





A mechanical analogy for synchronization/transient stability

Synchronization: common frequency and settled relative rotor angles.

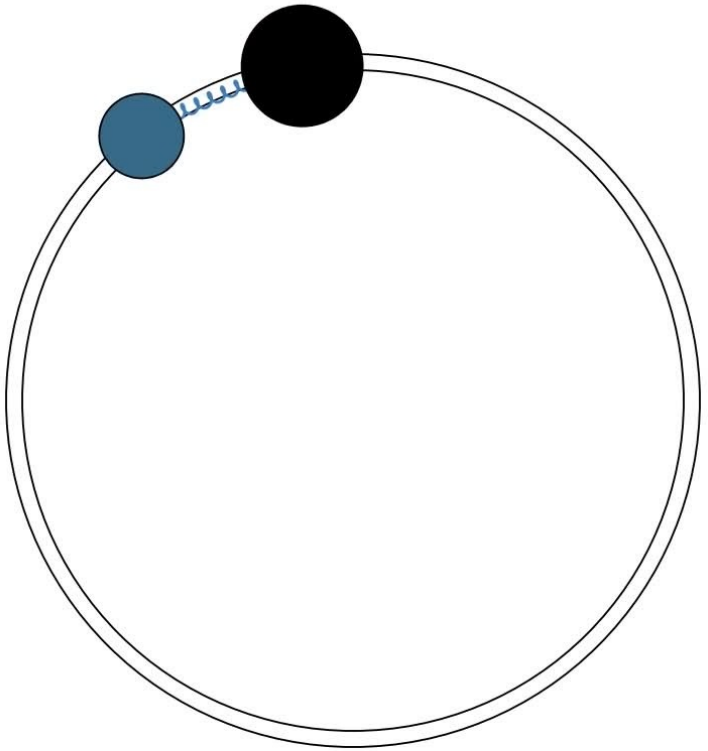


The inductive Thevenin reactance create synchronizing torque, modeled as a spring.

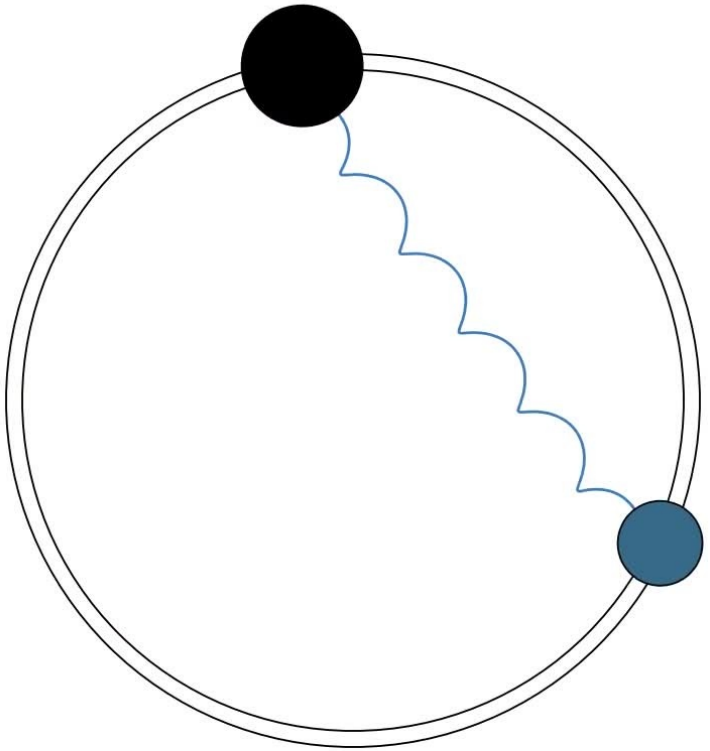
https://doi.org/10.1016/j.automatica.2014.04.012

What is a “pole slip”?

Initial conditions A:

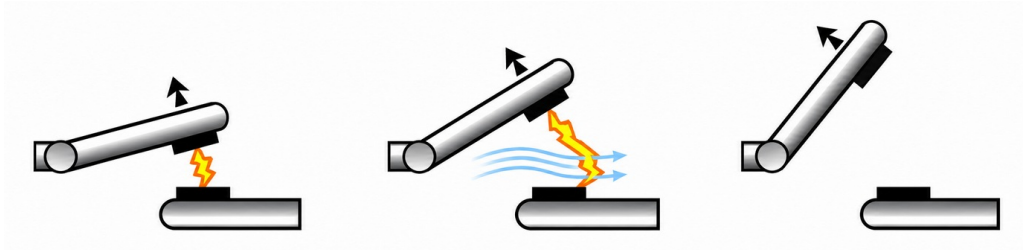


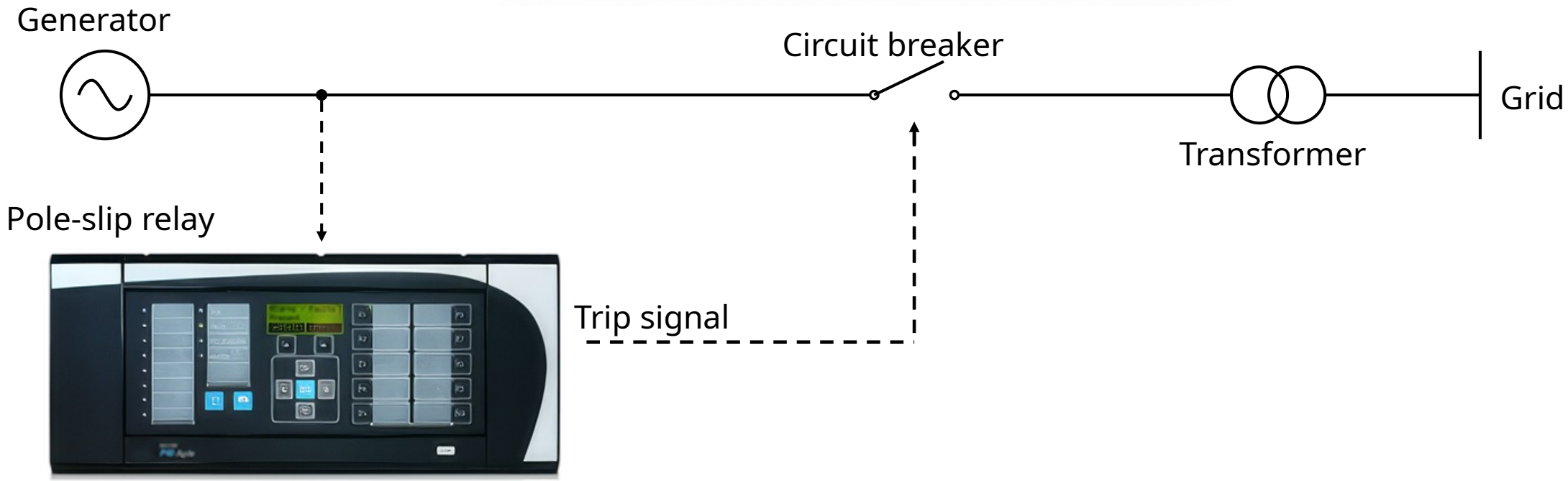
Initial conditions B:



The rotor can sometimes regain synchronism after a slip due to damping, but often it does not.

How is catastrophic damage avoided when a generator slips a pole?





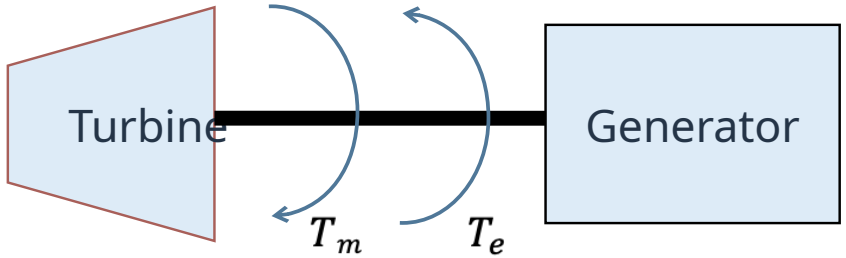
https://www.gevernova.com/electrification/sites/default/files/2026-02/P34x-TM-EN-Ia8-GEV.pdf
https://www.gevernova.com/electrification/automation/protection-control-metering/generator-protection/micom-agile-p34x
https://www.hitachienergy.com/products-and-solutions/generator-circuit-breakers/generator-circuit-breakers/hec-10-up-to-210-ka

Protection disconnects the generator if a pole slips… unless it doesn't.



Callide C4, Queensland, 25 May 2021

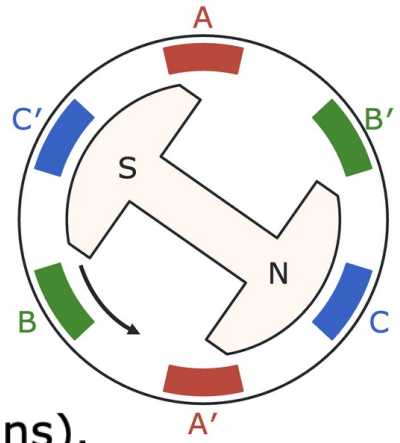
https://www.csenergy.com.au/news/cs-energy-releases-photo-of-unit-c4



The turbine and generator share a shaft.

Loss of synchronism causes large current and torque oscillations that can cause damage.

Protection disconnects the generator if a pole "slips."



A pole slip is when the generator's rotor and stator phase angles pass $180°$ (π radians).

The term comes from the generator rotor's magnetic poles "slipping" past the stator's poles.
(the stator poles come from the grid's 3-phase AC signal)

A pole slip can cause large current and torque stresses.

180° is when pole slip occurs but it is not the stability limit!
→ **a disturbance can make loss of synchronism inevitable before that angle is reached**.

Mundra, India, 13 July 2016

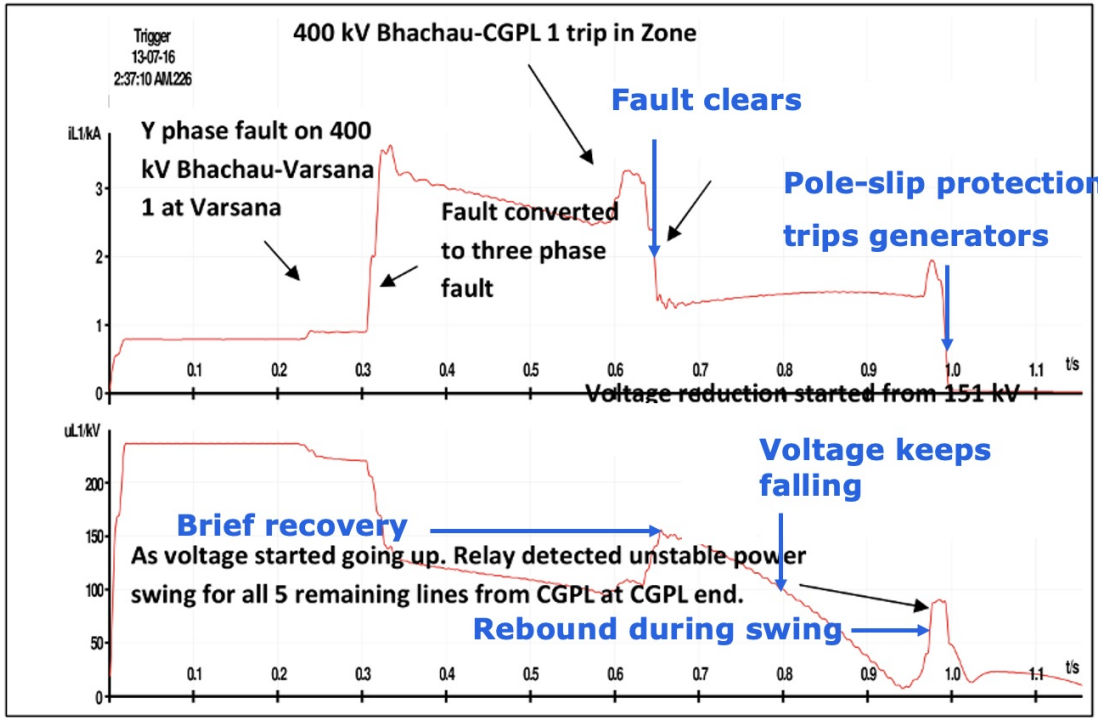
A fault and line trips reduced the generators' electrical output.

The fault cleared before a pole slipped.

Some time after, the pole slipped and the generator disconnected.

Why?



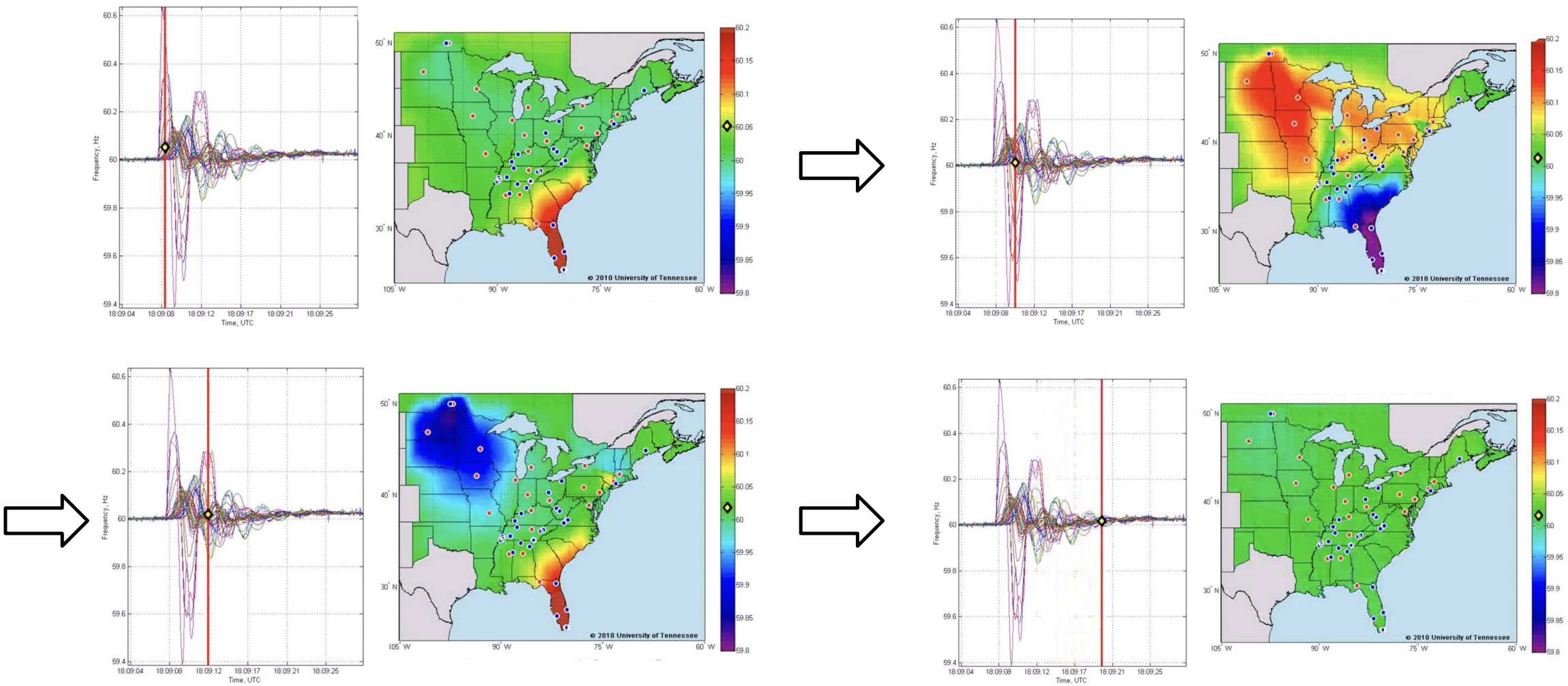


Recorded current and voltage

https://www.tatapower.com/energy-solutions/thermal-energy/mundra-plant

https://www.wrpc.gov.in/allfile/WRPC_Report_on_CGPL_Blackout_13-07-2016.pdf

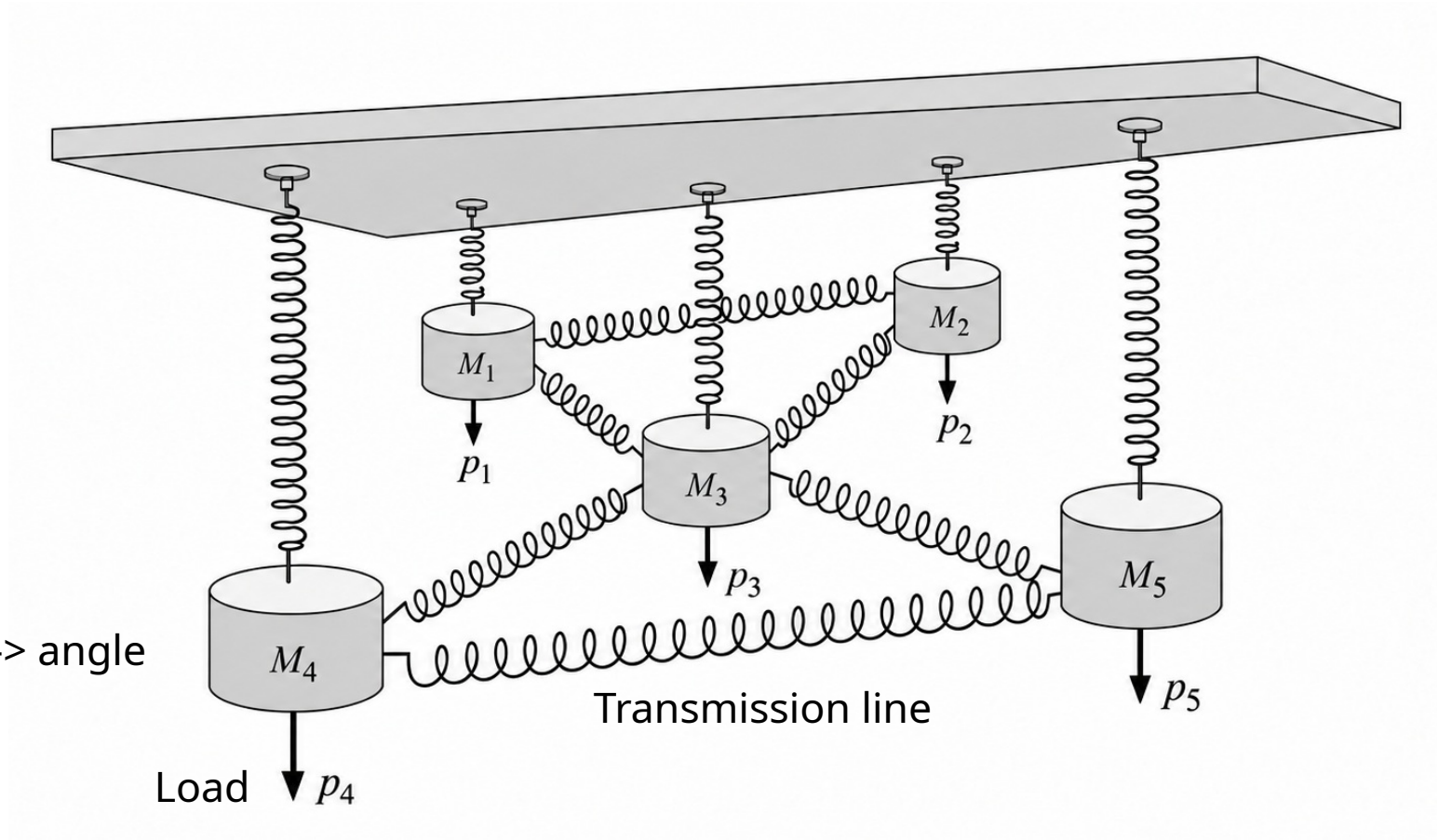
What was different about that fault from this fault that settled back to equilibrium?



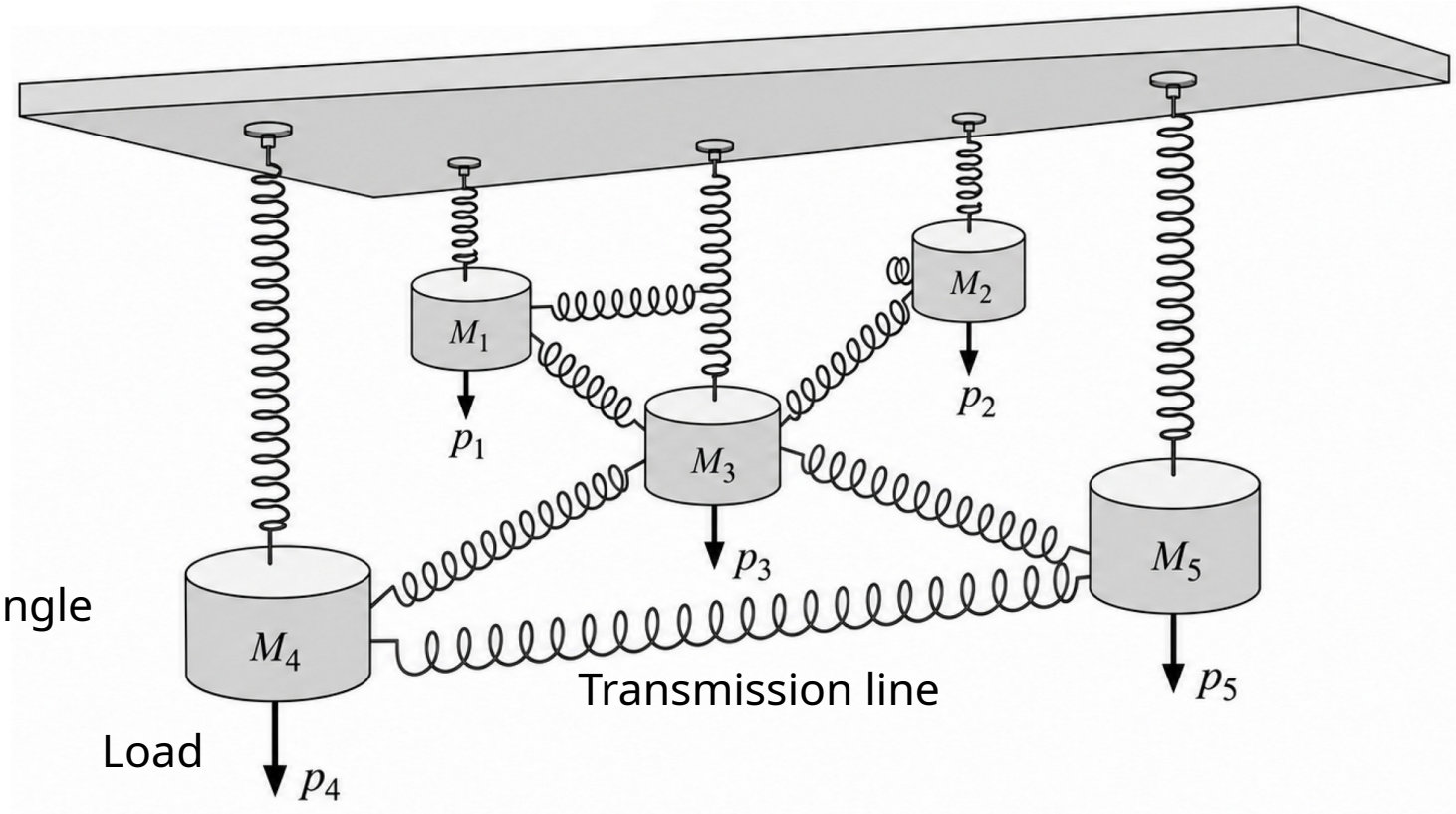
https://www.youtube.com/watch?v=bdBB4byrZ6U

That is the purpose of *Transient Stability Analysis* and today's lecture.

A mechanical analogy for transient power system stability:



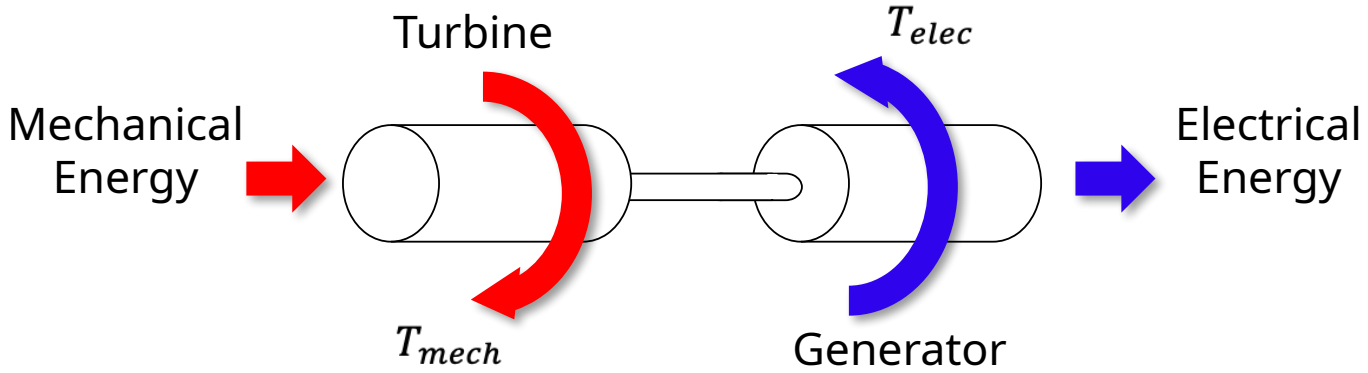
A mechanical analogy for transient power system stability:

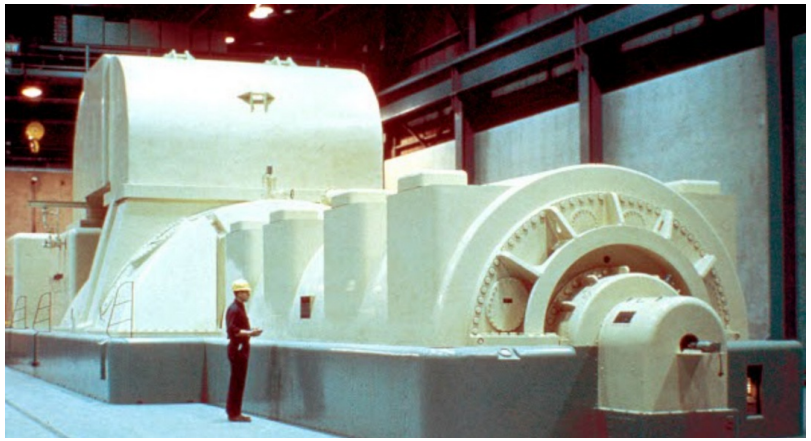


The transmission line spring provides the synchronizing force

Can the grid settle after a disturbance?

Power balance and generator dynamics





T_m: Mechanical torque applied by the turbine
T_e: Electromagnetic torque developed by the generator
J: Moment of inertia of the turbine/generator set
δ: Rotor angle in synchronous frame
ω: Actual rotor speed
P_m: Mechanical power delivered by the turbine (per unit)
P_e: Electrical power injected into the network (per unit)
ω_s: Rated synchronous rotor speed

Kinetic energy: $W = \frac{1}{2} J \omega^2$

Inertia constant: $H = \frac{W^\circ}{S_{base}} = \frac{J\omega_s{}^2}{2S_{base}}$

$M = \frac{2H}{\omega_s}$

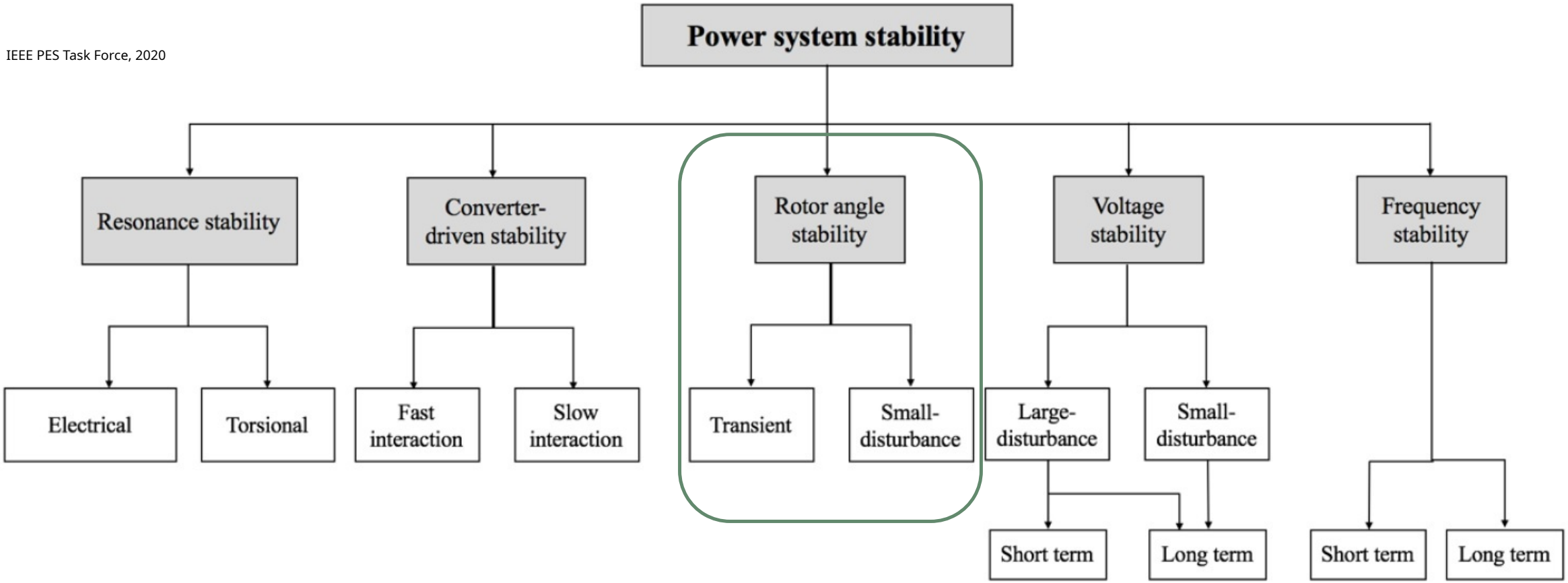
The swing equation:

$$J\frac{d^2\delta}{dt^2} = T_m - T_e \quad \Rightarrow \quad \frac{J\omega}{S_{base}}\frac{d^2\delta}{dt^2} = P_m - P_e \quad \Rightarrow \quad M\frac{d^2\delta}{dt^2} \approx P_m - P_e$$

$$J\frac{d\omega}{dt} = T_m - T_e \qquad \frac{J\omega}{S_{base}}\frac{d\omega}{dt} = P_m - P_e \qquad \omega \approx \omega_s$$

Classical power system stability classification

IEEE PES Task Force, 2020

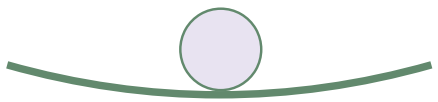


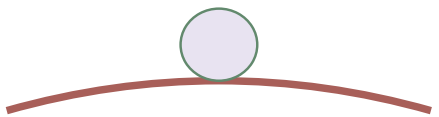
Small-disturbance stability: The ability of a power system to maintain synchronism following a “small” disturbance

Transient stability: The ability of a power system to maintain synchronism when subjected to a “large” disturbance

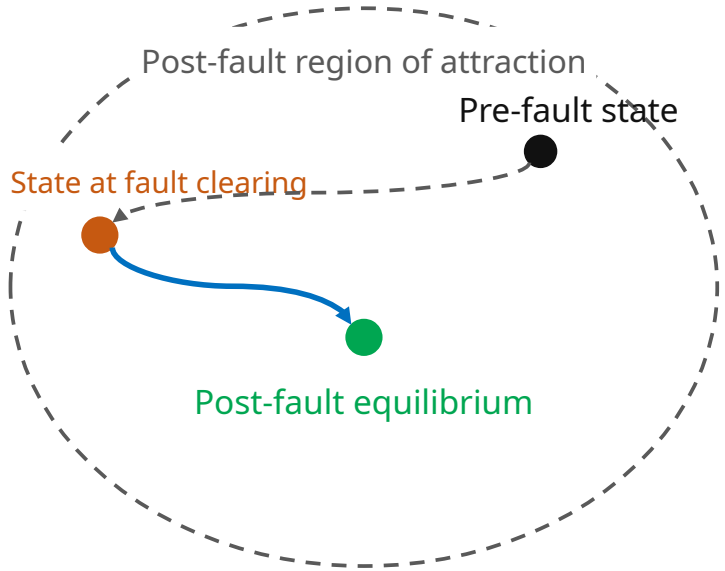
Small-Signal stable equilibrium:

the system returns to the steady state equilibrium after a small perturbation

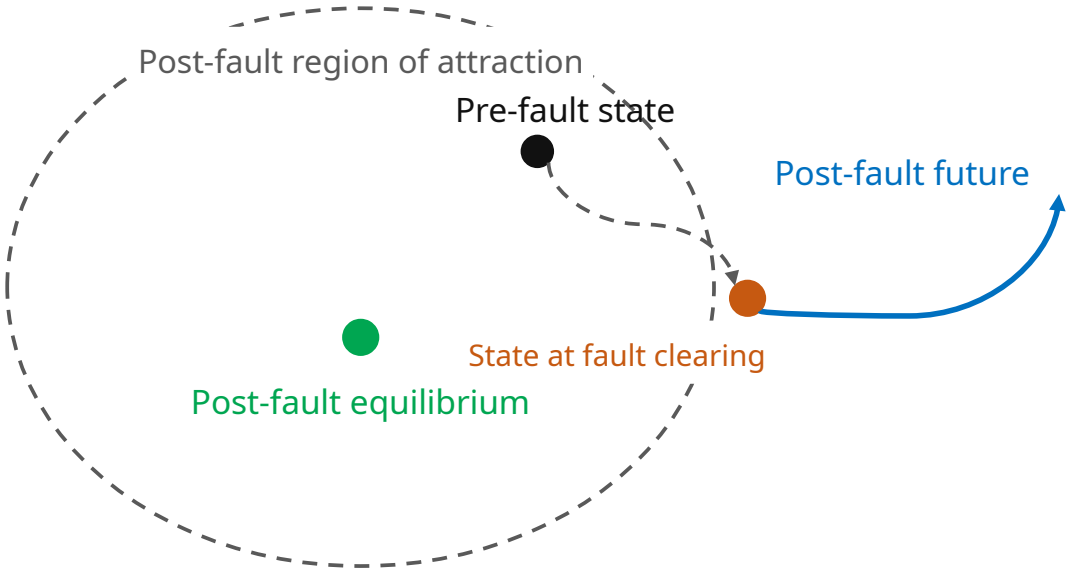




Transient Stability: the system must stay within a region of attraction after a fault/transient.



Stable



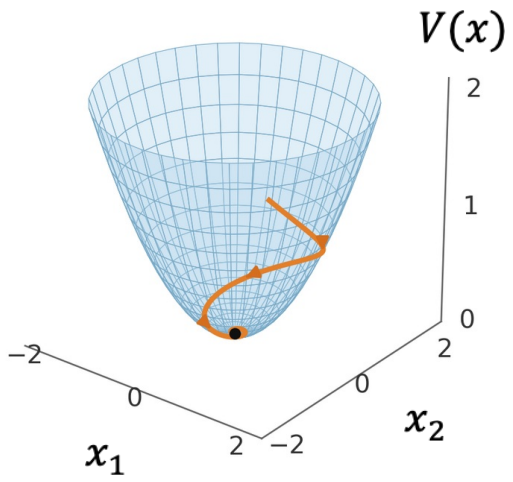
Unstable

Of course, Transient Stability depends on the specific fault/transient.

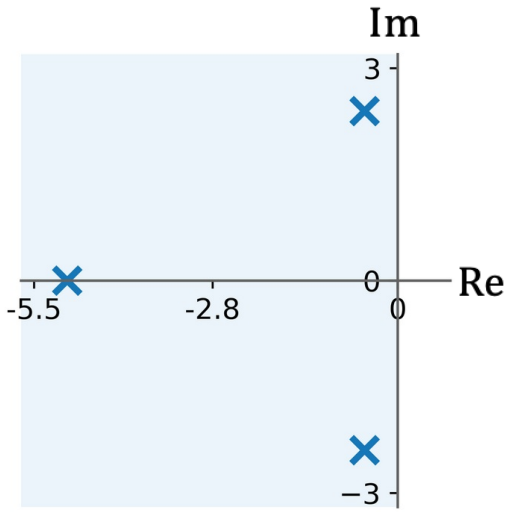
Control theorists understand stability as small signal stability and large signal stability

Large signal/Transient

Lyapunov

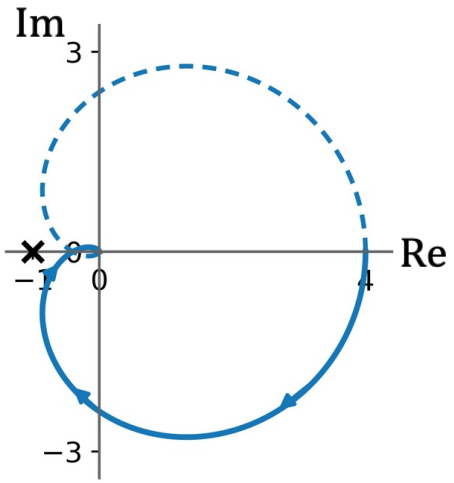


Pole Plot

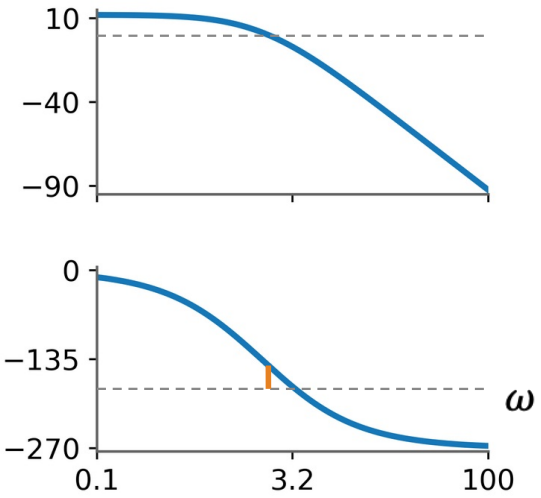


Small signal

Nyquist



Bode



How do we analyze power system stability?



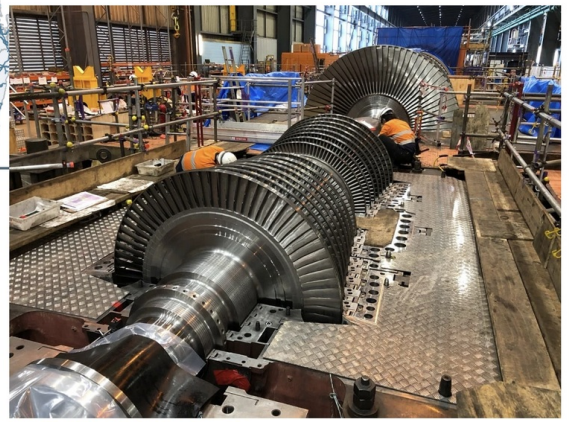
https://theenergy.co



https://www.csenergy.com.au



https://portal.engineersaustralia.org.au



https://tic.toshiba.com.au

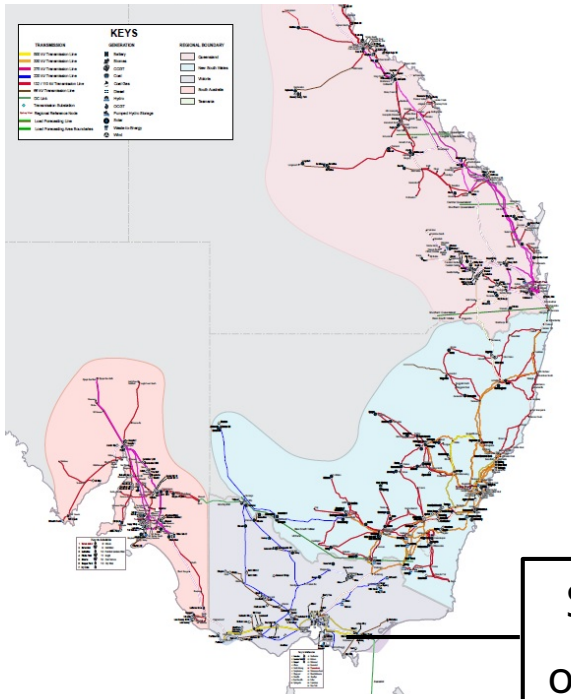
The grid a large interconnected system with many connected devices.

We look at simplified systems to develop intuition and guidelines for the real-world grids.

An infinite bus keeps its voltage magnitude and frequency fixed

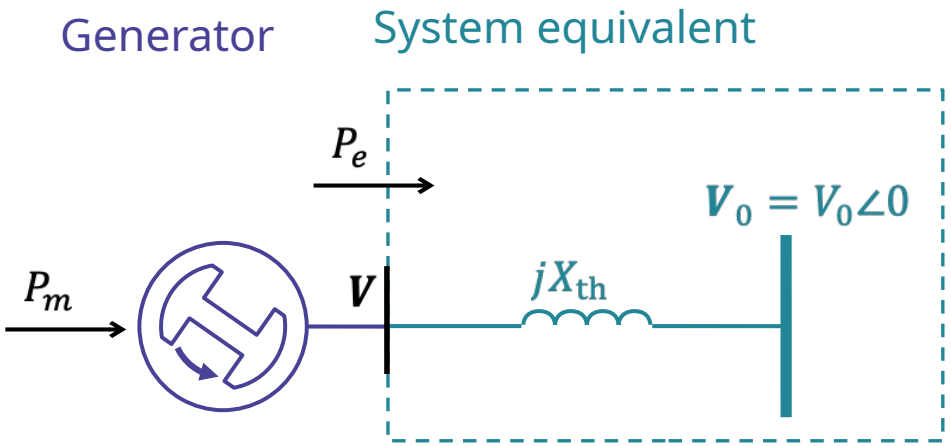
The grid can be approximated by an infinite bus/voltage source behind an impedance.

This is an approximation and does not represent the true system dynamics.

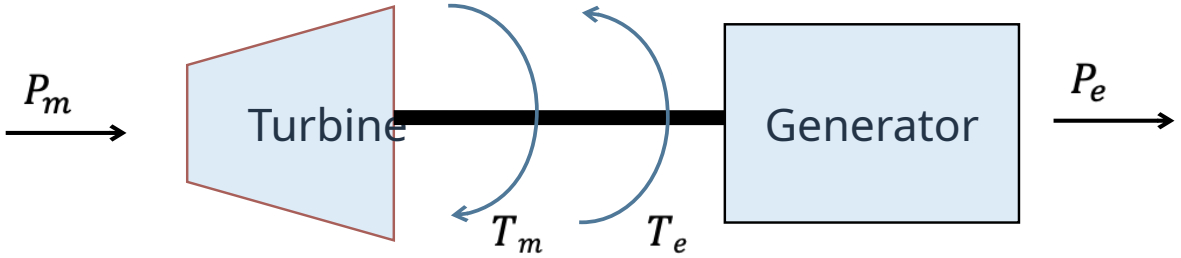


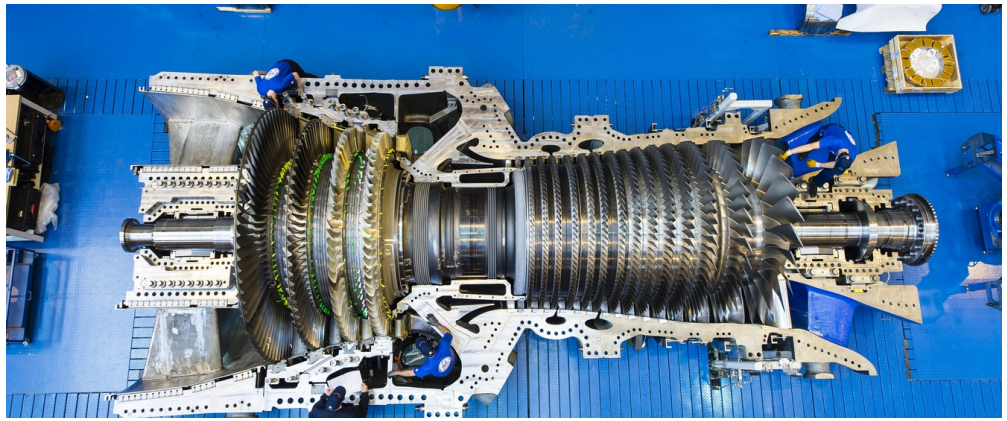
small compared with the grid

Single load
or generator

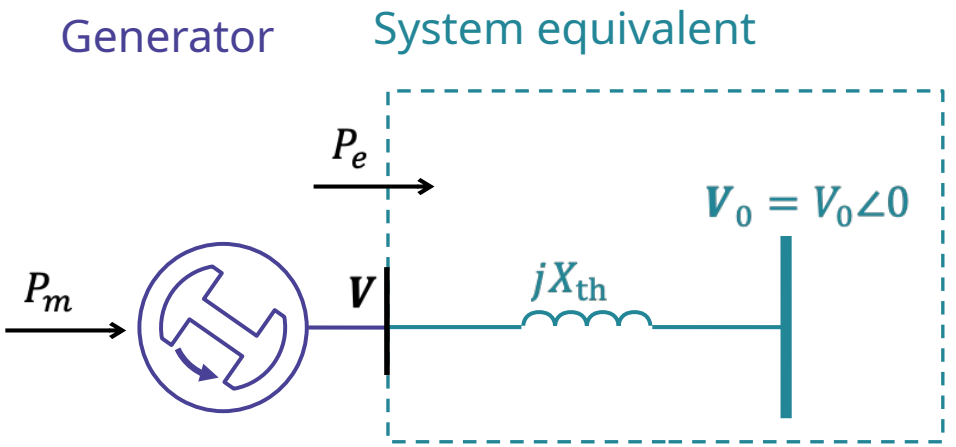


(Rotor) Angle Stability: After a disturbance, will the generator stay synchronized with the grid?



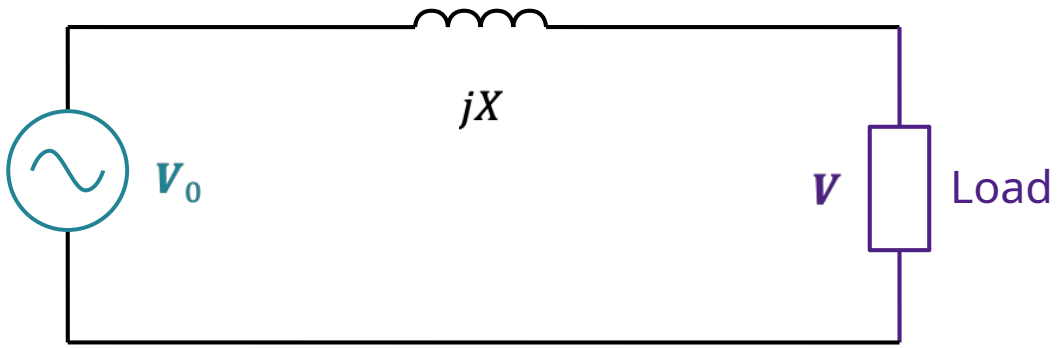


https://www.ge.com/news/reports/its-only-natural-new-ohio-power-plant-to-use-ges-record-setting-natural-gas-turbines



AC circuit maximum power transfer

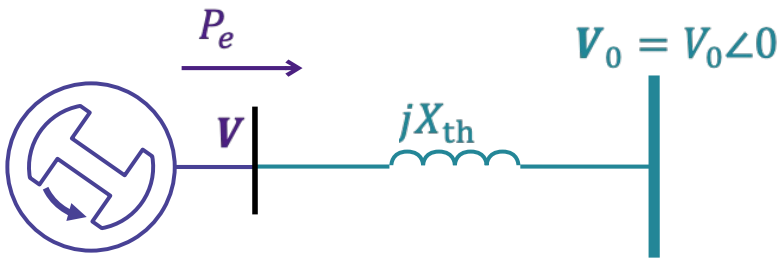
Recall the power transfer equations for the two-bus, voltage source network



From $\boldsymbol{S = V\bar{I}} = \frac{-j}{X}[V_0\, V(cos\ \theta\ -\ j\ sin\ \theta)\ -\ V^2]$,

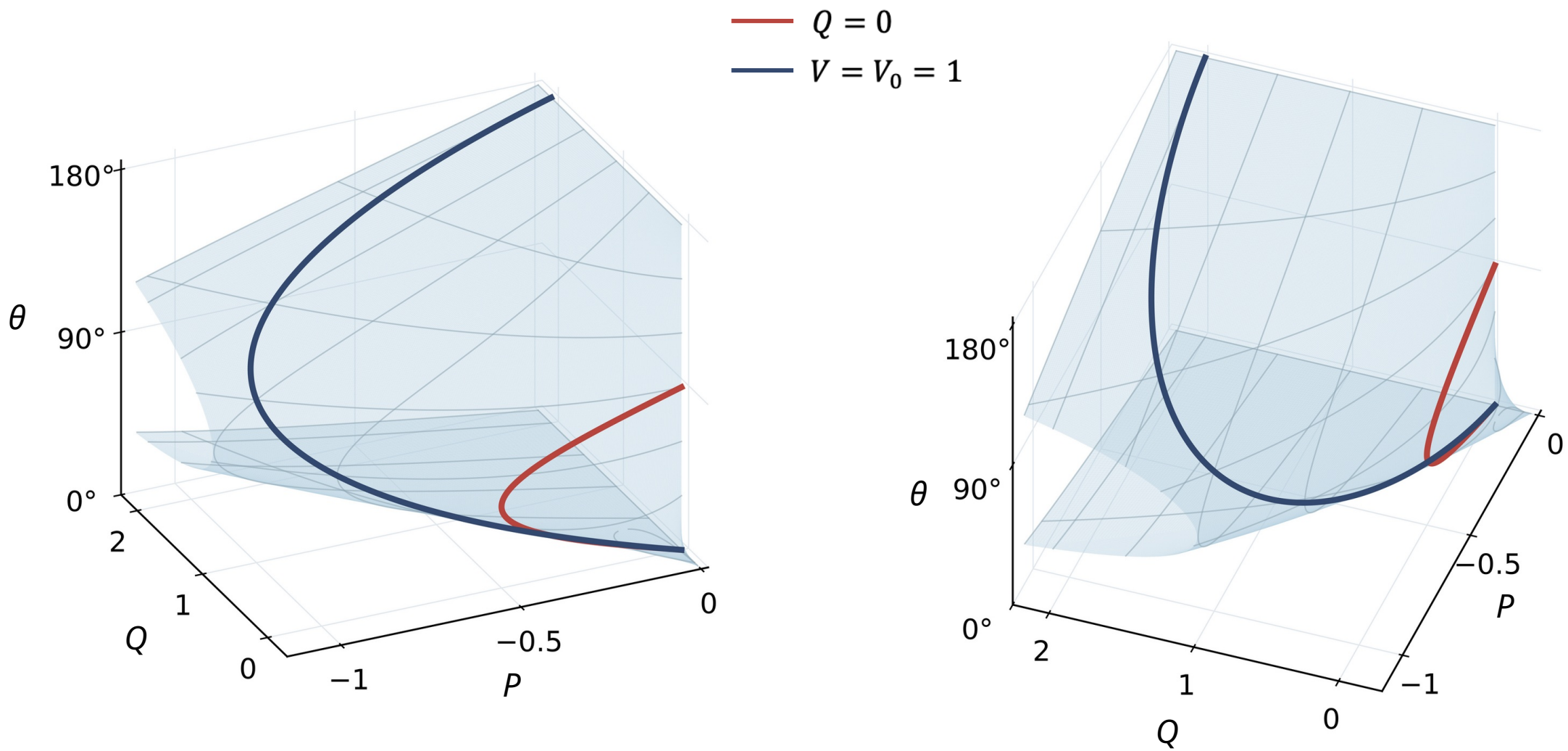
$$P\ =\ -\frac{V_0\, V}{X}\ sin\ \theta, \qquad Q\ =\ -\frac{V_0\, V}{X}\ cos\ \theta\ +\ \frac{V^2}{X}$$

The same equation applies to the electric power being sent from the generator to the infinite bus



$$P_e\ =\ -\frac{V_0\, V}{X}\ sin\ \theta$$

The Power Flow manifold

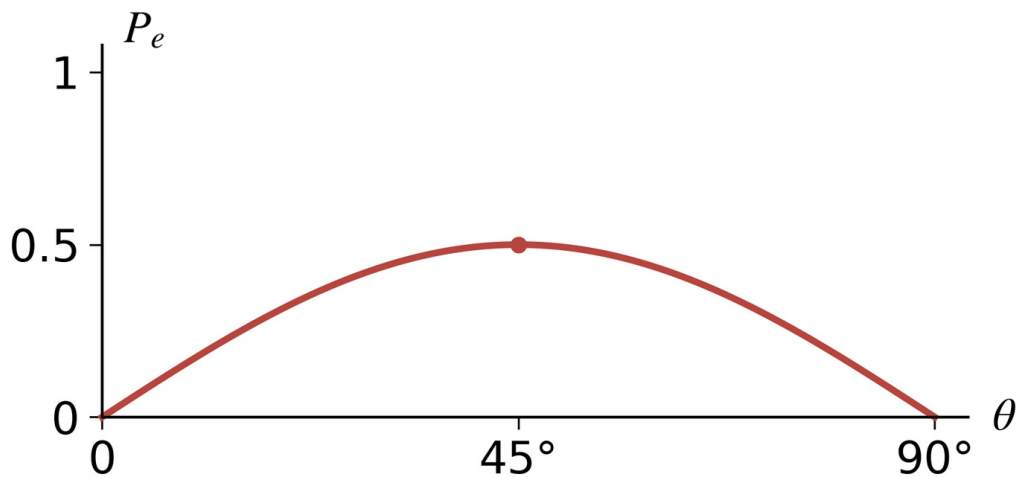


For a lossless line, the power–angle curve depends on what is held fixed.

These equations are always true: $P_e = \frac{V_0 V}{X}\sin(\theta),\ Q_e = \frac{V^2 - V_0 V\cos\theta}{X}$

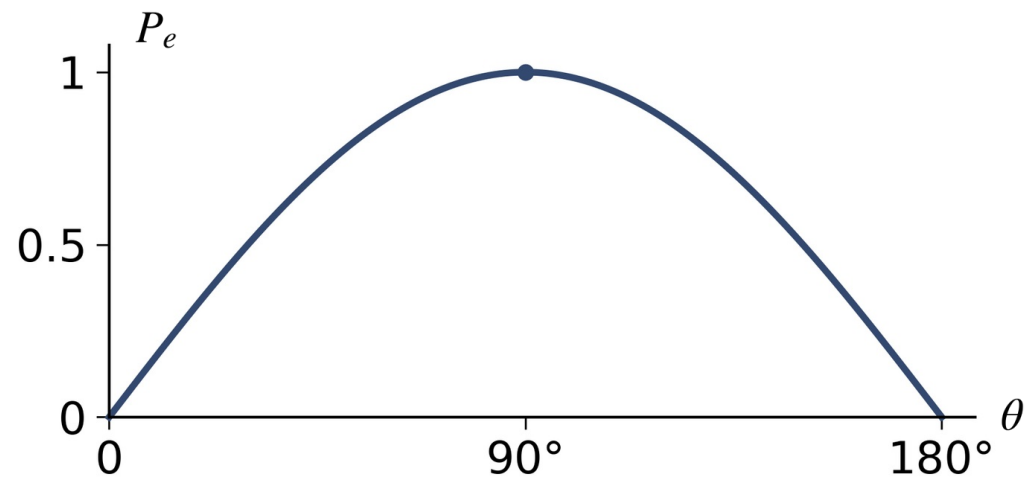
Fixed reactive power: $Q = 0,\ V = V_0\cos(\theta)$

$$P_e = \frac{V_0^2}{2X}\sin(2\theta)$$



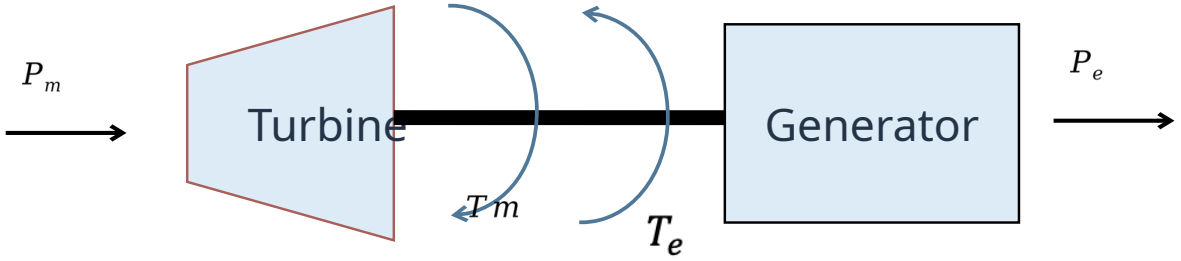
Fixed voltage magnitude: $V = V_0$

$$P_e = \frac{V_0 V}{X}\sin(\theta)$$



$V_0 = X = 1$. θ is the voltage-angle separation.

For angle dynamics/stability, we are interested in the turbine rotor angle, which varies with time.



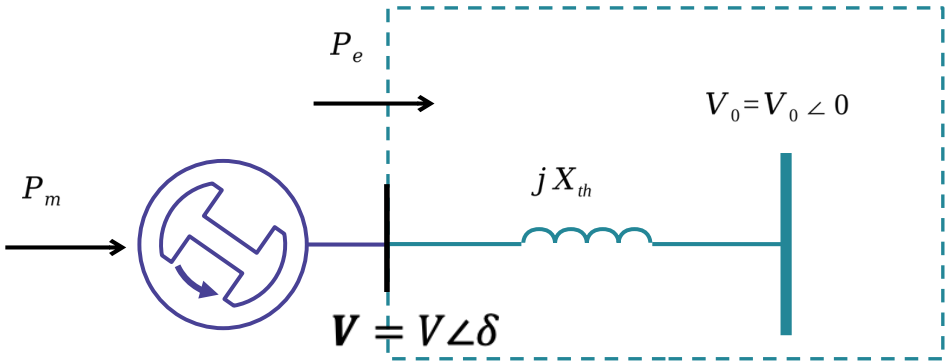
$$M\frac{d^2\delta}{dt^2} \approx P_m - P_e$$

ω: rotor electrical speed

ω_s: grid electrical speed

$$\dot{\delta} = \omega - \omega_s$$

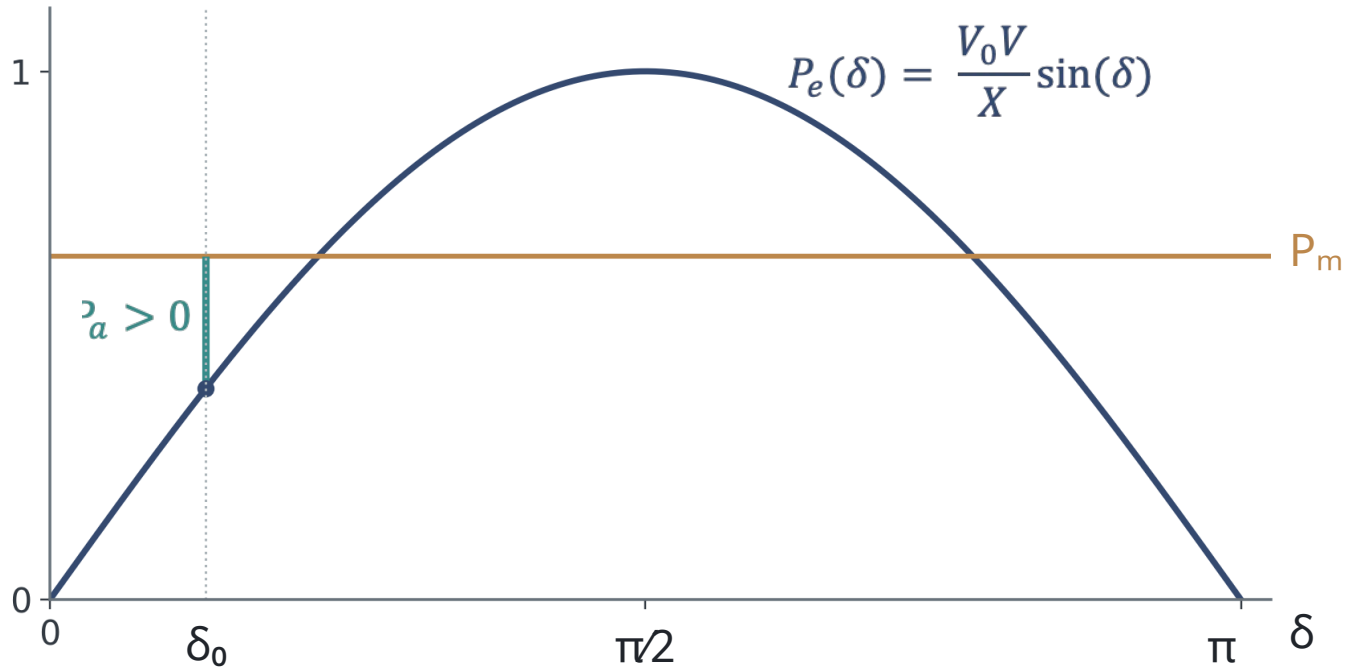
Generator System equivalent



Here, $\delta = \theta$. We use δ for rotor-angle dynamics.

(We replaces θ with the rotor position δ as it will give us more flexibility in generator modeling)

Power balance accelerates the rotor



$$M\frac{d^2\delta}{dt^2} \approx P_m - P_e = P_a$$

$$P_a = P_m - P_e$$

P_m: mechanical input from the prime mover.

P_e: electrical output

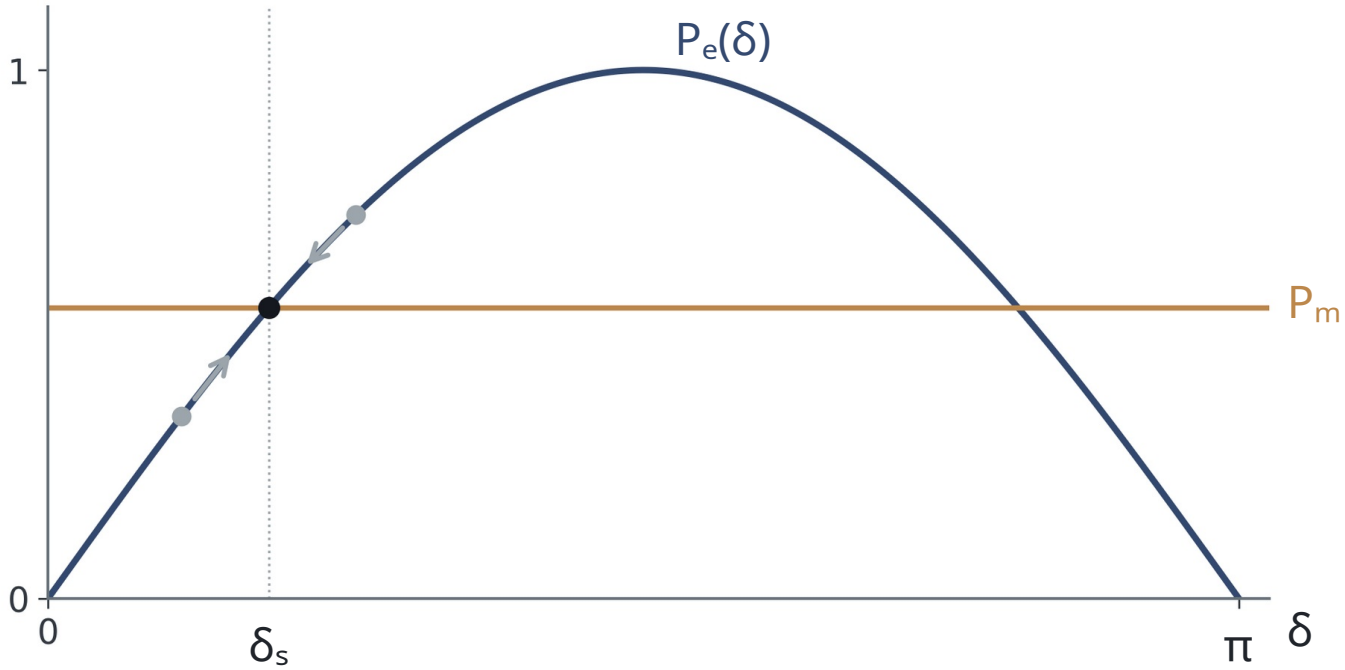
$P_a > 0$: the rotor accelerates relative to the grid.

→ Moves to the right

$P_a < 0$: the rotor decelerates relative to the grid.

→ Moves to the left

Stable equilibrium



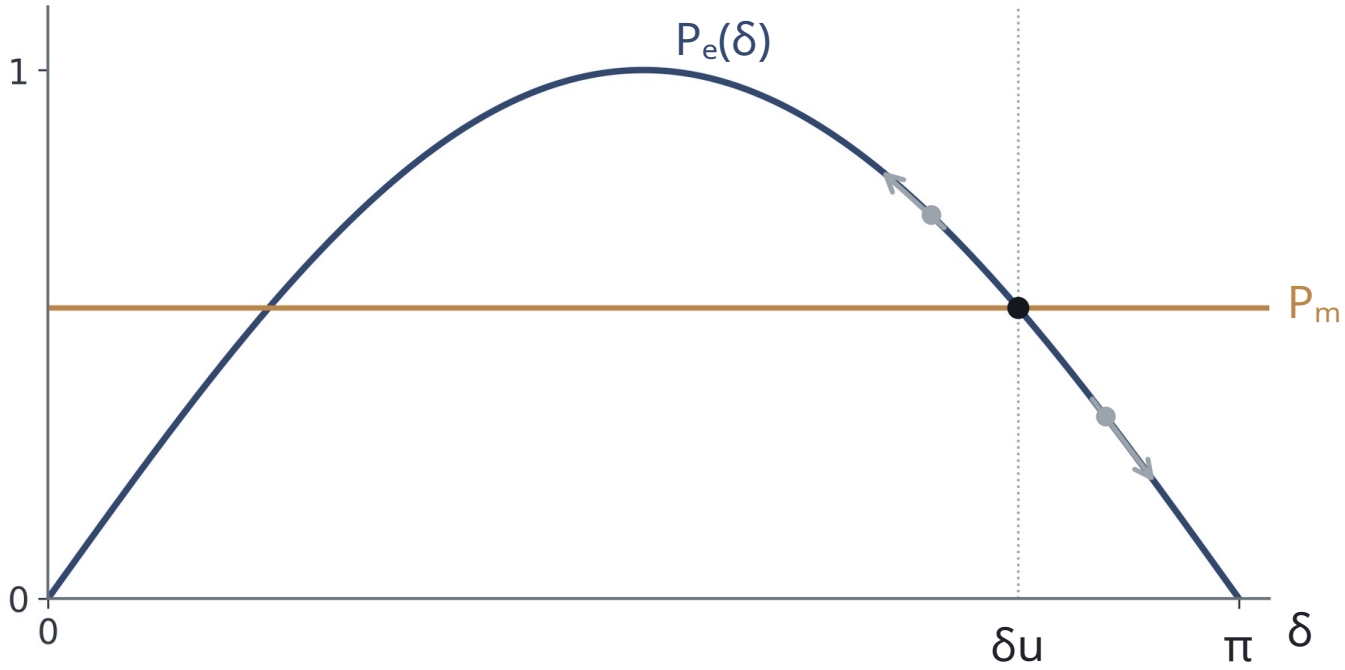


At the left intersection, $P_e(\delta_s) = P_m$

If the angle moves slightly, the power imbalance pushes it back toward δ_s.

→ Stable equilibrium

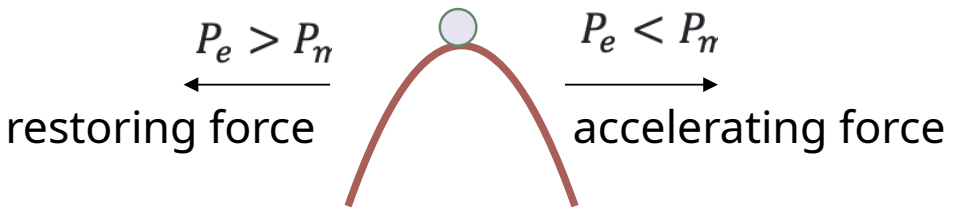
Unstable operating point



At the right intersection, $P_e(\delta_s) = P_m$

If the angle moves slightly, the power imbalance pushes away from δ_s.

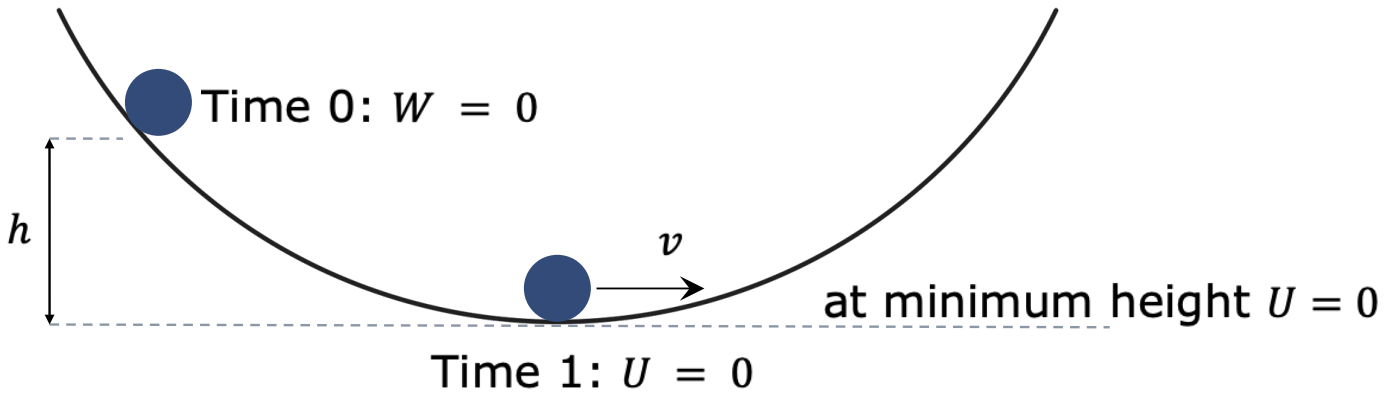
→ Unstable equilibrium



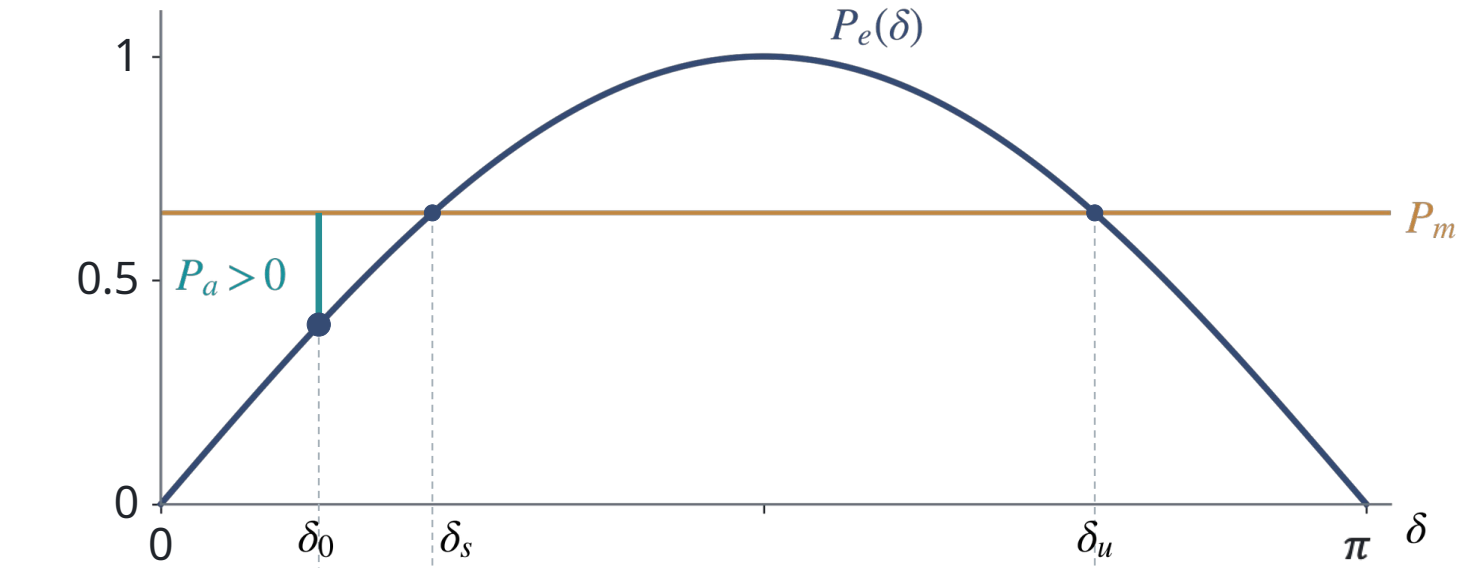
Frictionless marble analogy for the exchange potential and kinetic energy

Potential energy: $U(\theta) = mgh$

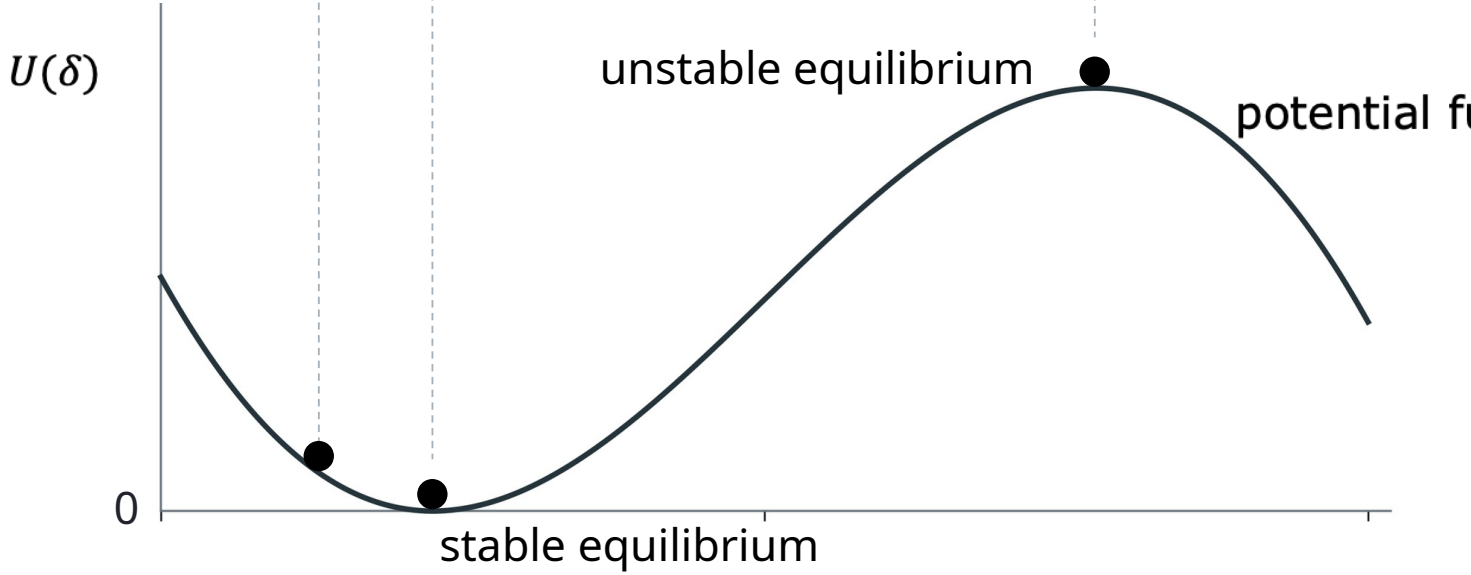
Kinetic energy: $W = \frac{1}{2}mv^2$



What is the potential-energy curve for the single machine system?



$$M\frac{d^2\delta}{dt^2} \approx P_m - P_e$$

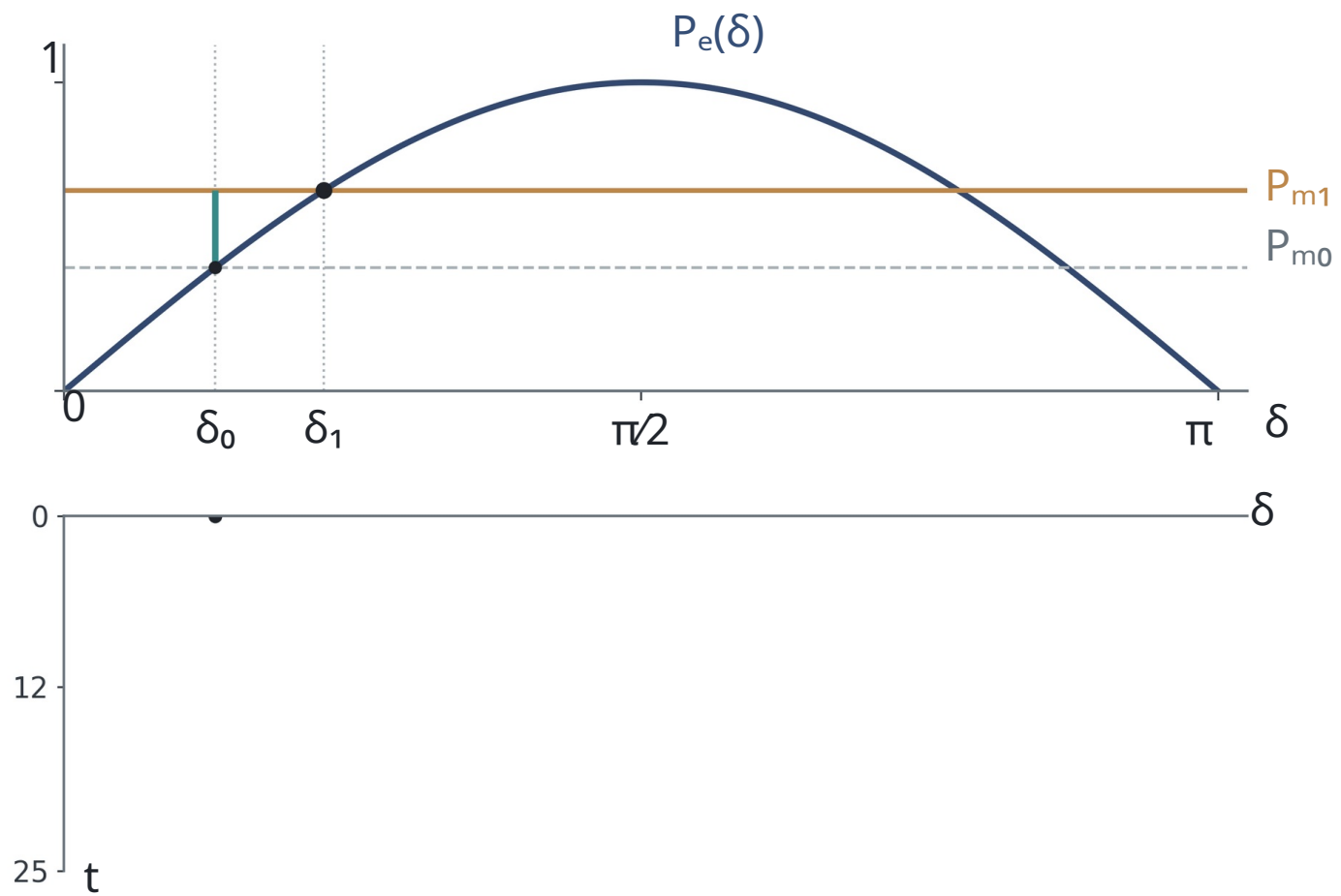


potential function: $U(\delta) = -P_{\max}\cos\delta - P_m\delta + C$

The rotor accelerates down the potential according to the "force"

$$\frac{\mathrm{d}U}{\mathrm{d}\delta} = P_e(\delta) - P_m$$

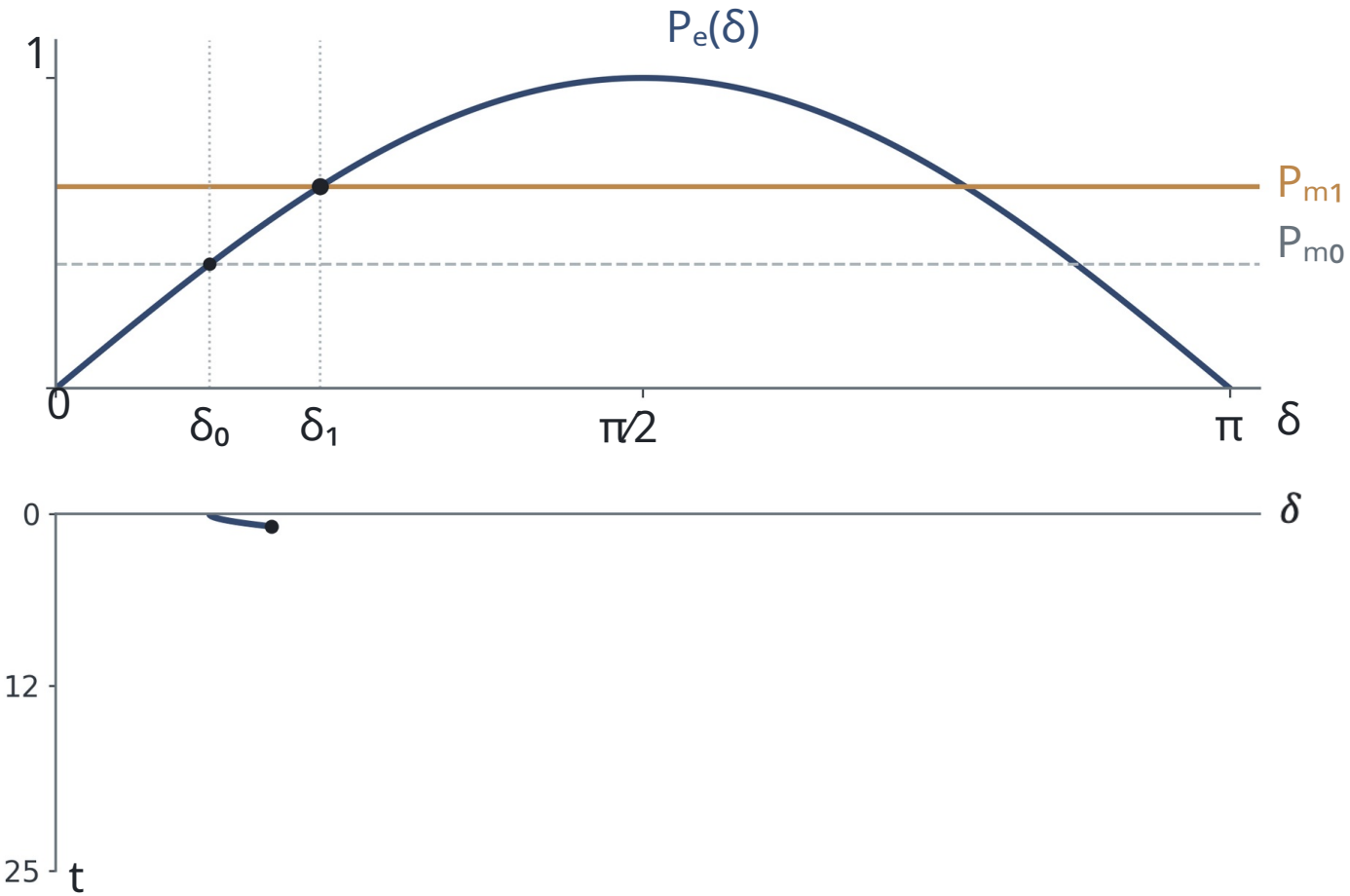
Exercise: a step in mechanical power



The generator initially satisfies $P_{m0} = P_e(\delta_0)$. Mechanical input suddenly rises.

1. Does δ change instantaneously? Where is it relative to the new δ_1?
2. What is the sign of $P_{m1} - P_e(\delta_0)$?
3. Which way does δ initially move on the power–angle plot?

Why does a step in mechanical power cause a swing?

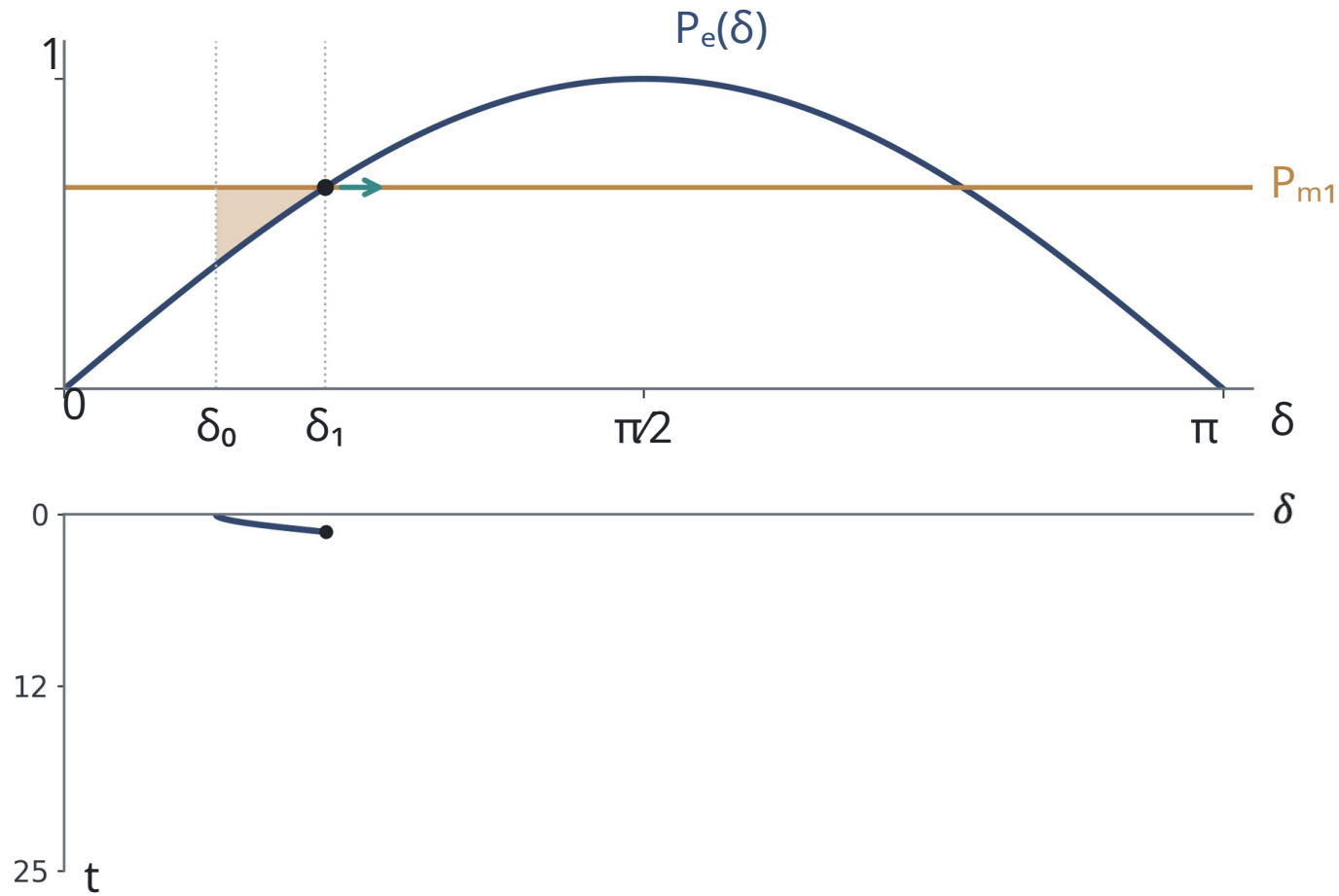


Before: $P_{m0} = P_e(\delta_0)$

After the step: $P_{m1} > P_e(\delta_0)$

The angle cannot jump---it starts at δ_0 and accelerates toward δ_1.

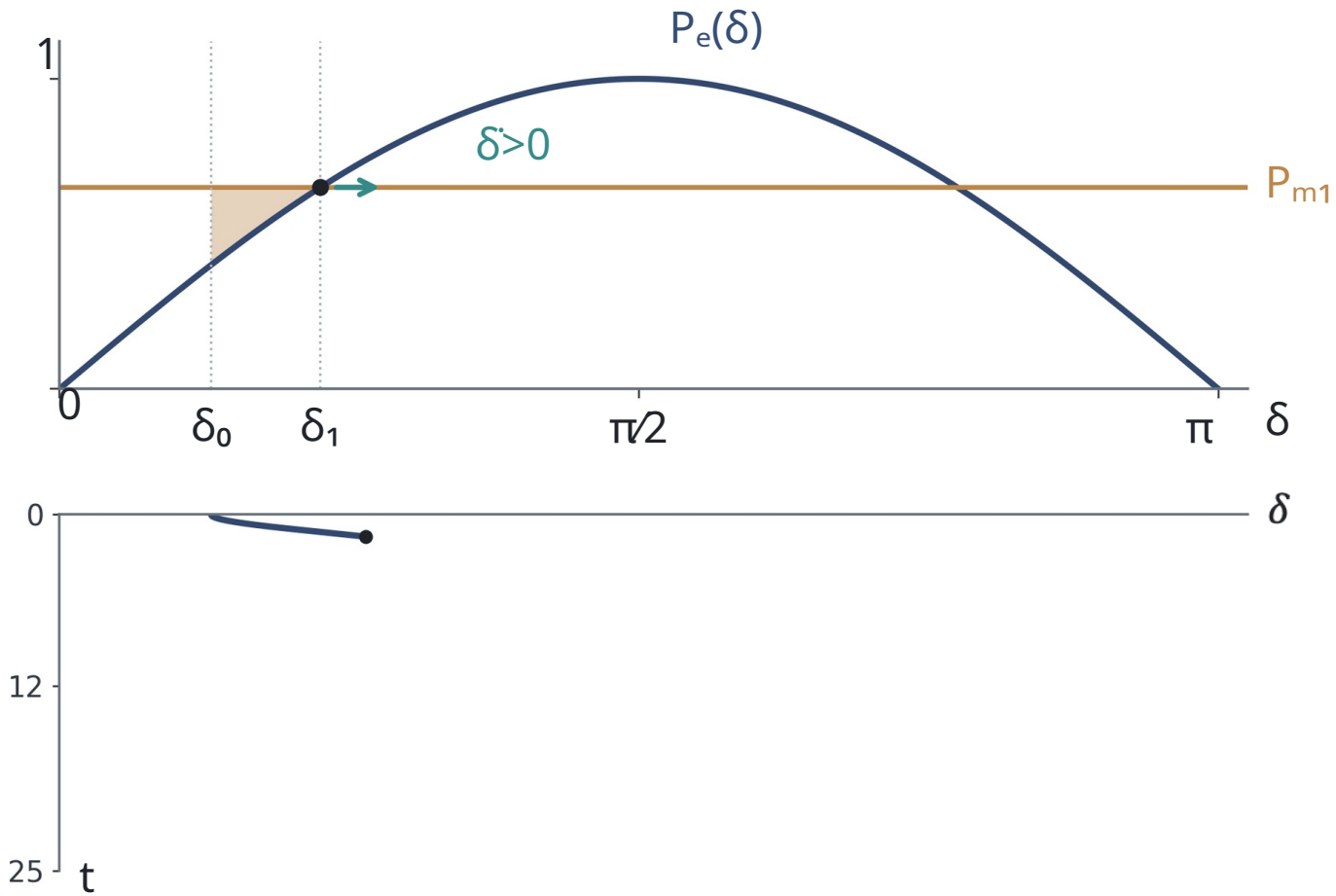
Exercise: crossing the new equilibrium



At δ_1, electrical output equals the new mechanical input.

1. Is the rotor stationary (relative to the grid) at that instant?
2. Why does δ continue to increase beyond δ_1?
3. What is the sign of $P_{m1} - P_e$ once $\delta > \delta_1$?

Why does the rotor pass the new equilibrium?

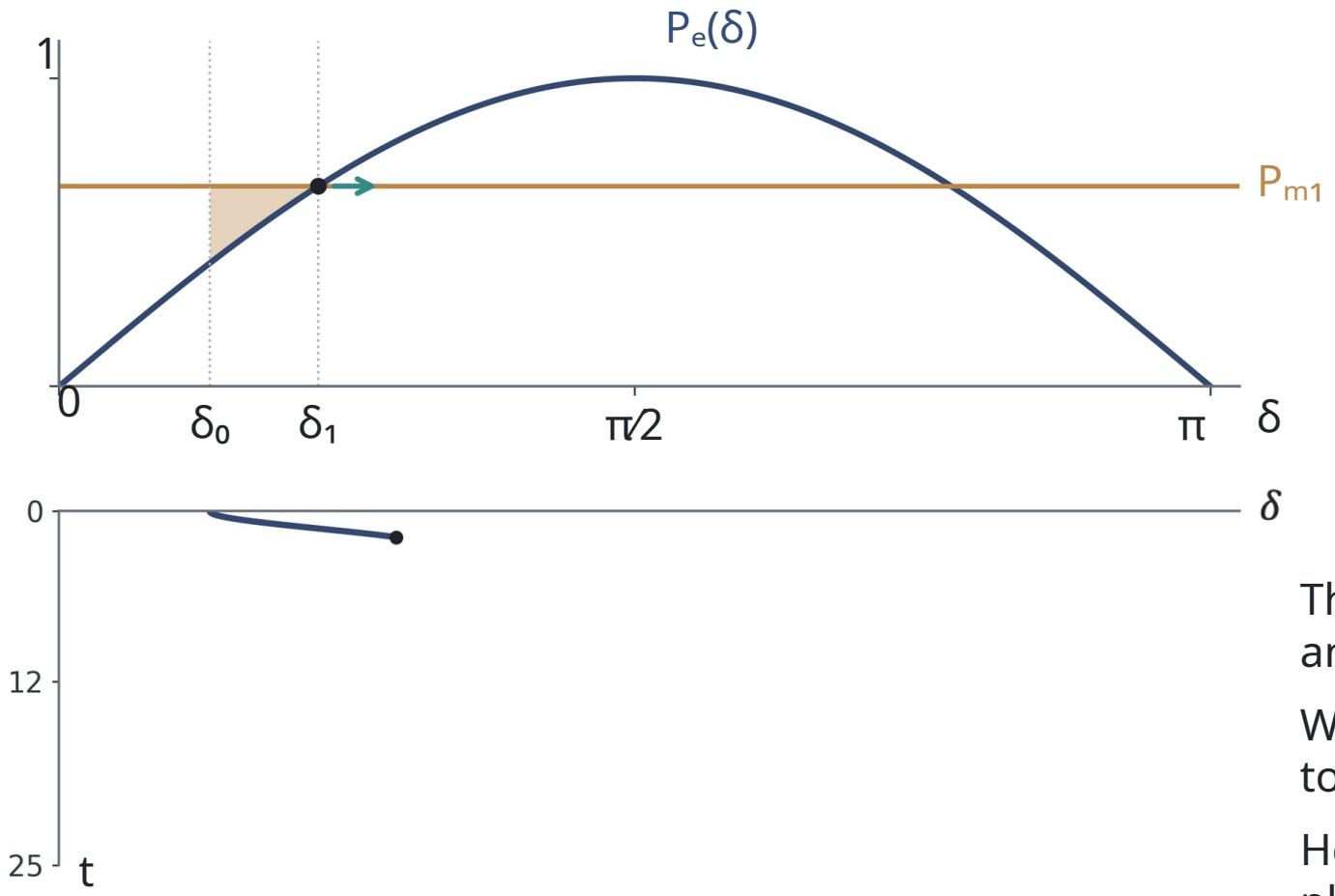


$$P_e(\delta_1) = P_{m1} \quad \rightarrow \quad \ddot{\delta} = 0 \; at \; \delta_1$$

But $\dot{\delta} > 0$. The rotor has gained kinetic energy/momentum and continues past δ_1.

Then $P_e > P_{m1}$, so it decelerates.

Exercise: where does the rotor stop?

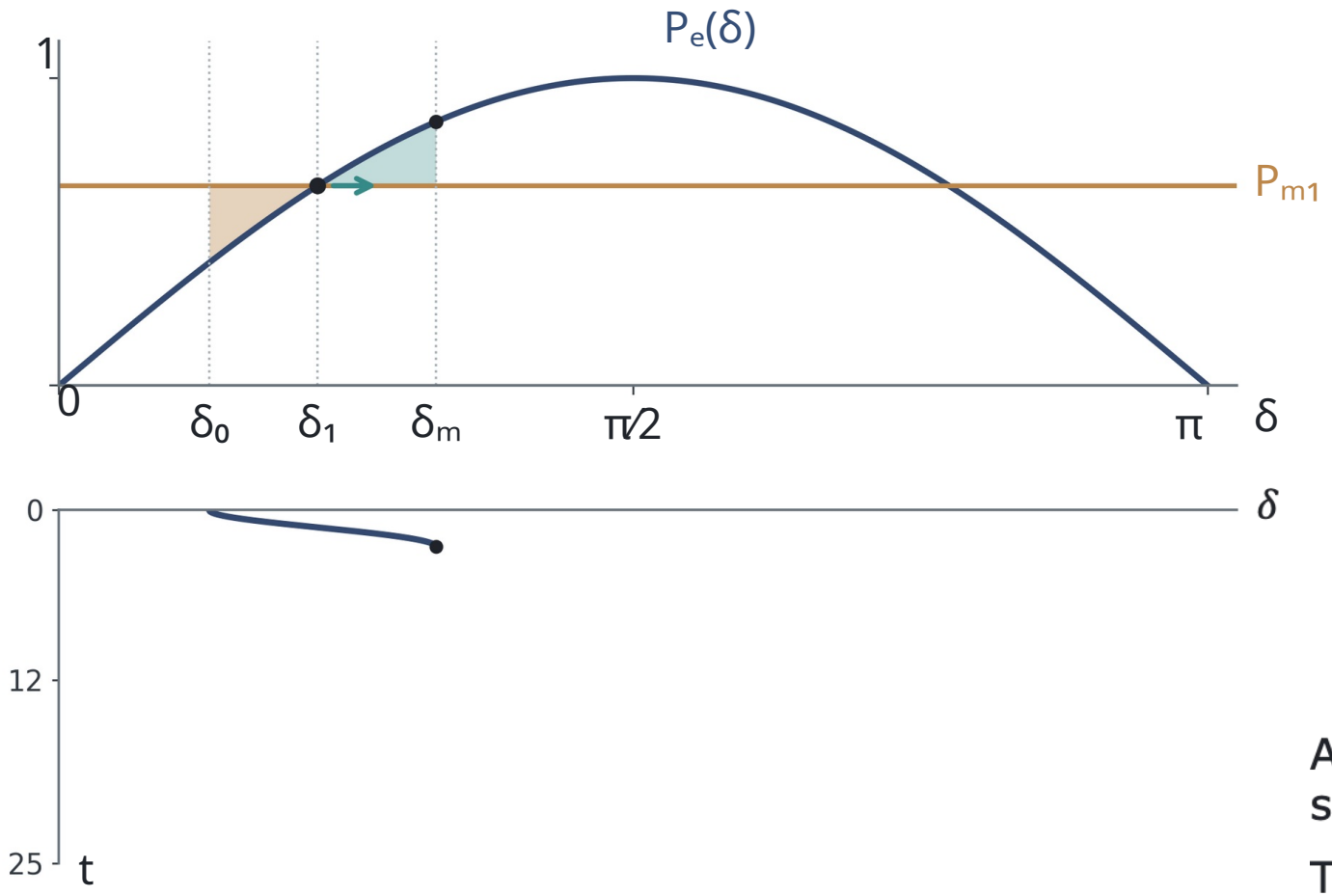


The rotor crosses the new equilibrium and begins to decelerate.

Where does it first stop moving relative to the grid?

How could you find that angle from this plot?

Where does the rotor stop?

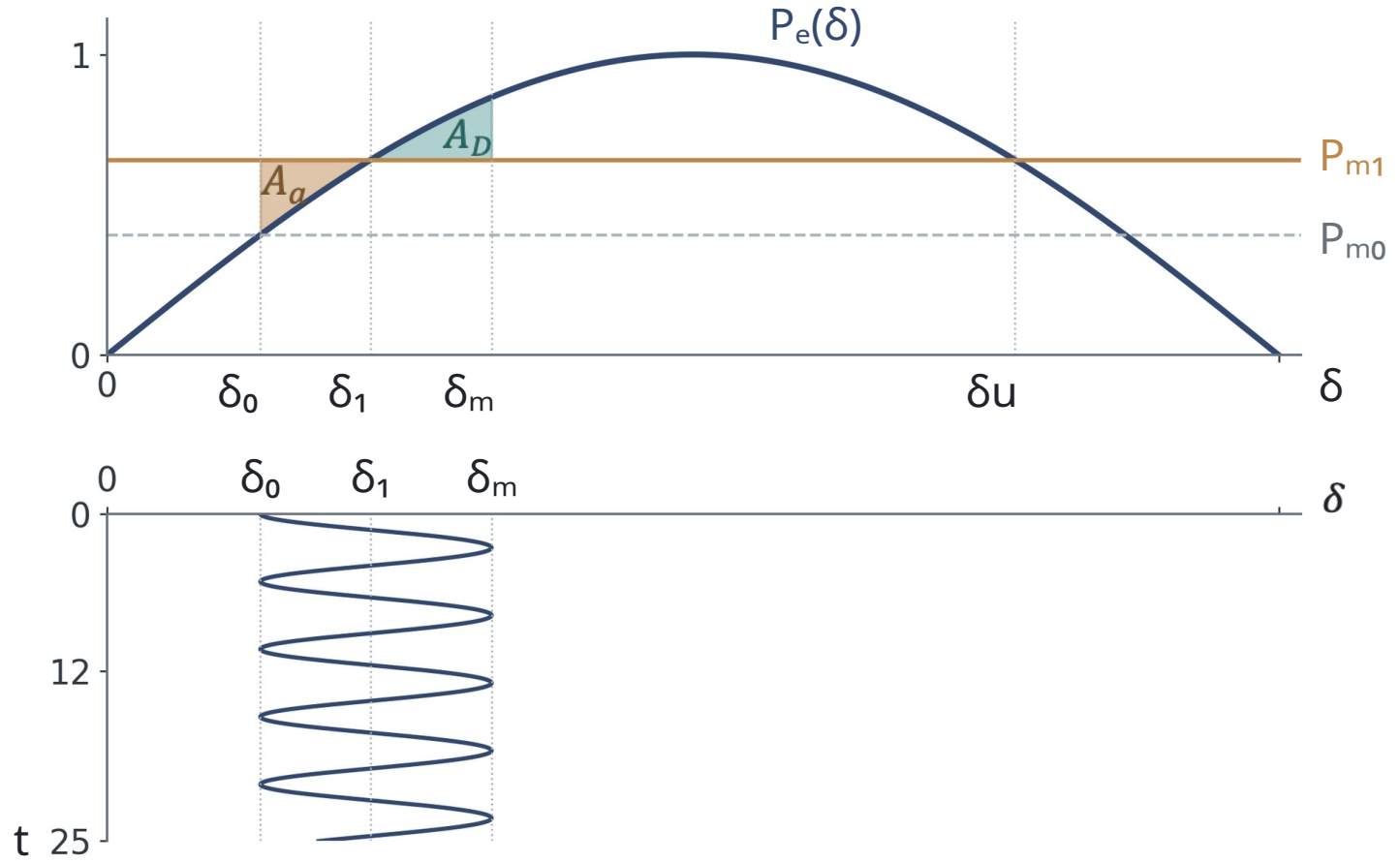


At its first turning angle δ_m, relative speed reaches zero.

The green area removes the kinetic energy gained in the orange area.

The Equal Area Criterion

For simple swing rotor dynamics $M\frac{d^2\delta}{dt^2} \approx P_m - P_e$, the acceleration area equals the deceleration area.



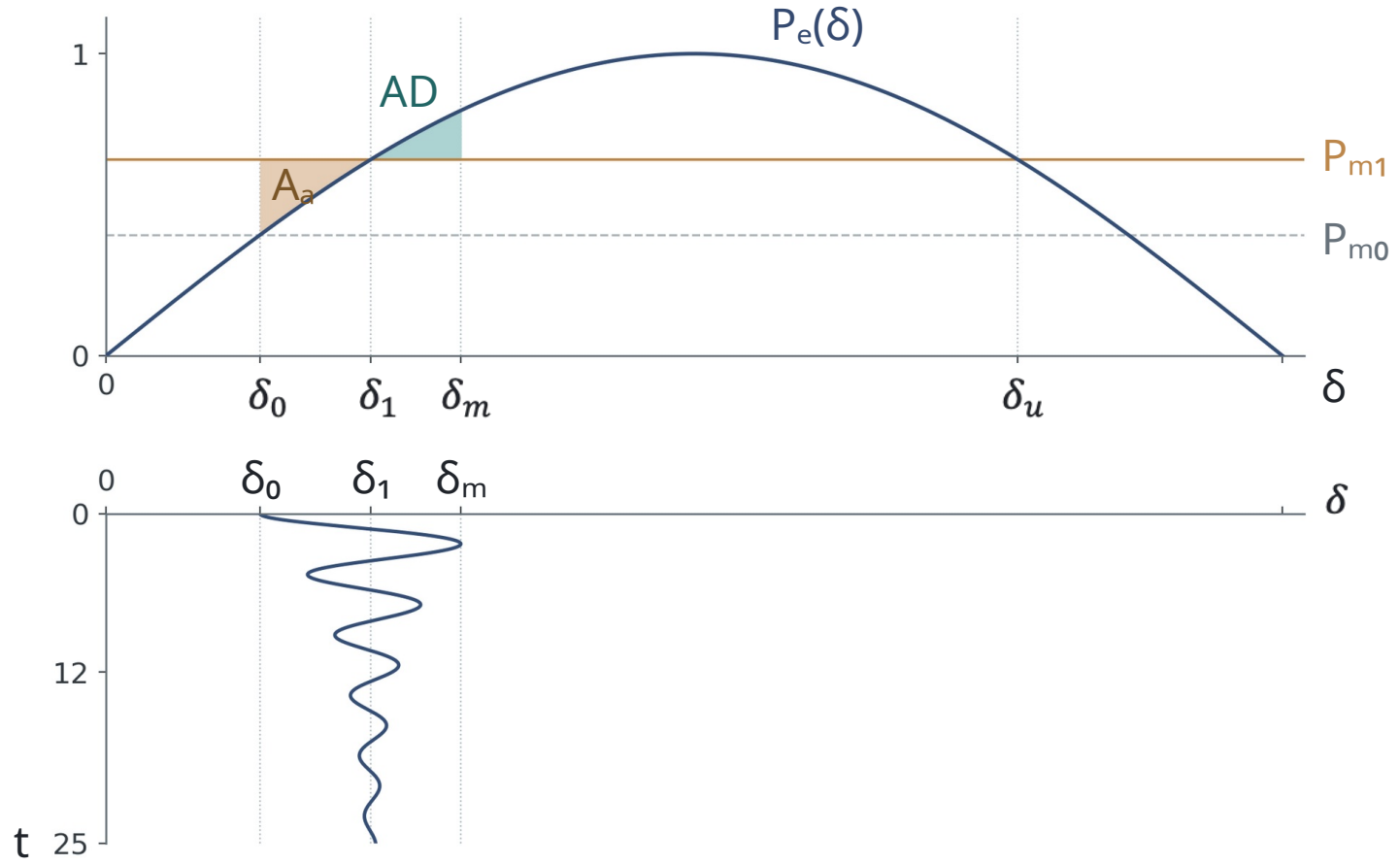
A_a: $P_{m1} > P_e$ (accelerating)

A_D: $P_e > P_{m1}$ (decelerating)

Proof in the appendix

Kundur, Power System Stability and Control, Fig. 13.4 (redrawn).

With damping, the oscillations will decay back to the stable operating point.

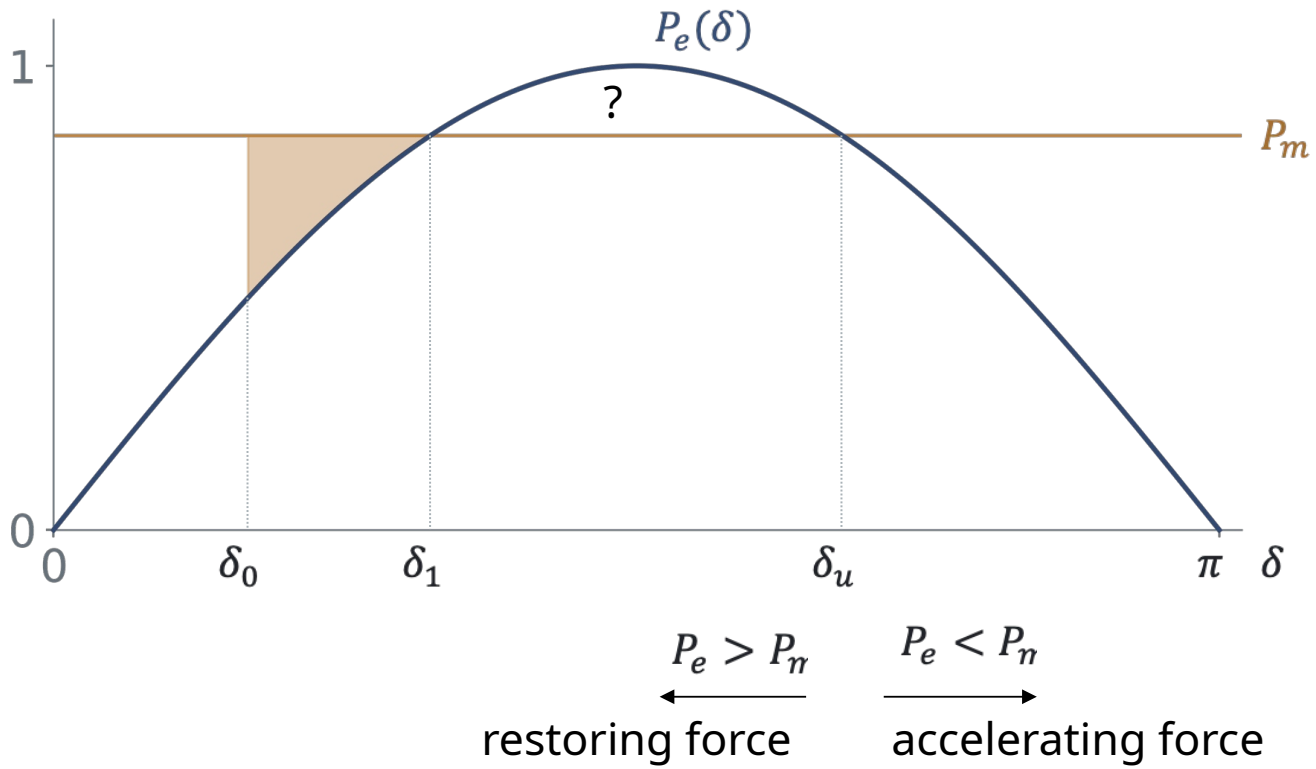


A_a: $P_{m1} > P_e$ (accelerating)

A_D: $P_e > P_{m1}$ (decelerating)

Kundur, Power System Stability and Control, Fig. 13.4 (redrawn).

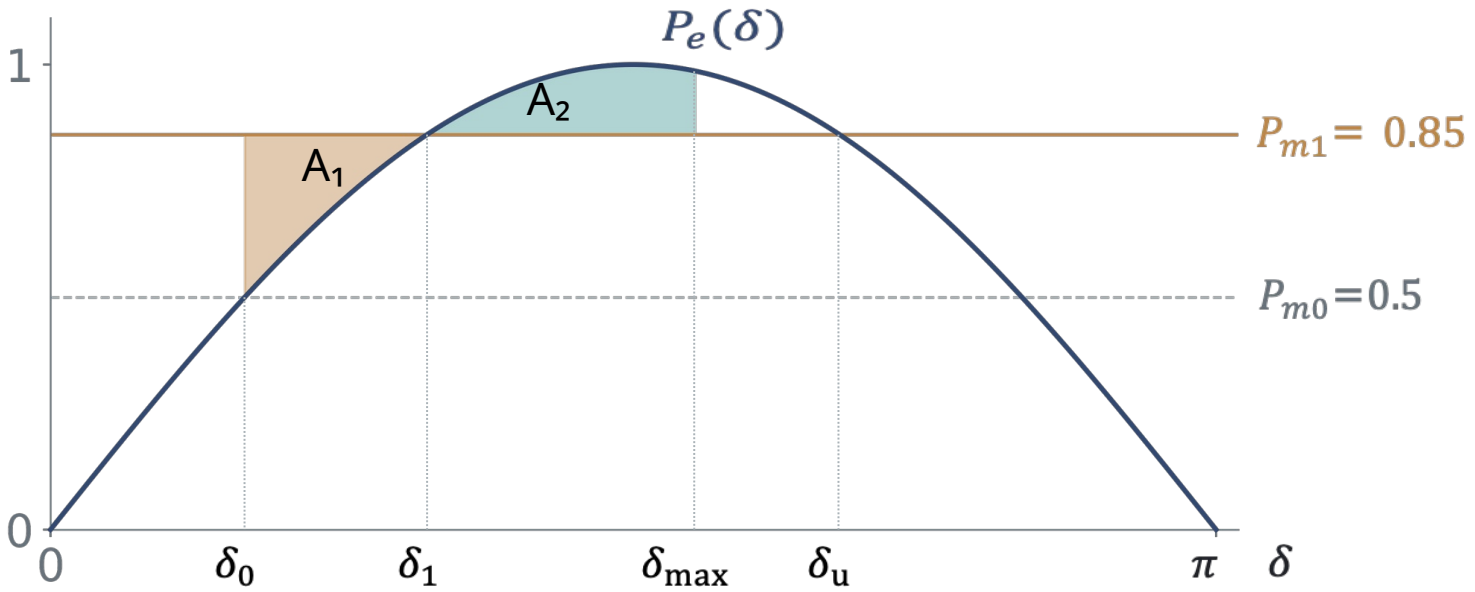
When is there enough deceleration area?



The first turning angle must precede the unstable equilibrium.

Compare the gained energy with the available deceleration area.

Is the mechanical-power step stable?



$P_{max} = 1$

$\delta_0 = \sin^{-1}(0.5) = 0.5$

$\delta_1 = \sin^{-1}(0.85) = 1.0$

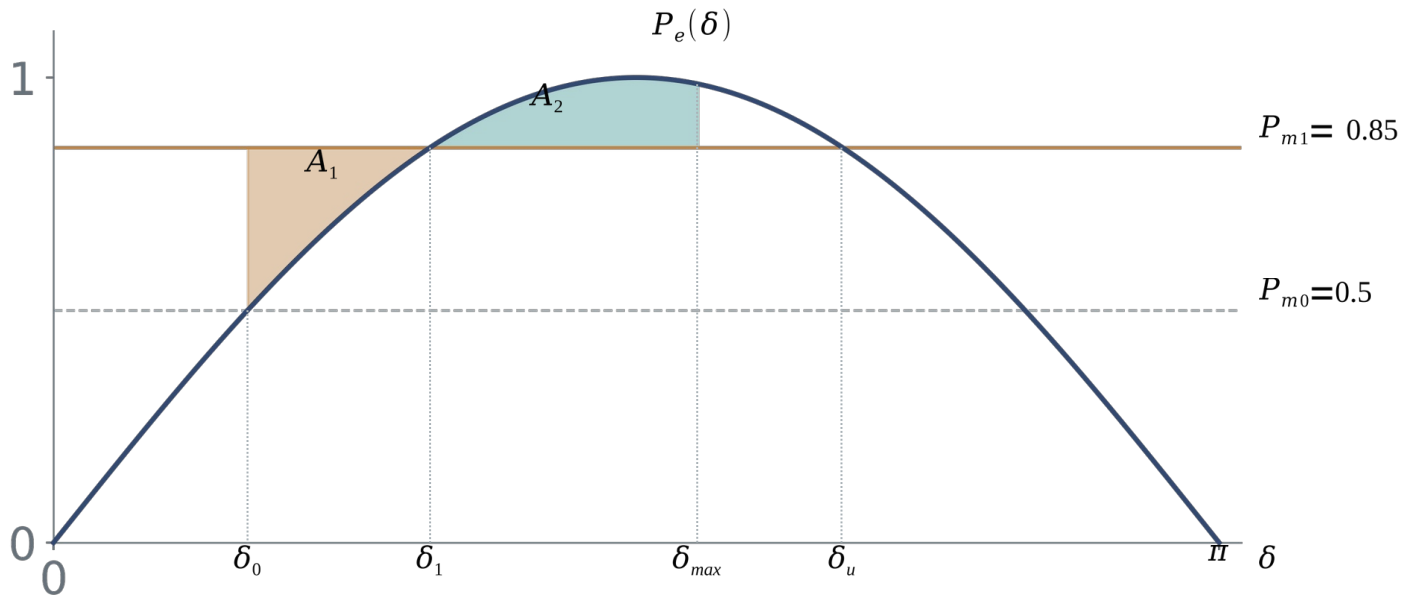
$\delta_{max} = 1.7$

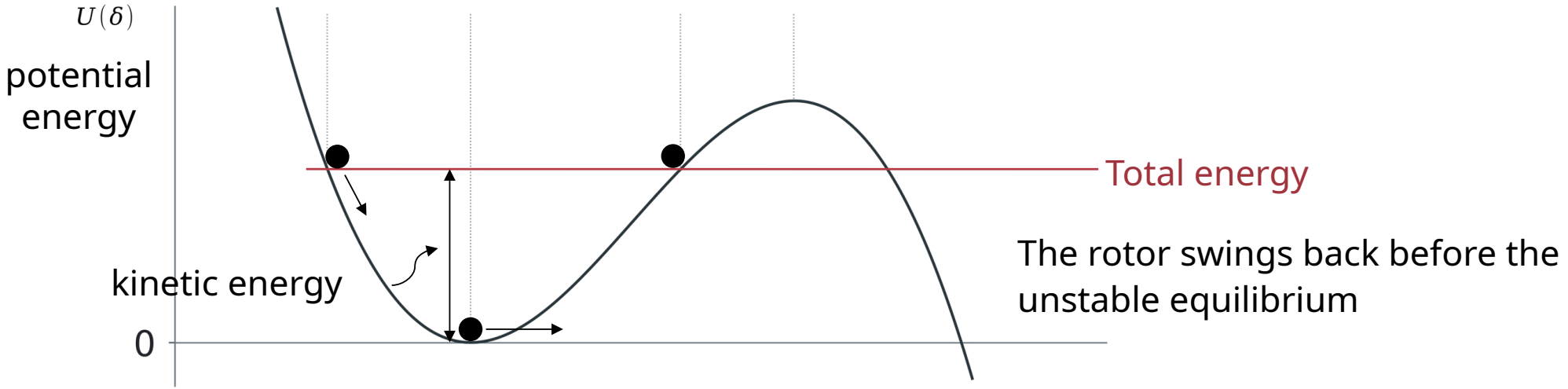
$\delta_u = \pi - \delta_1 = 2.1$

$\delta_{max} < \delta_u$

→ The turn at δ_{max} occurs before the right intersection.

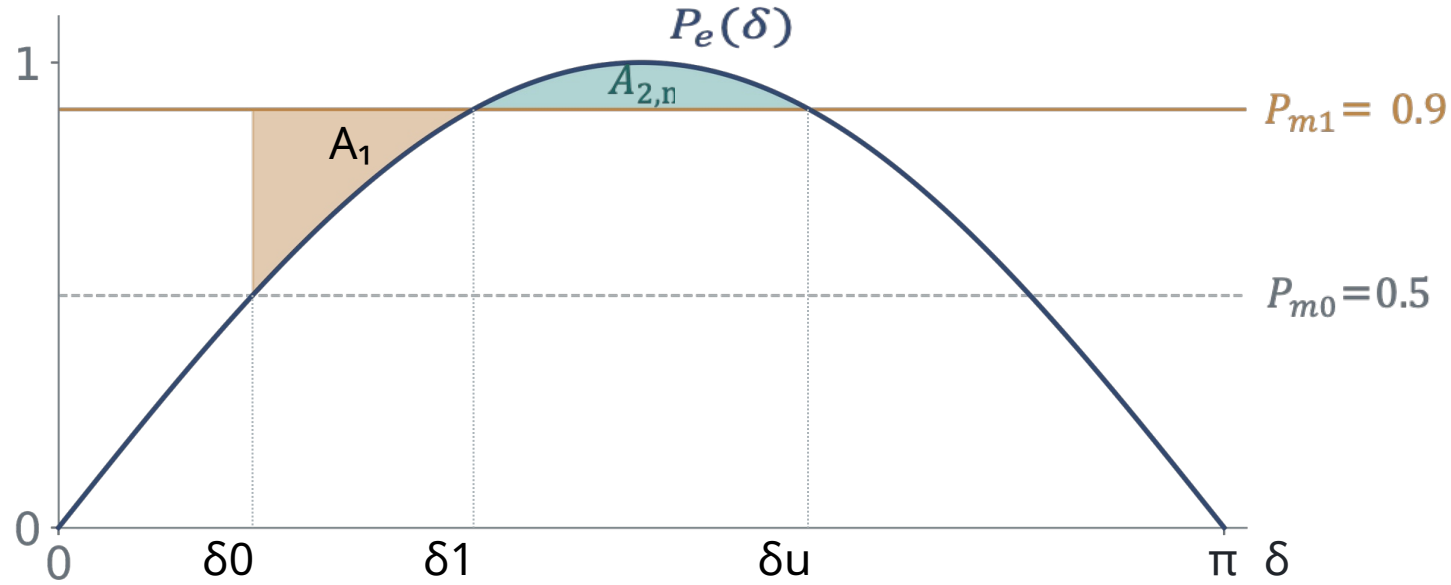
Is the mechanical-power step stable?



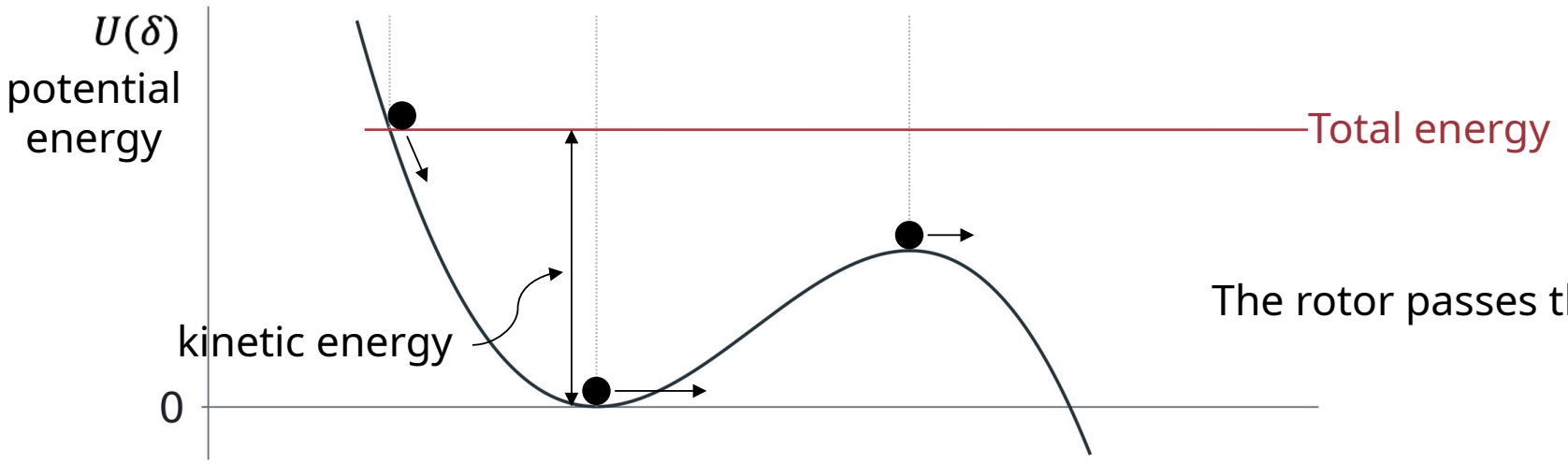


The rotor swings back before the unstable equilibrium

Is the larger mechanical-power step stable?

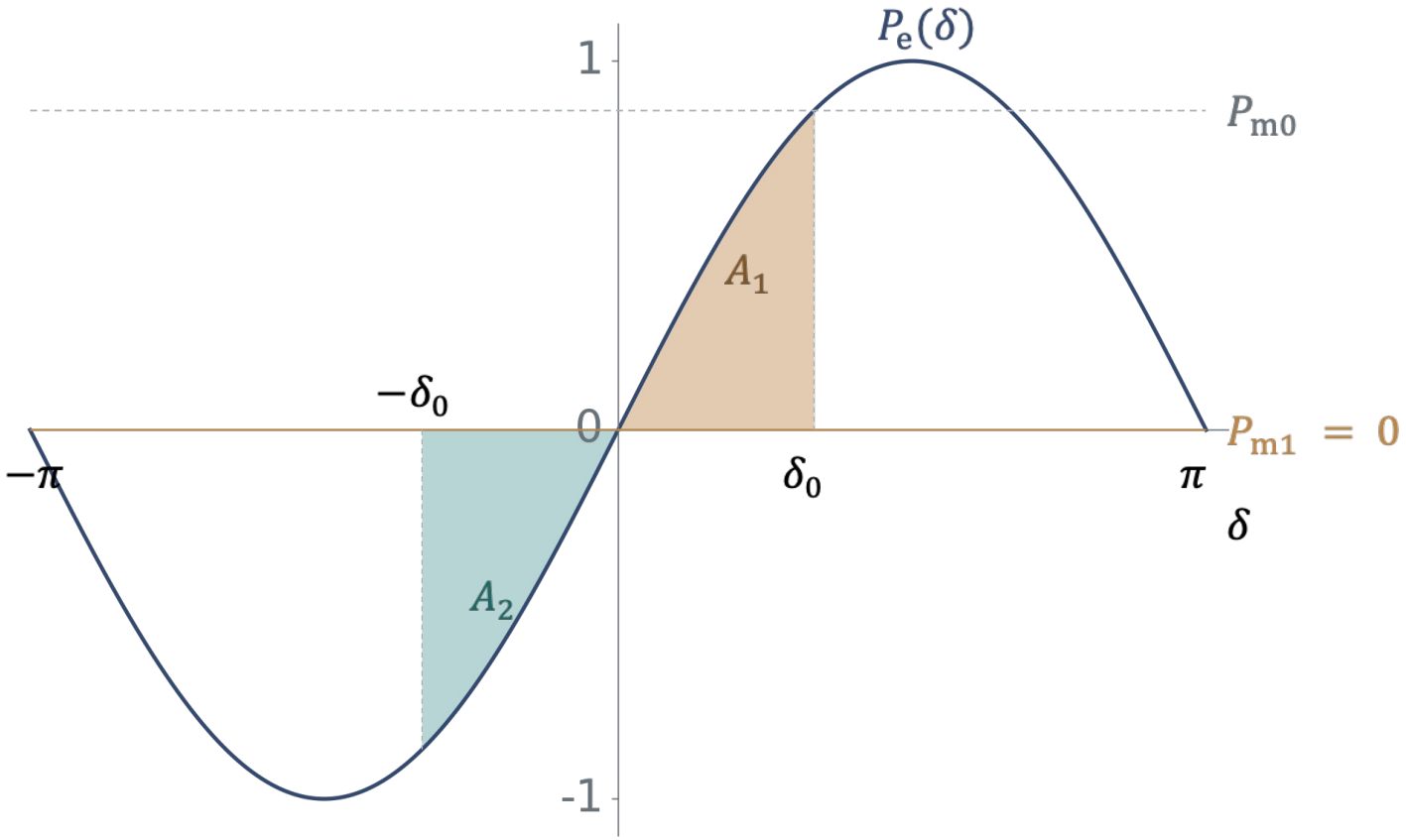


$A_1 > A_{2,\max}$



The rotor passes the unstable equilibrium

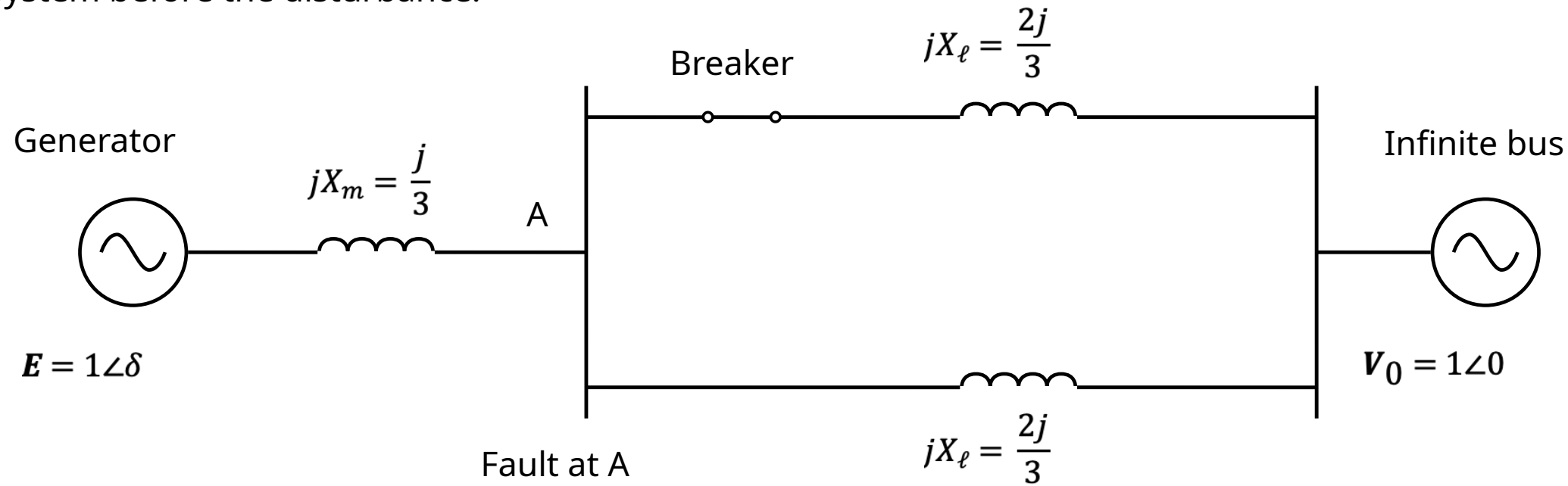
What happens if mechanical power suddenly drops to zero?



The Equal Area Criterion shows that losing mechanical power does not result in a pole slip

Exercise: Analyzing a fault and a circuit trip using the Equal Area Criterion

System before the disturbance:



Two cases:

1. A bolted three-phase fault occurs at A, persists and then is cleared.
2. The upper-line breaker opens.

In both cases, the mechanical input is constant.

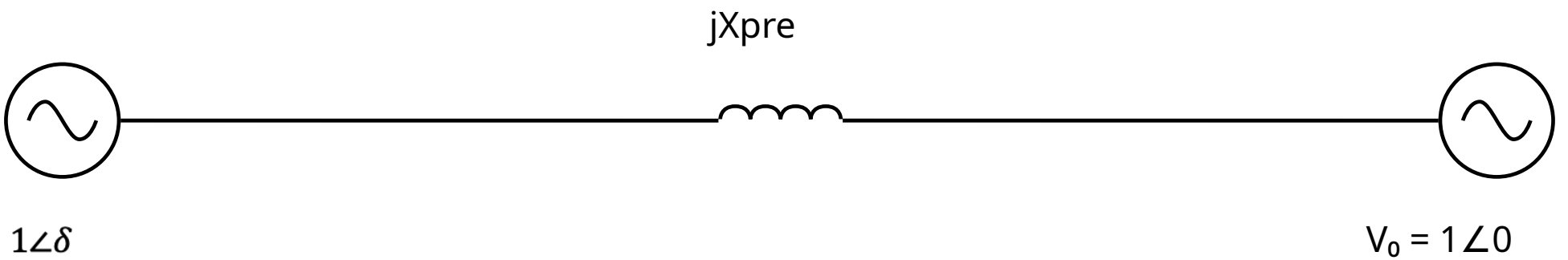
Both a fault or a trip will change the electric power transfer equation

$$P_e(\delta) = \frac{V_0 V}{X}\sin(\delta)$$

A **fault** reduces electrical transfer to $P_e(\delta) = 0$. Clearing the fault restores the original $P_e(\delta)$.

A **line trip** increases the equivalent reactance and lowers the $P_e(\delta)$power curve.

How much power is transferred prior to the disturbance?

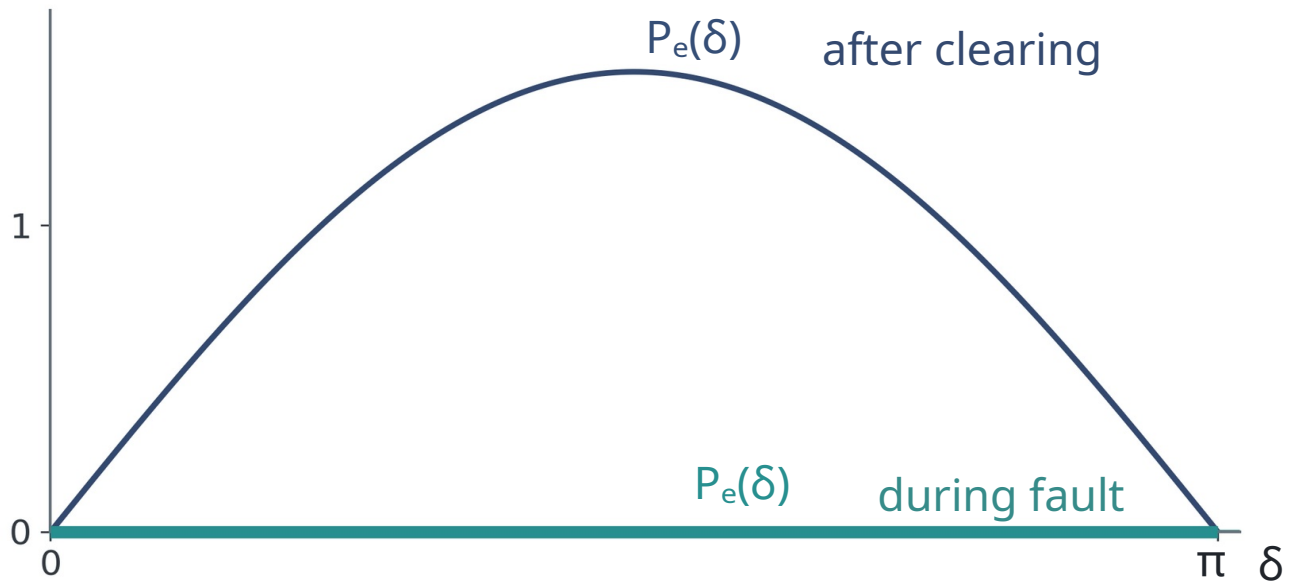


$$X_{\text{pre}} = X_m + \frac{X_\ell}{2} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$P_{\text{max,pre}} = \frac{EV_0}{X_{\text{pre}}} = 1.5$$

$$P_{\text{e,pre}}(\delta) = 1.5 \sin\delta$$

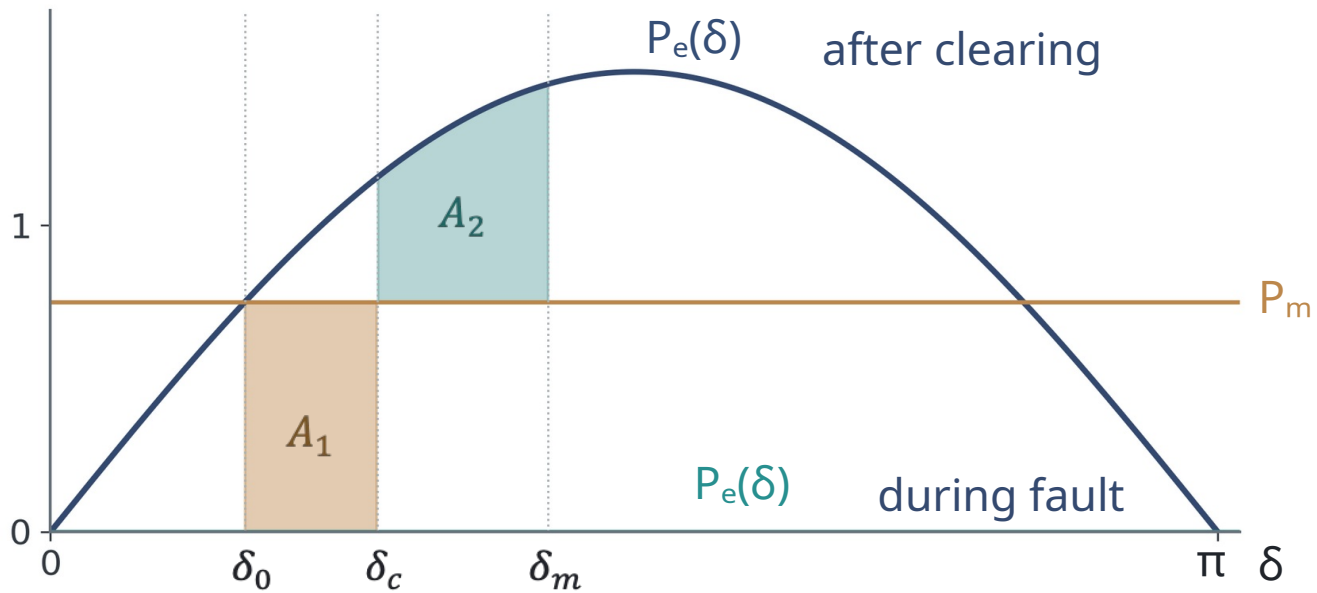
How does a fault at A change electrical power?



$$P_e^{\text{fault}} = 0$$

$$P_e^{\text{cleared}} = 1.5 \sin \delta$$

What happens when the fault is cleared?

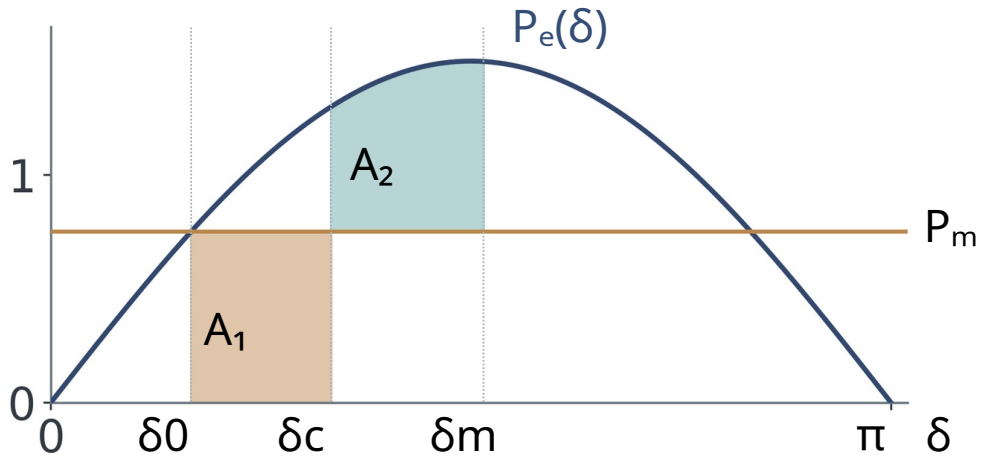


Clearing restores the original curve, but the rotor continues forward.

In this stable case, it turns when the returning area equals the gained area.

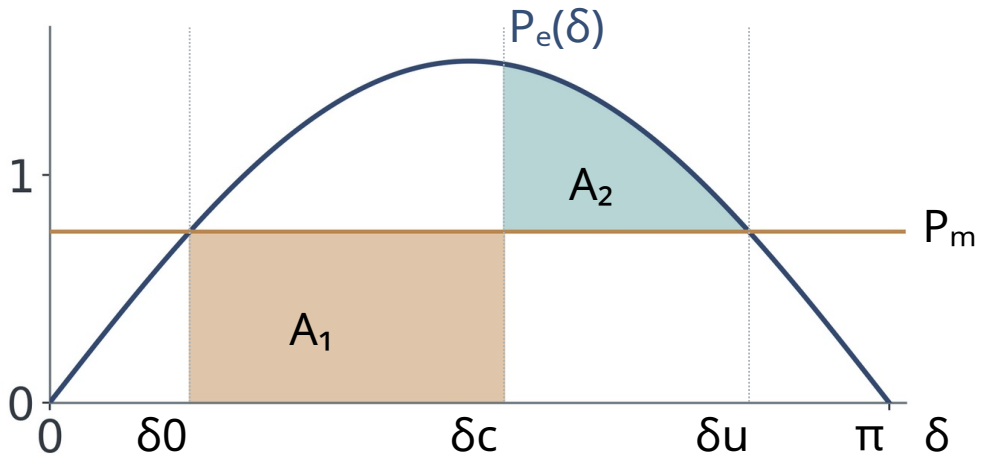
What happens if the fault persists for a bit longer?

Stable



$A_1 = A_2$, $\delta_m < \delta_u$

Unstable



$A_1 > A_2$,available

The rotor passes the unstable equilibrium angle with positive relative speed.

Mundra, India, 13 July 2016

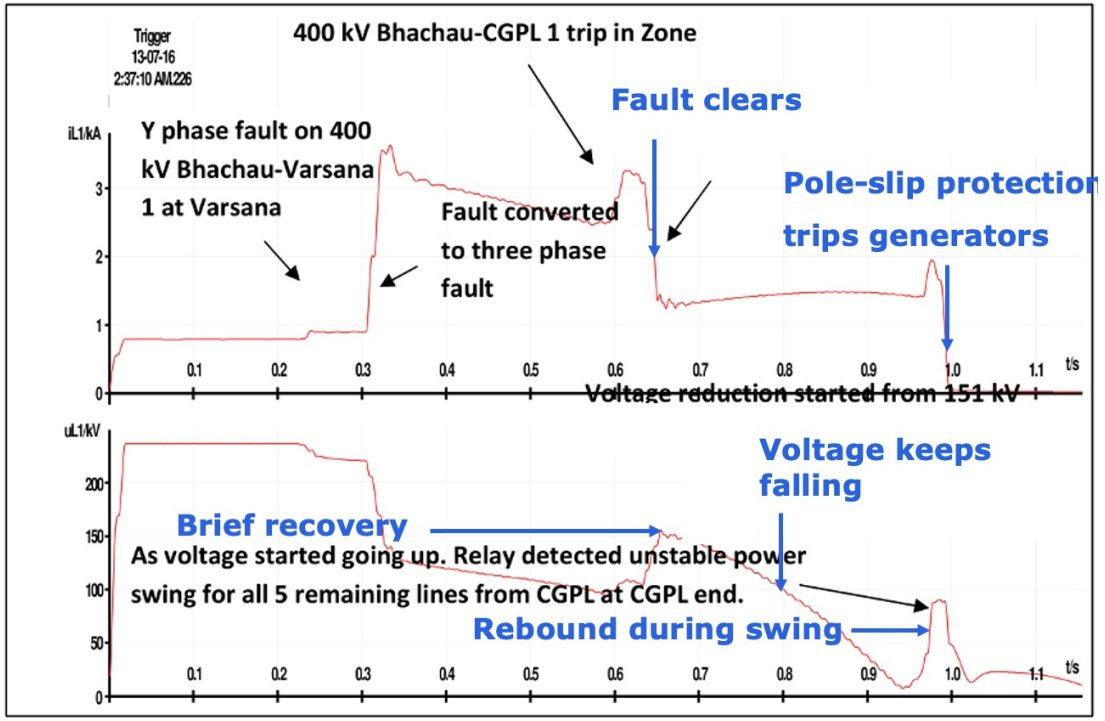
A fault and line trips reduced the generators' electrical output.

The fault cleared before a pole slipped.

Some time after, the pole slipped and the generator disconnected.

Why?



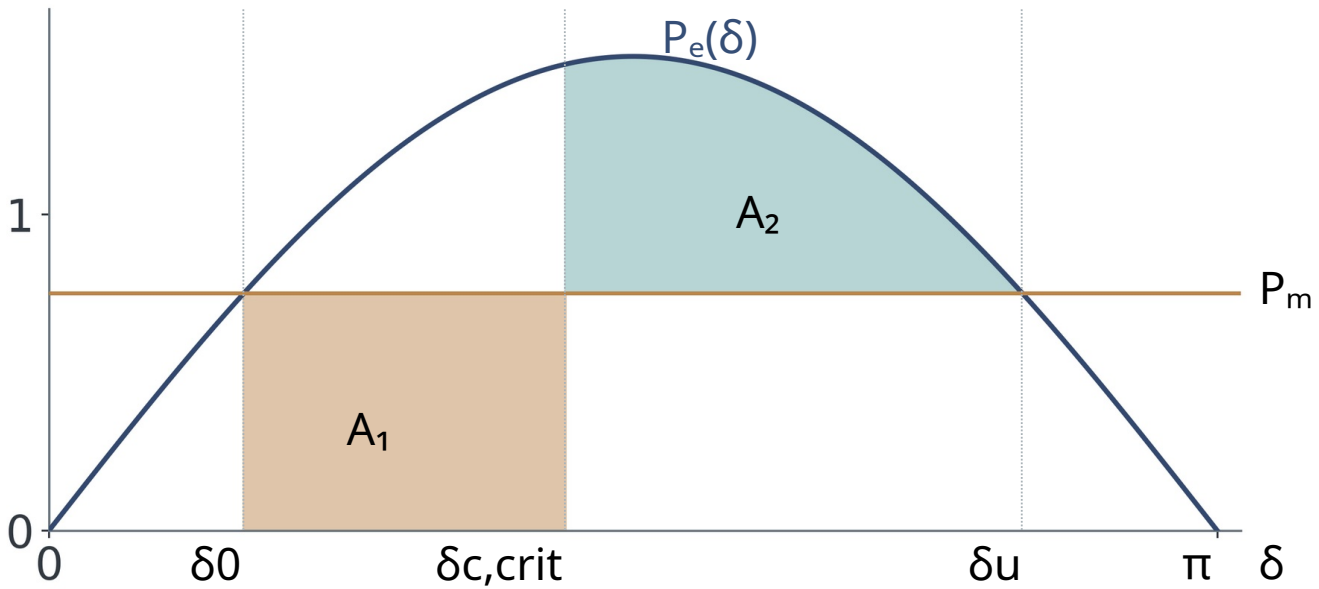


Recorded current and voltage

https://www.tatapower.com/energy-solutions/thermal-energy/mundra-plant

https://www.wrpc.gov.in/allfile/WRPC_Report_on_CGPL_Blackout_13-07-2016.pdf

What is the critical clearing angle?



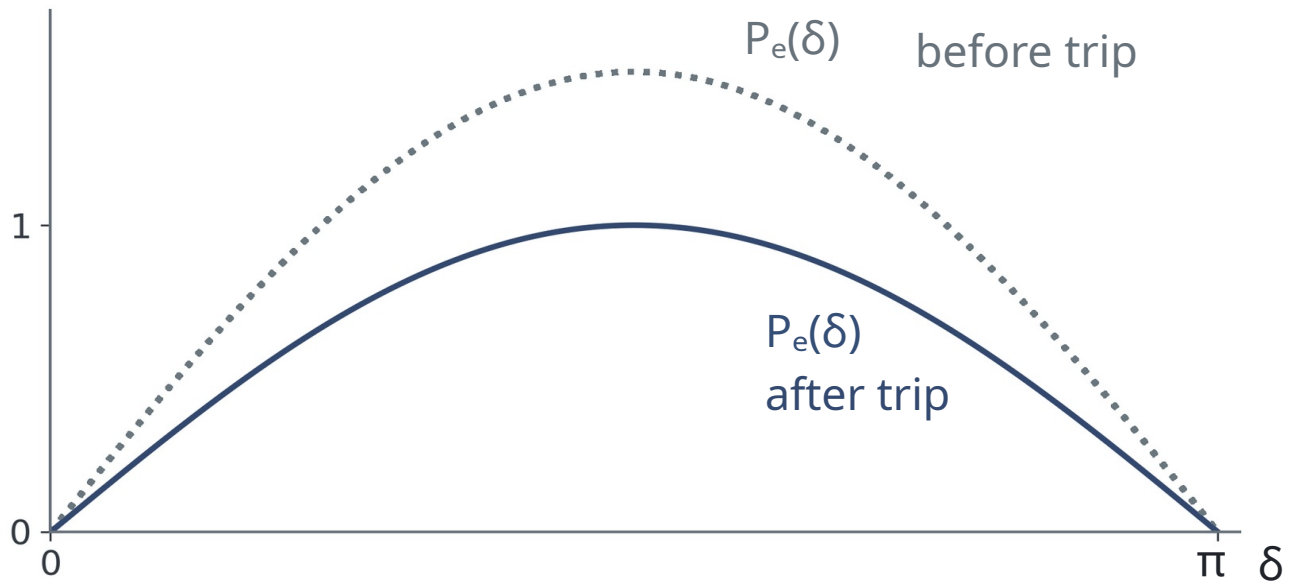
$P_m = 0.75$

$\delta_{c,crit} \approx 1.389$ rad

$A_1 = A_{2,available}$

This is the boundary between stable oscillations and loss of synchronism.

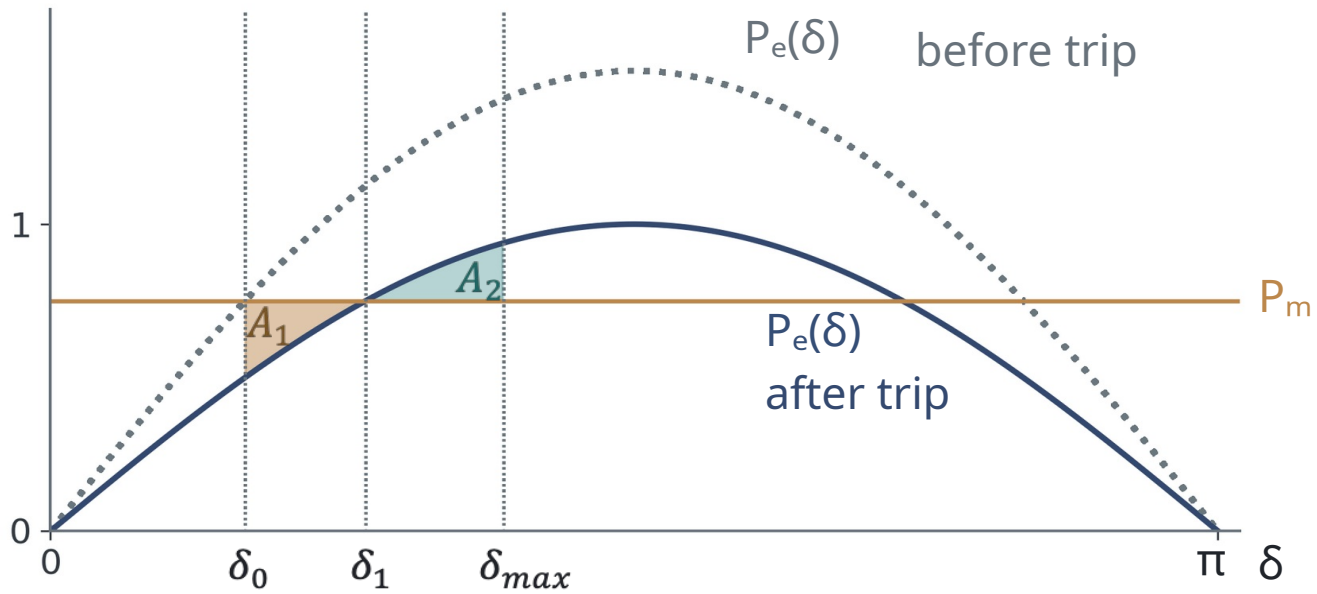
How does opening one line change the power curve?



$X_{trip} = 1/3 + 2/3 = 1$

$P_e,trip = \sin \delta$

Is the line trip stable?



$\mathbf{P_m = 0.75}$

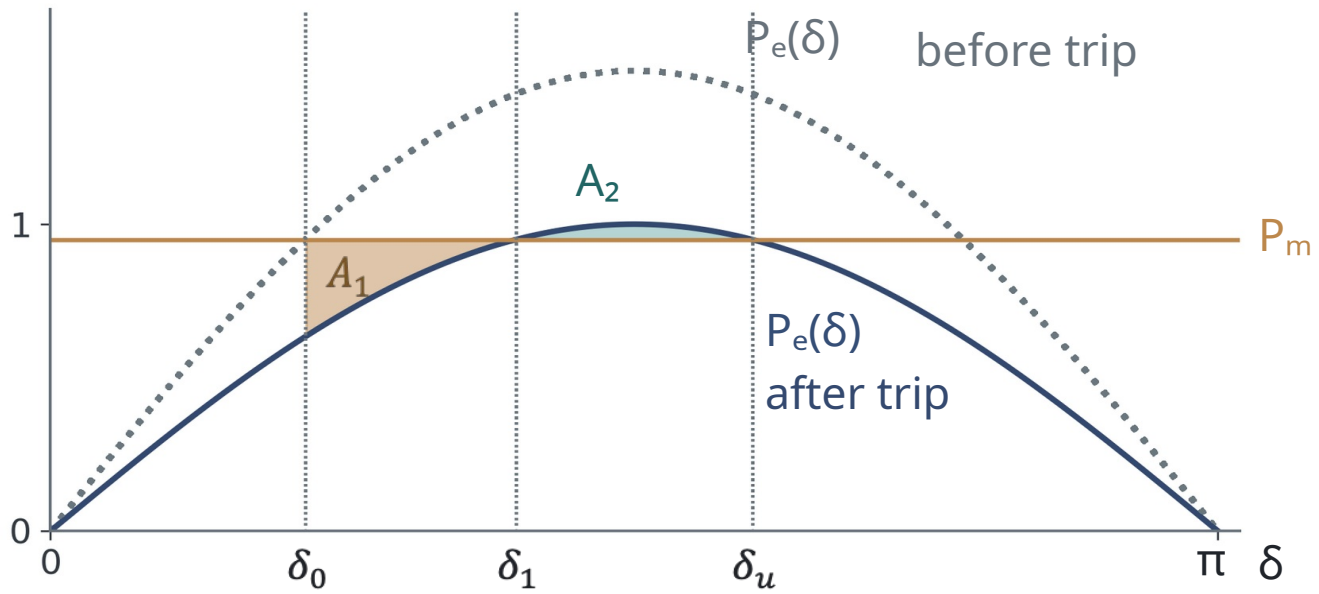
A1 ≈ 0.039

A2,max ≈ 0.239

The initial angle is 0.524 rad. The new left intersection is 0.848 rad.

There is enough area to stop the forward motion before the right intersection.

Can a line trip cause instability despite a stable equilibrium existing?



$\mathbf{P_m = 0.95}$

A1 ≈ 0.077

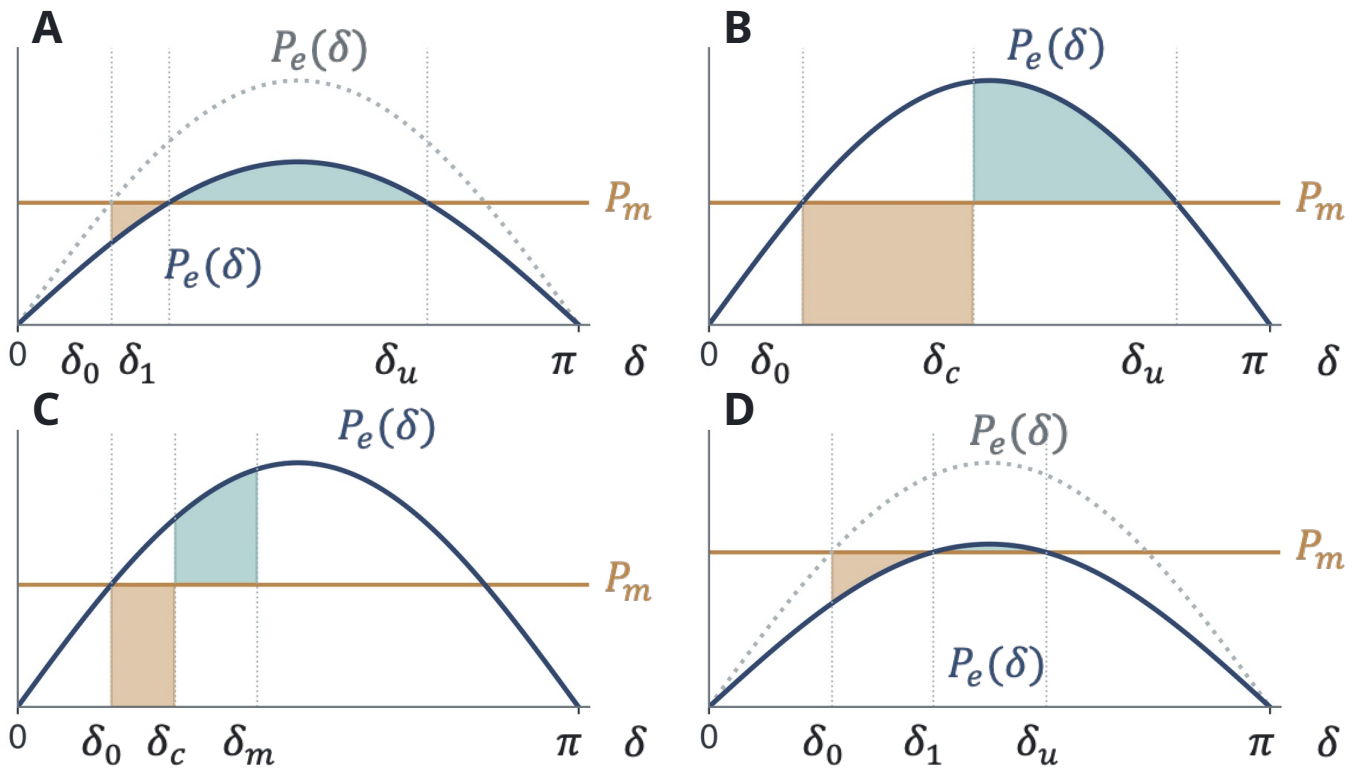
A2,max ≈ 0.021

An equilibrium still exists, but the gained energy exceeds the available deceleration area.

The rotor passes the unstable equilibrium.

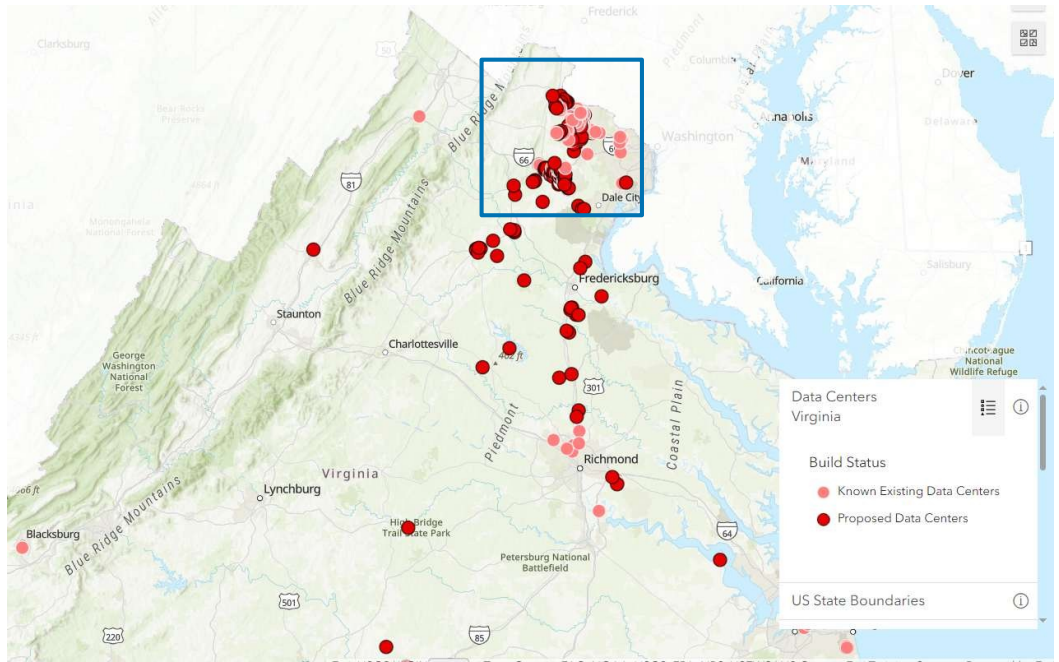
Exercise: Which event and stability outcome does each panel show?

fault or line trip, stable or unstable?

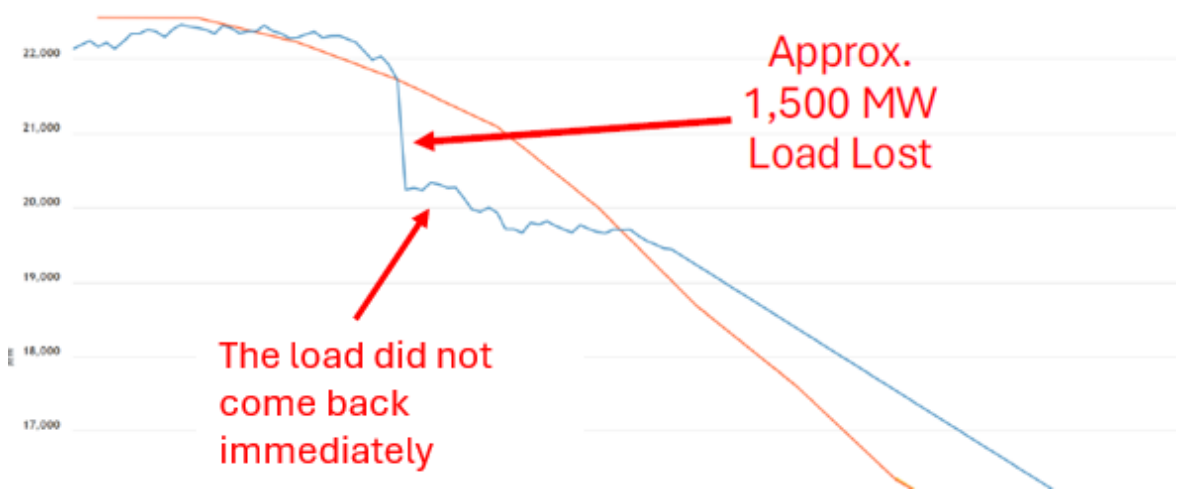


22 July 2026, Norther Virginia, USA:

A $230\,kV$ line fault triggered $1.5\,GW$ of data center load to suddnely disconnect from the grid.



https://www.pjm.com/-/media/DotCom/committees-groups/committees/oc/2026/20260806/20260806-item-05---dominion-july-loss-of-load-event---presentation.pdf



https://www.nerc.com/globalassets/our-work/reports/event-reports/incident_review_large_load_loss.pdf#page=3

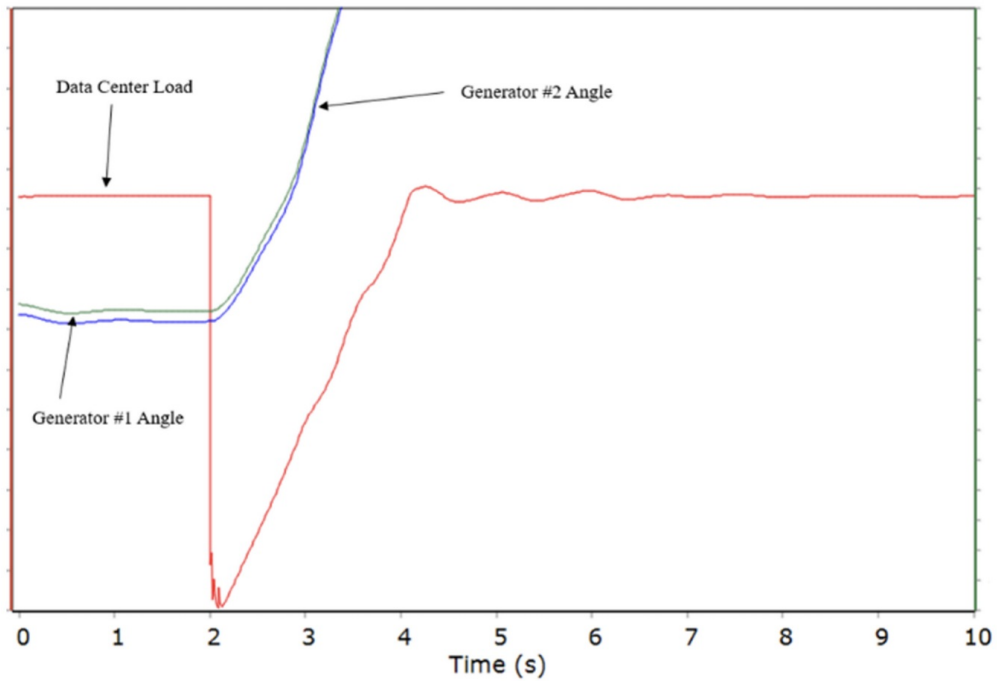
($1.5\,GW$ is a lot of power)

A large load tripping offline can result in loss of synchronism.

Risk Mitigation for Emerging Large Loads

NERC

May 2026



https://www.nerc.com/globalassets/our-work/guidelines/reliability/RG_Risk-Mitigation-For-Emerging-Large-Loads.pdf

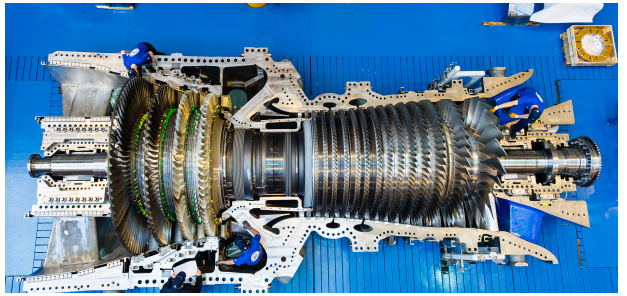
→ How?

The Yallourn coal power plant is scheduled to retire in mid-2028.



Consider a new use of the high-voltage grid connection

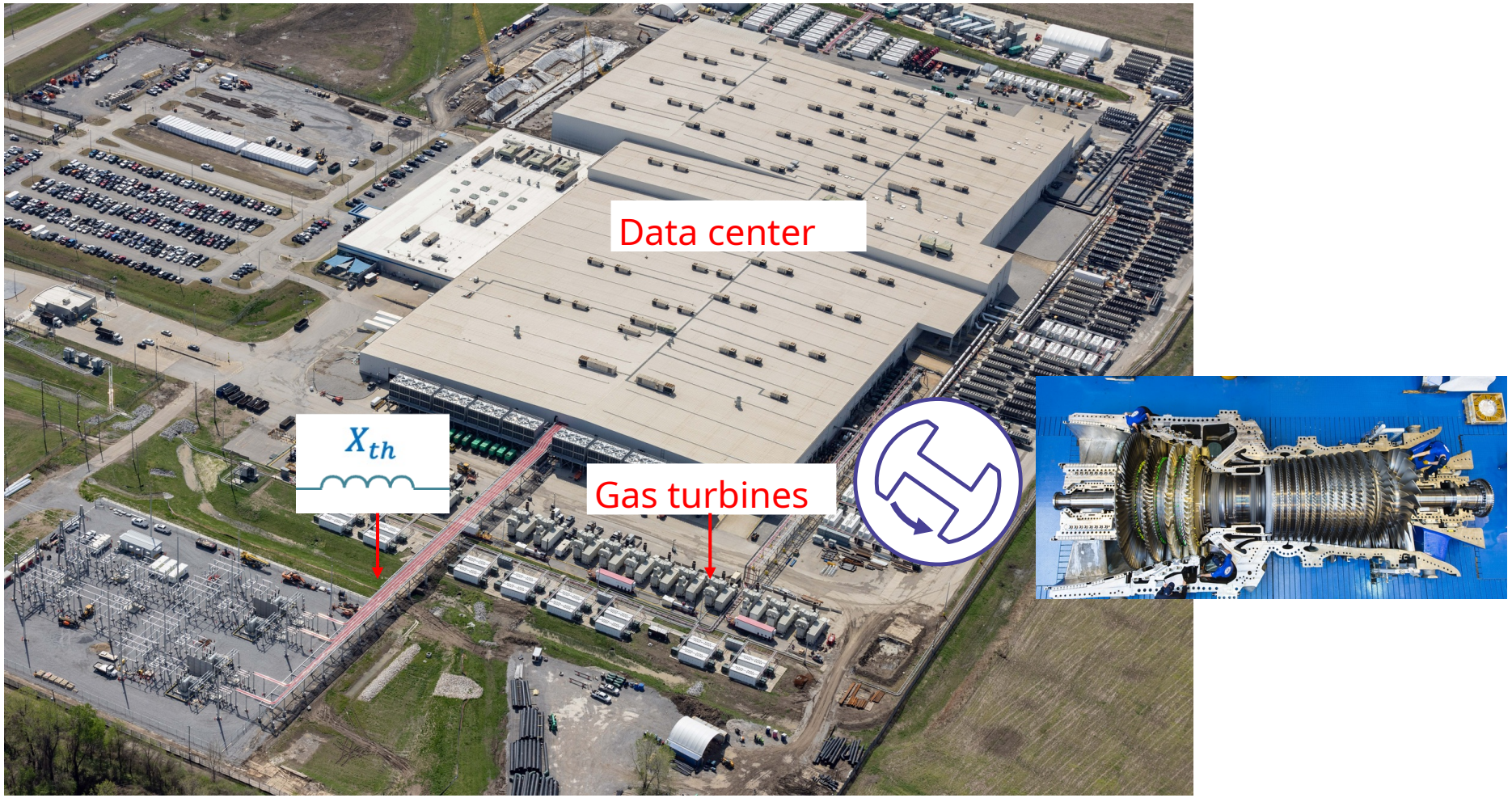




Large data center + Synchronous generator

What could happen if the data center suddenly trips offline?

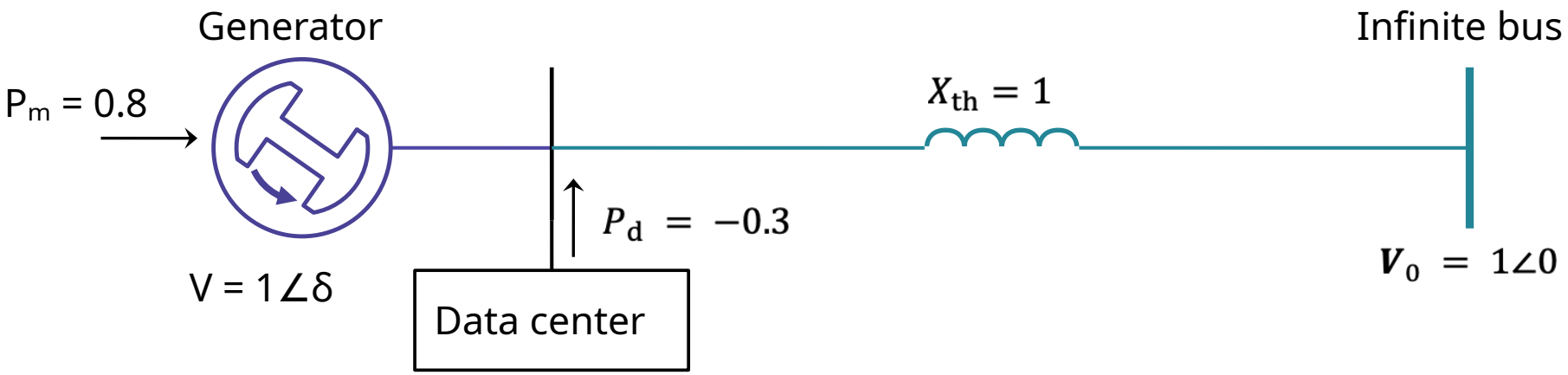
https://www.technologyreview.com/2026/01/14/1131253/data-centers-are-amazing-everyone-hates-them/
https://www.transformingyallourn.com.au/about/yallourn-energy-security-precinct
https://aemo.com.au/-/media/files/electricity/nem/planning_and_forecasting/maps/nem-regional-boundaries-map-web.pdf



https://substack-post-media.s3.amazonaws.com/public/images/44645f53-d1f1-4007-8fa3-76aa4fcf642e_8192x5464.jpeg

What could happen if the data center suddenly trips offline?

Exercise: Transient stability when a large data center trips offline



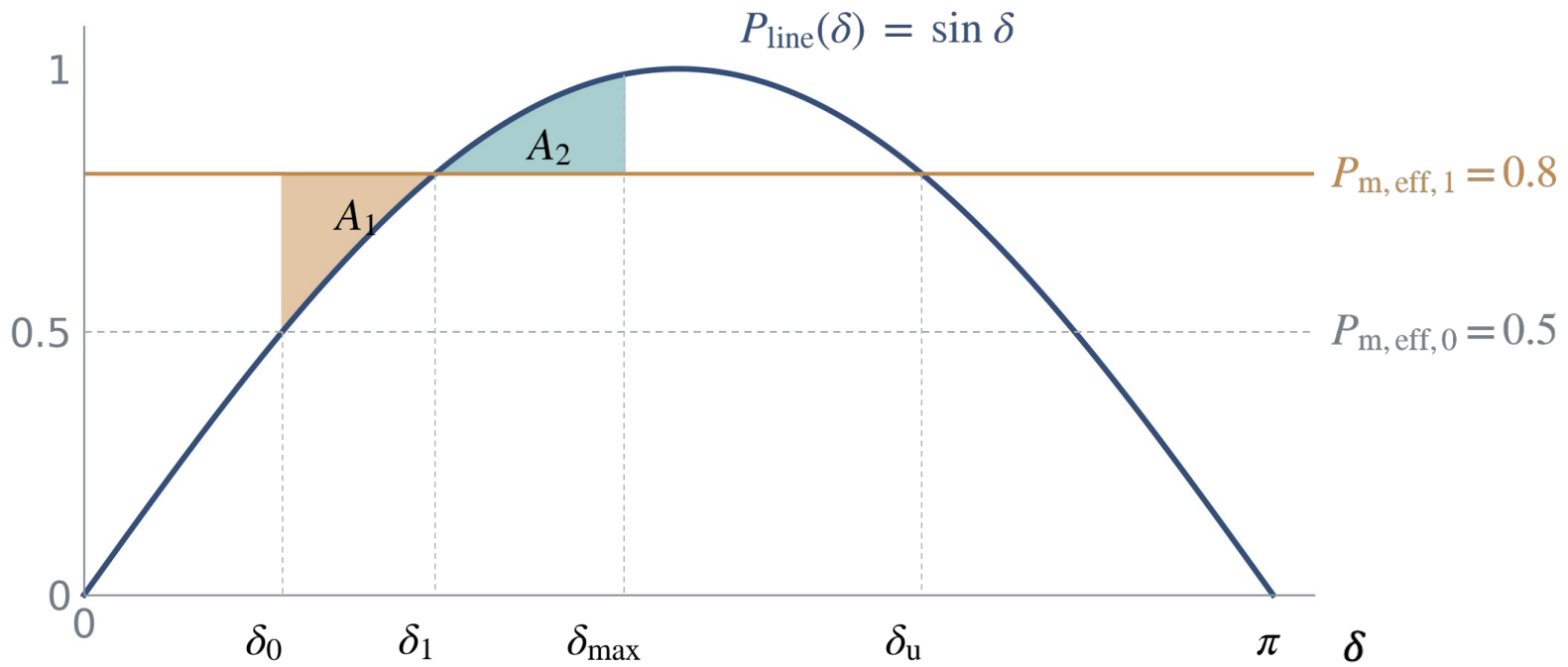
Fixed voltage magnitudes and mechanical input. Neglect damping.

The data center disconnects completely. **Does the generator slip a pole?**

$$\textit{Hint}: P_{m,eff} = P_m + P_d$$

Exercise solution:

$P_{\mathrm{m,eff}} = P_{\mathrm{m}} + P_{\mathrm{d}}$ rises from 0.5 to 0.8. The rotor accelerates according to $M\ddot{\delta} = P_{\mathrm{m,eff}} - P_{\mathrm{line}}(\delta)$

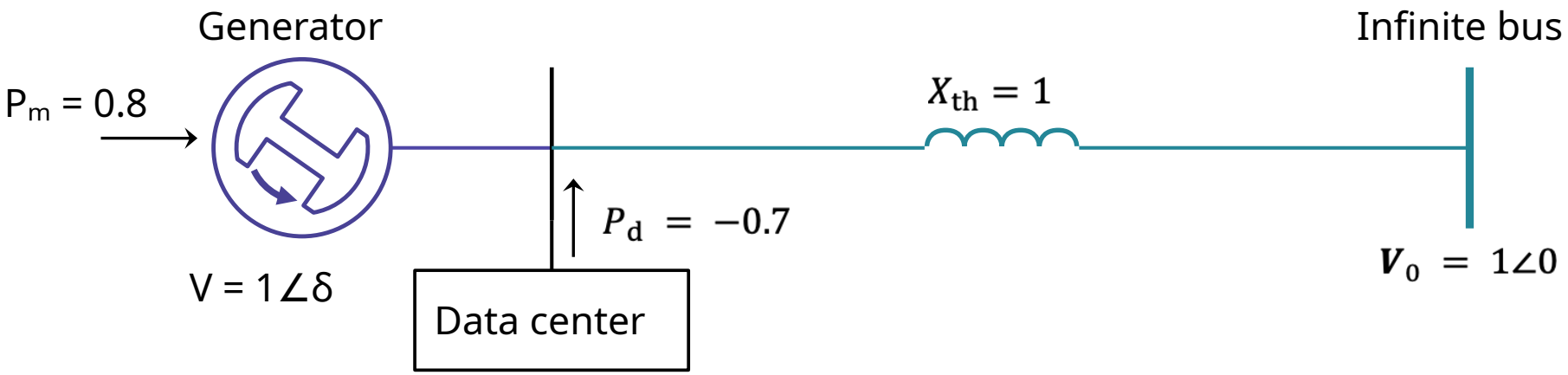


$A_1 < A_{2,\max}$

$\delta_{\max} \approx 1.4 < \delta_{\mathrm{u}}$

No pole slip. The rotor oscillates about the new stable equilibrium.

Exercise: Transient stability when a larger data center trips offline



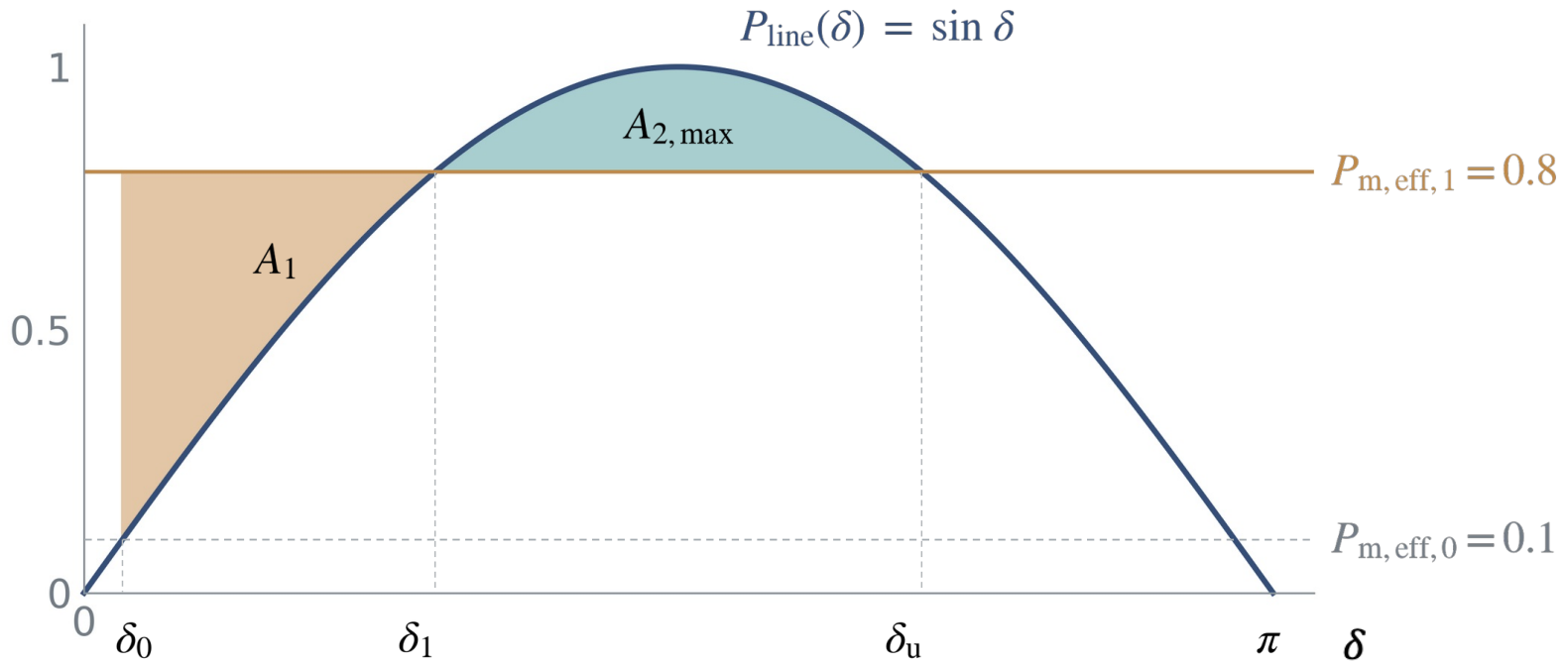
Now the data center load is 0.7.

The data center disconnects completely. **Does the generator slip a pole?**

$$\textit{Hint: } P_{m,eff} = P_m + P_d$$

Exercise solution:

$P_{\mathrm{m,eff}} = P_{\mathrm{m}} + P_{\mathrm{d}}$ rises from 0.1 to 0.8. The rotor accelerates according to $M\ddot{\delta} = P_{\mathrm{m,eff}} - P_{\mathrm{line}}(\delta)$



$A_1 > A_{2,\max}$

The pole slips. The rotor passes the unstable equilibrium with positive relative speed.

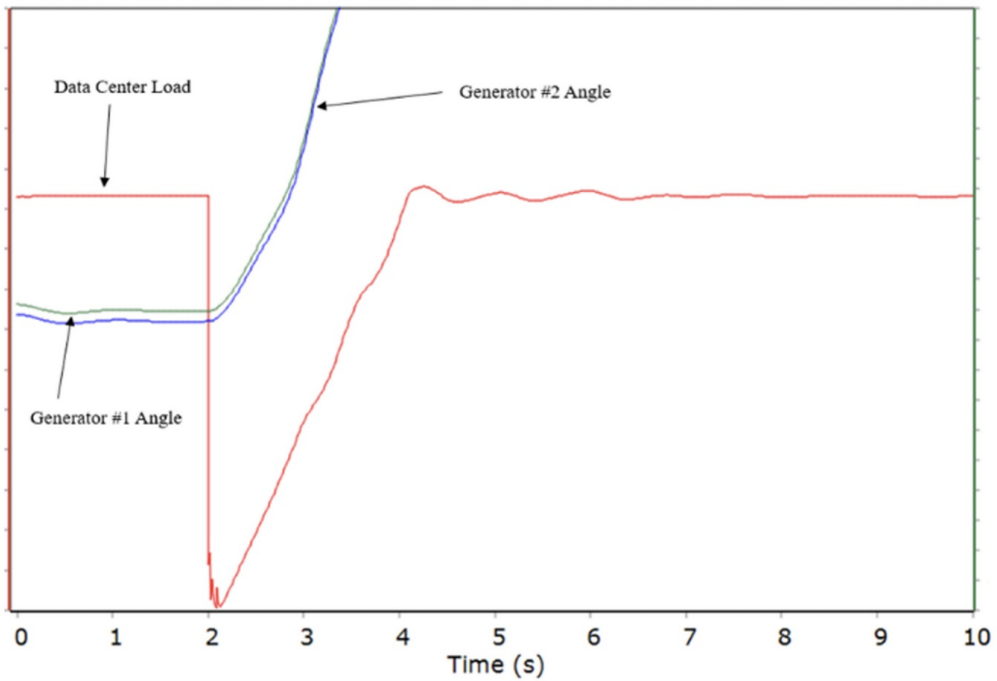
Protection disconnects a generator if it slips a pole… unless it doesn't.



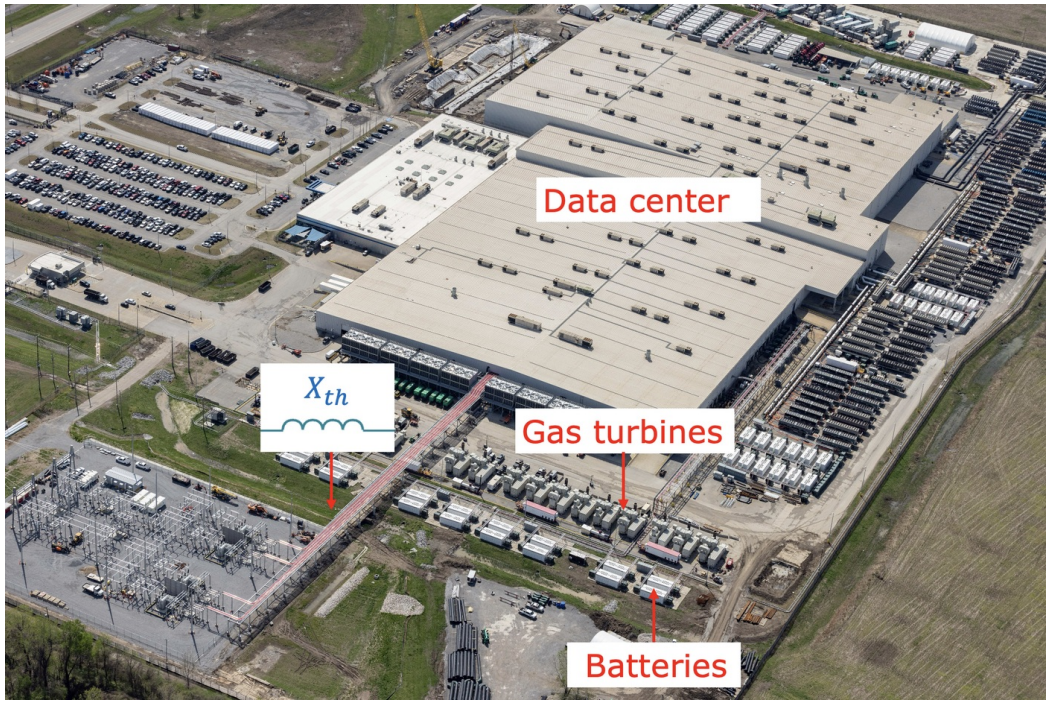
Callide C4, Queensland, 25 May 2021

https://www.csenergy.com.au/news/cs-energy-releases-photo-of-unit-c4

Take-home lesson: It is best to avoid massive, sudden changes in load on the grid.



https://www.nerc.com/globalassets/our-work/guidelines/reliability/RG_Risk-Mitigation-For-Emerging-Large-Loads.pdf

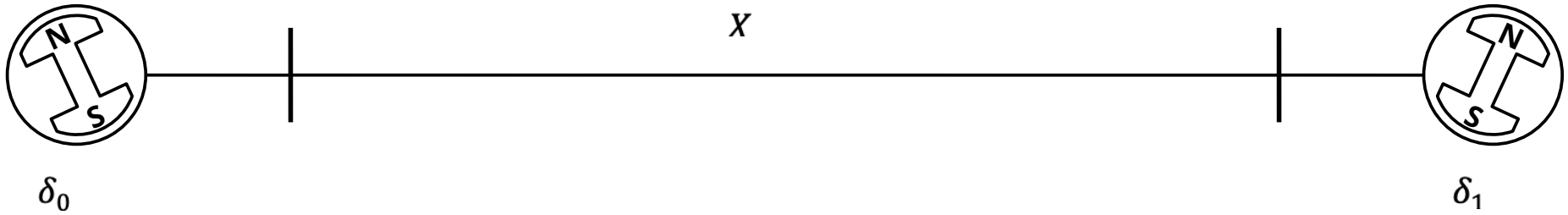


https://substack-post-media.s3.amazonaws.com/public/images/44645f53-d1f1-4007-8fa3-76aa4fcf642e_8192x5464.jpeg

One way to avoid this is to build collocated batteries to smooth compute fluctuations.

Transient stability

We can generalize the energy-based methods from pole slip to system-wide synchronization.

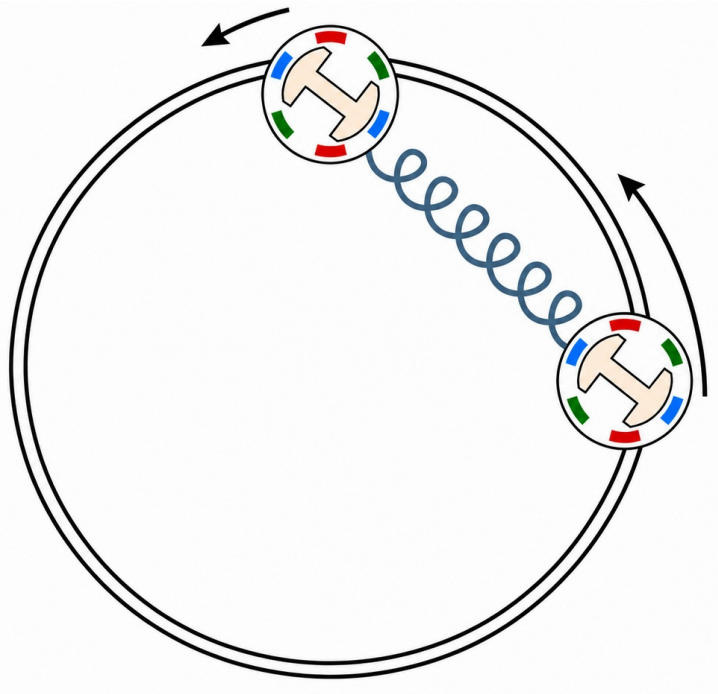


Two moving rotors still have only one relative angle $\delta = \delta_0 - \delta_1$

The same equal-area argument applies as in the single-machine infinite bus case.

A mechanical analogy for synchronization/transient stability

Synchronization: common frequency and settled relative rotor angles.

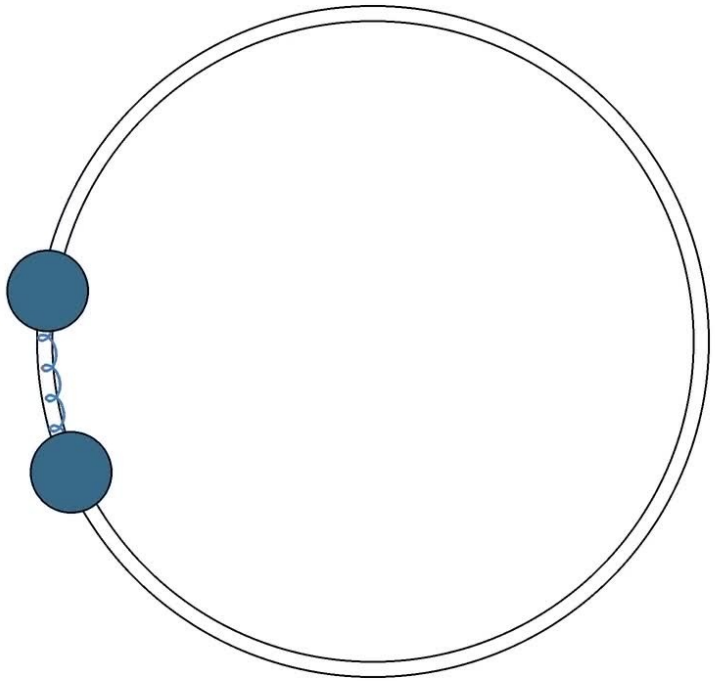


Inductive lines create synchronizing torques between generators, modeled as springs.

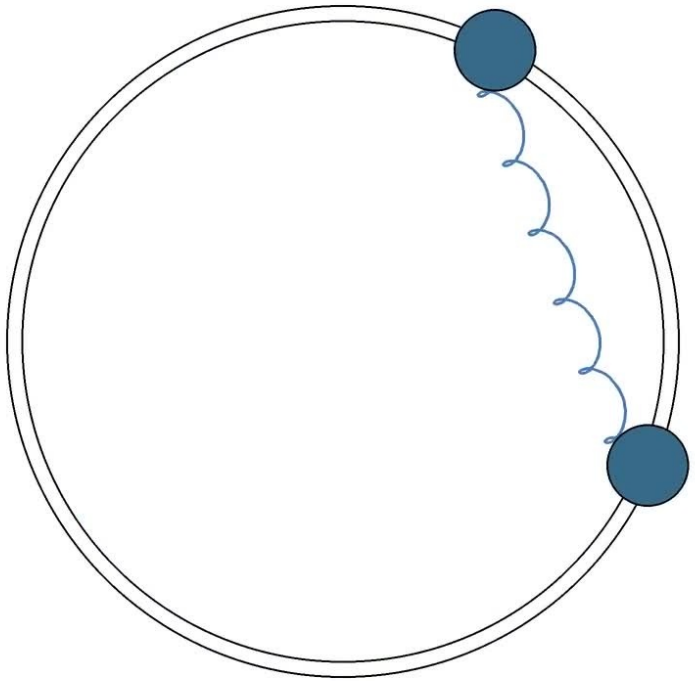
https://doi.org/10.1016/j.automatica.2014.04.012

What is transient stability/system-wide synchronization for three generators?

Initial conditions A:



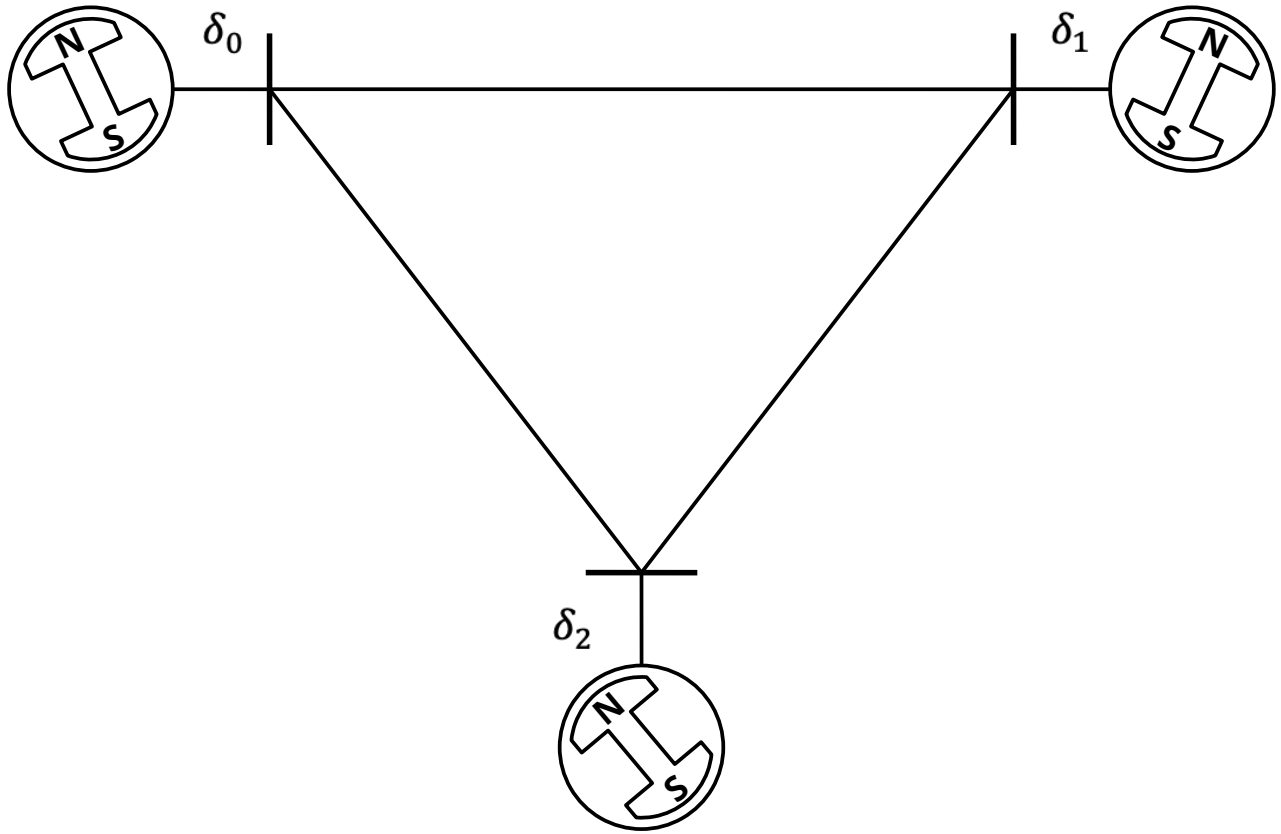
Initial conditions B:



The rotor can sometimes regain synchronism after a slip due to damping, but often it does not.

https://motion.me.ucsb.edu/pdf/2013b-db.pdf

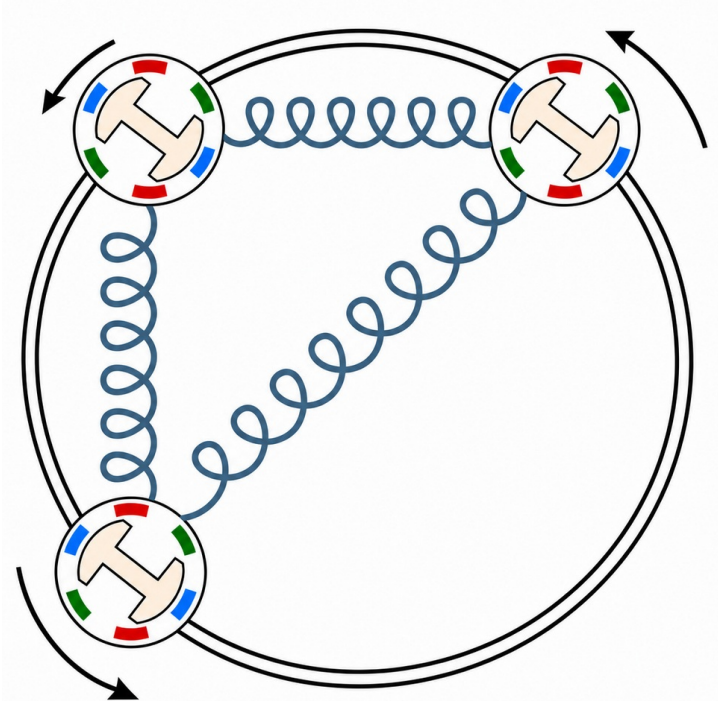
Three generator/area systems provide additional intuition



Three rotors have two independent angle differences, $\delta_1 - \delta_0$ and $\delta_2 - \delta_0$

A mechanical analogy for synchronization/transient stability

Synchronization: common frequency and settled relative rotor angles.

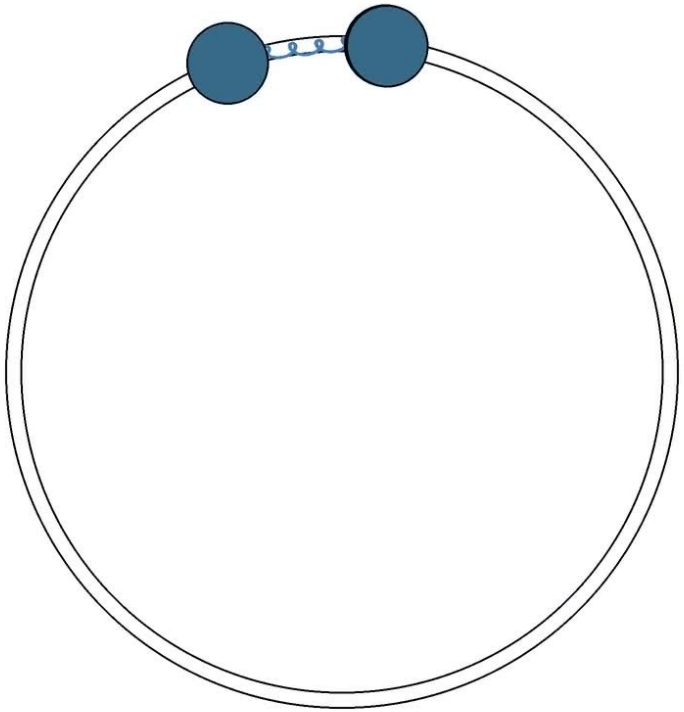


Inductive lines create synchronizing torques between generators, modeled as springs.

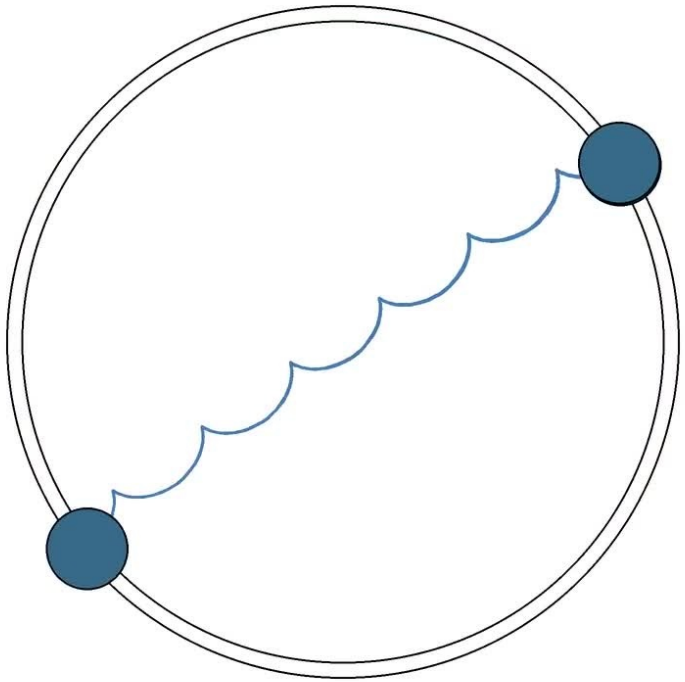
https://doi.org/10.1016/j.automatica.2014.04.012

What is transient stability/system-wide synchronization for three generators?

Initial conditions A:

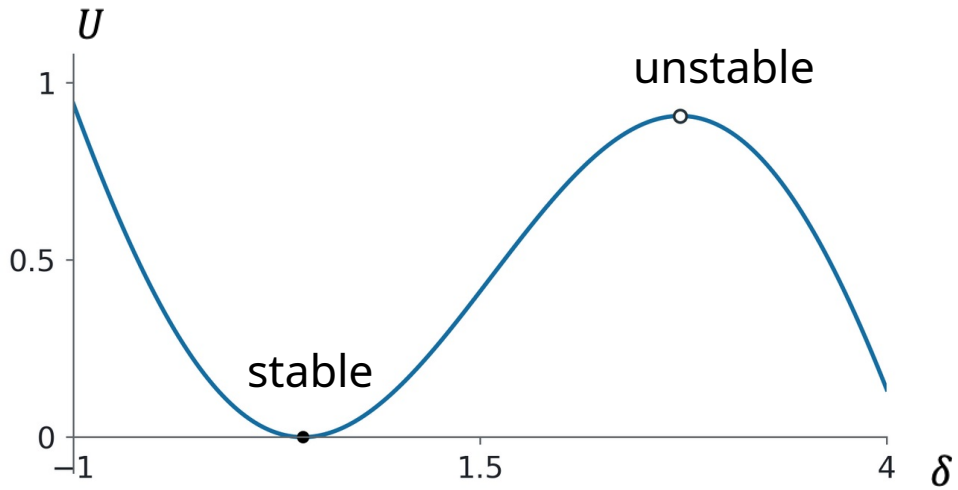


Initial conditions B:

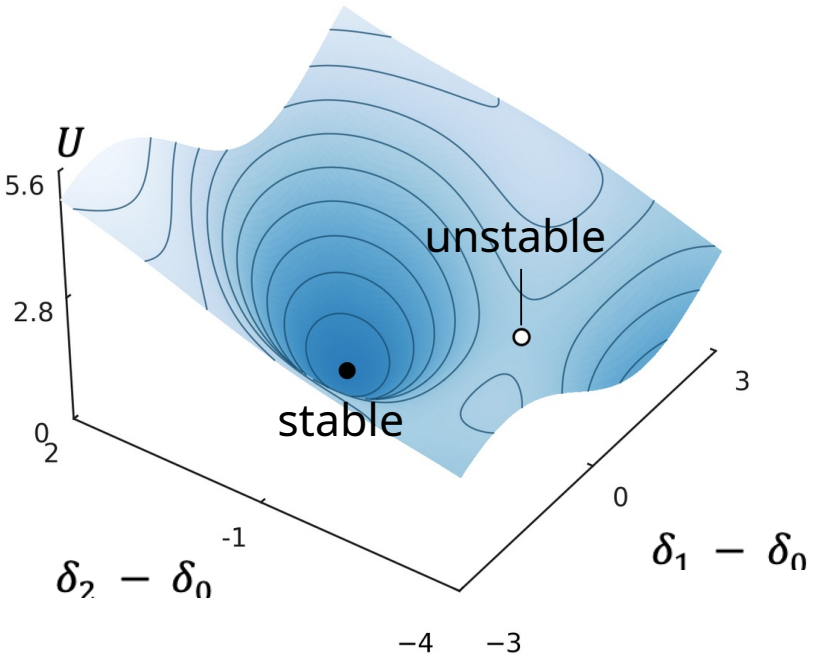


https://motion.me.ucsb.edu/pdf/2013b-db.pdf

Two rotors



Three rotors



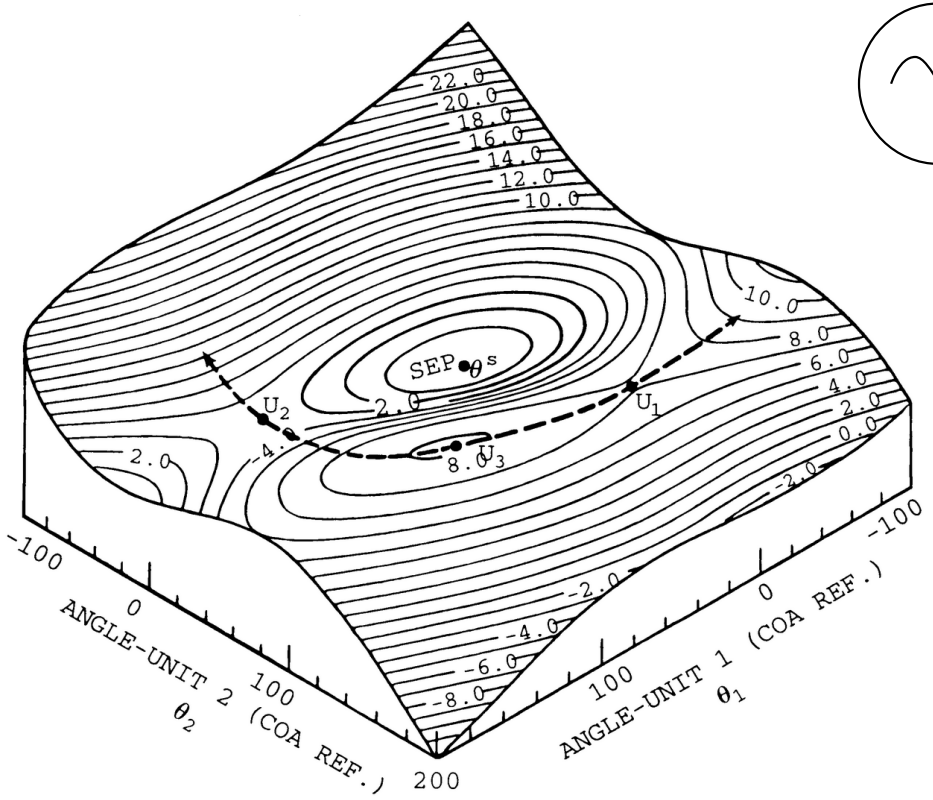
Total energy: $F = W + U$

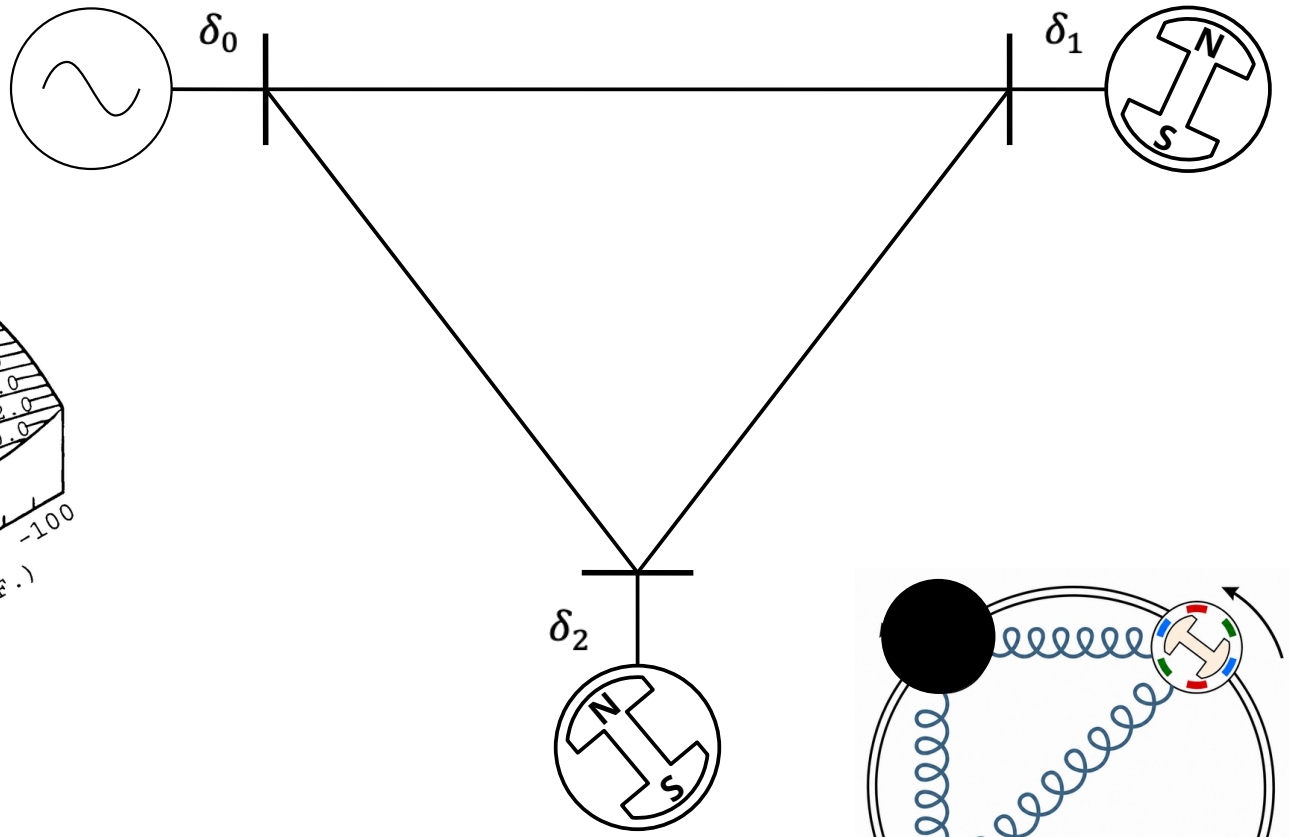
W : kinetic energy in the rotors' motion relative to one another.

U : potential energy associated with the rotor angle differences.

The restoring force is given by the derivative of the potential energy:

$$\tau_{\text{rest}} = -\frac{dU}{d\delta}$$



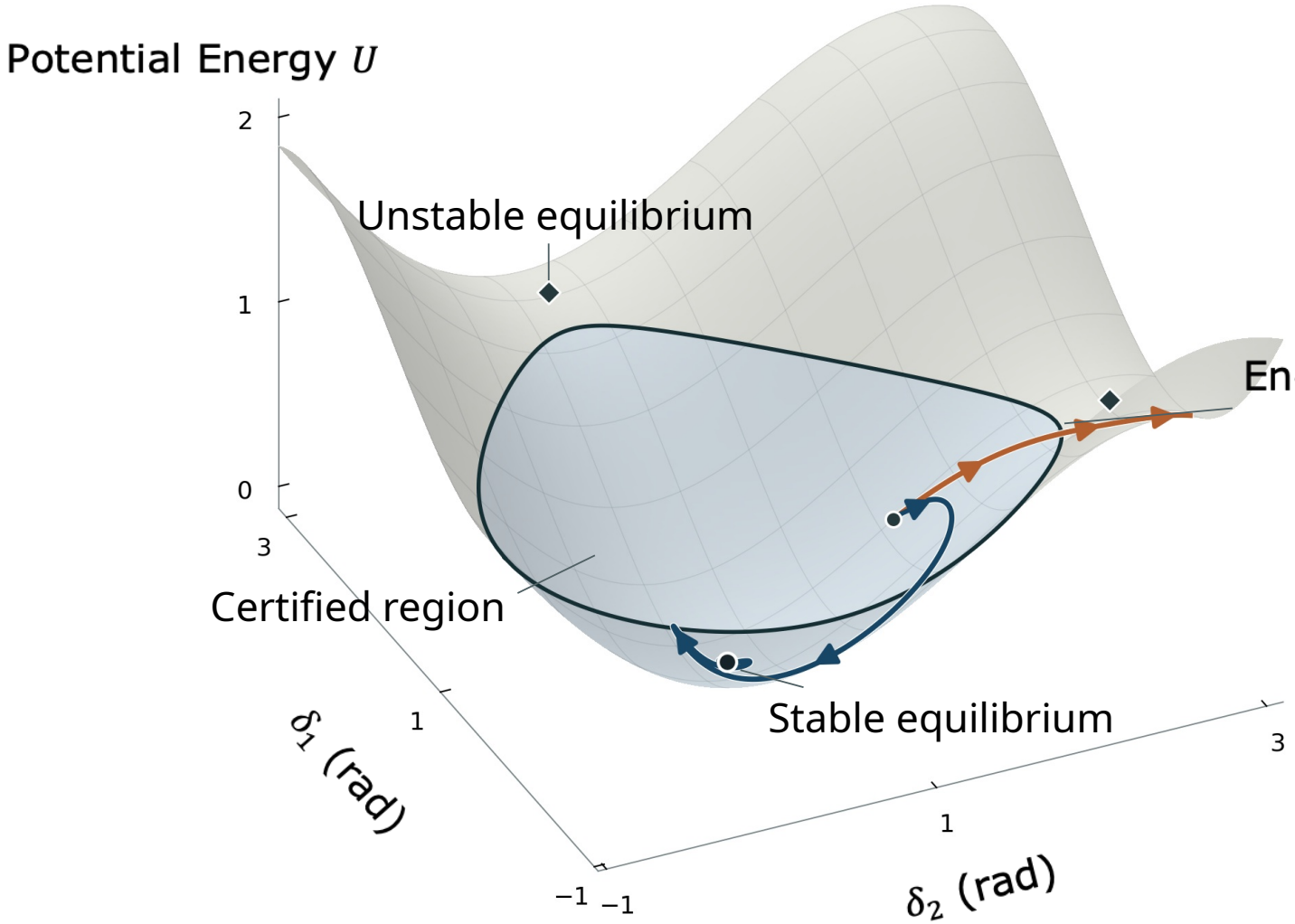


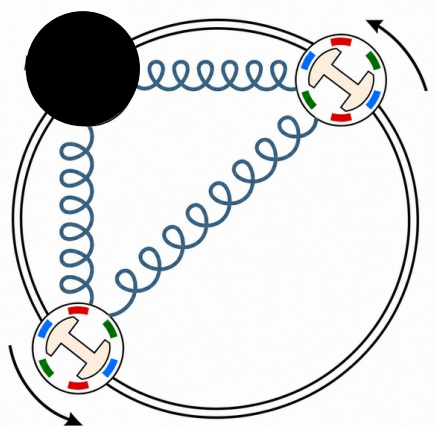
The potential energy defines a boundary surface

Potential energy contours surround the stable operating point (SEP).

The dashed curve traces the edge of the stable region.

Sauer & Pai, Power System Dynamics and Stability, author manuscript (2007), Fig. 9.10. Reproduced from T. Athay, R. Podmore & S. Virmani, "A Practical Method for the Direct Analysis of Transient Stability," IEEE Trans. Power Apparatus and Systems, PAS-98(2), 573–584 (1979). https://doi.org/10.1109/TPAS.1979.319407



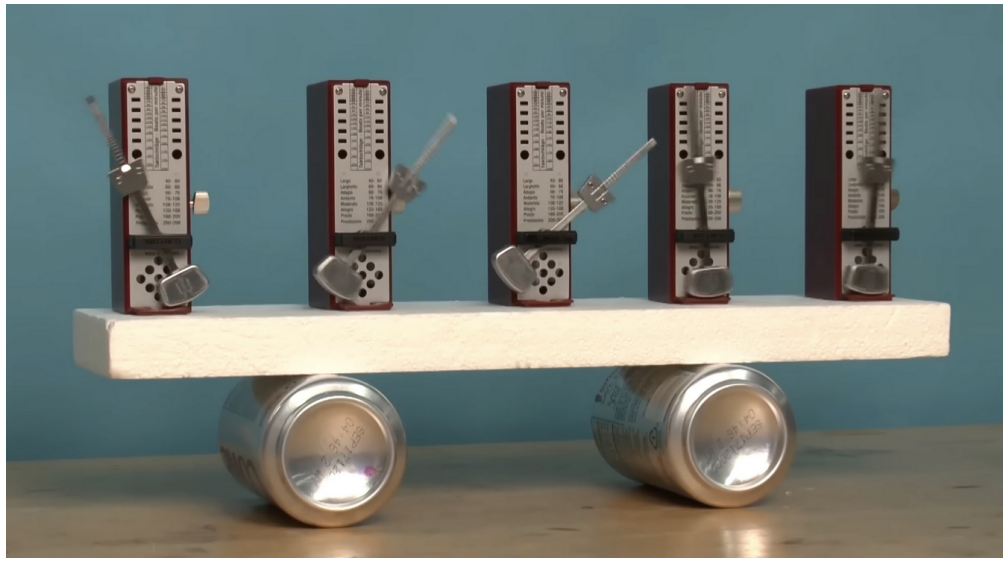


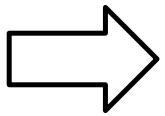
Escapes this well

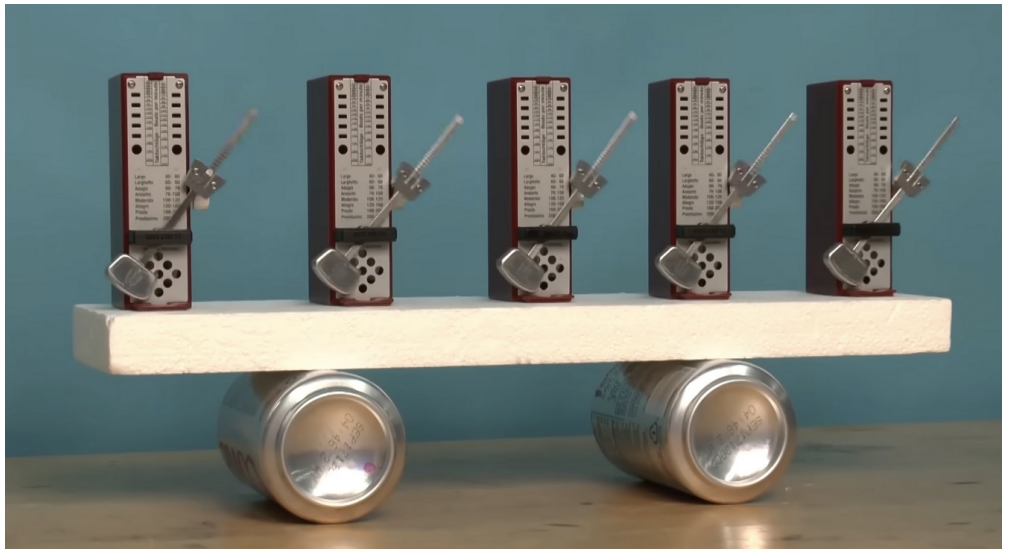
Returns to equilibrium

https://www2.eecs.berkeley.edu/Pubs/TechRpts/1984/ERL-m-84-72.pdf

Have we seen this type of behavior before?







Synchronization is one way of interpreting energy-function based transient stability analysis methods such as the Equal Area Criterion.

https://www.youtube.com/watch?v=T58lGKREubo&t=1s

Transient stability

We can generalize the energy-based methods from pole slip to system-wide synchronization.

But, these methods have their limitations and are generally restricted to textbook examples

Real-world challenges:

Scale: *Real-world grids have more than the generators.*

Assumptions: *Are lines lossless? Can we neglect voltage magnitude? (No)*

Modeling accuracy: *Do we know the dynamics of every device connected to the grid?*



https://theenergy.co



https://www.csenergy.com.au



https://portal.engineersaustralia.org.au



https://tic.toshiba.com.au

Modern girds have new challenges that are not just based on rotating machine dynamics.

- Solar (possibly + battery)

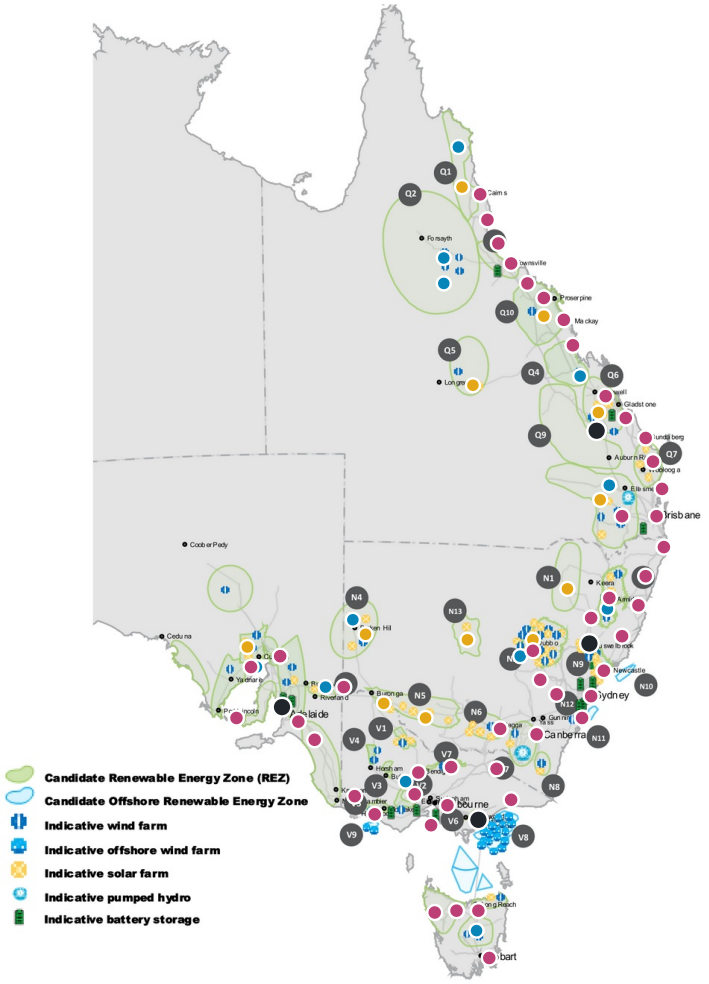


https://www.csiro.au

- Wind (possibly + battery)



https://arena.gov.au



aemo.com.au (indicative locations)

- Synchronous generators



https://www.csenergy.com.au

- Adaptive loads, e.g., EVs

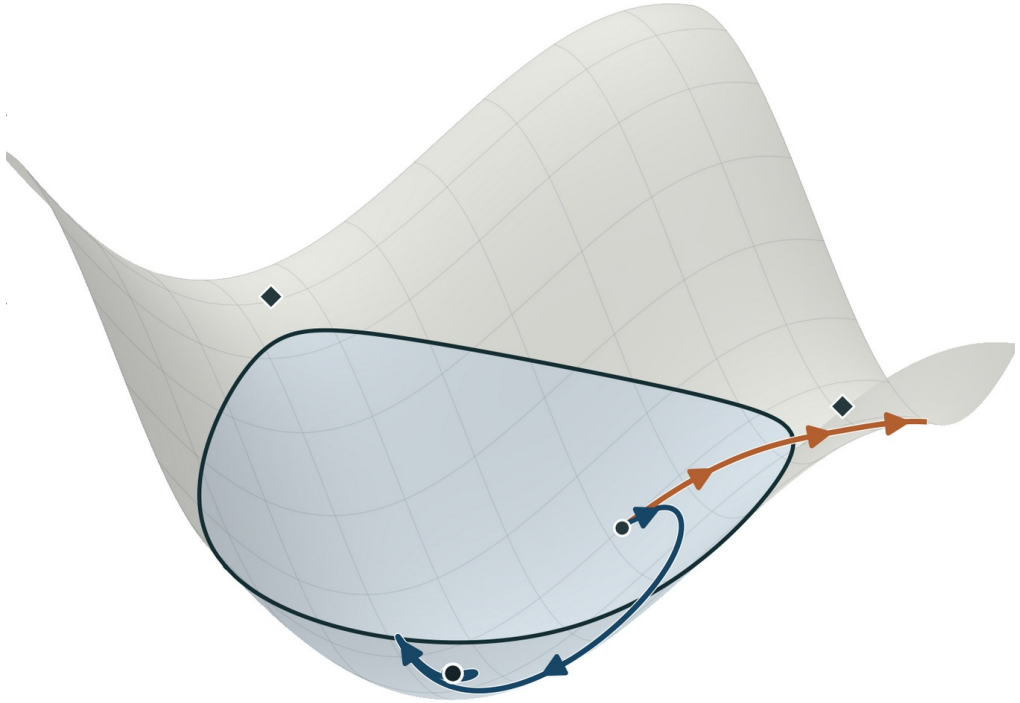


https://newsroom.porsche.com

New stability analysis tools and engineers are needed in Australia and throughout the world.

Coming soon... a new course taught with Prof. Steven Low of CalTech:

The Mathematics of Power Systems



Power flow & optimization

Dynamics & stability

Email me for more info: keith.moffat@unimelb.edu.au

Transient Stability

ELEN90060 Power System Analysis

Keith Moffat

Appendix I: Deriving the equal-area equation

Neglect damping and integrate the swing equation from δ_0 to δ_m.

$M = 2H/\omega_s, \quad M\, d^2\delta/dt^2 = P_{m1} - P_e(\delta)$

$\int_{\delta_0}^{\delta_m} M(d^2\delta/dt^2)d\delta = \int_{\delta_0}^{\delta_m} (P_{m1} - P_e(\delta))d\delta$

Set u equal to the square of relative angular speed.

$u = (d\delta/dt)^2, \quad du/d\delta = 2d^2\delta/dt^2$

$du = 2(d^2\delta/dt^2)d\delta$

Appendix I: Deriving the equal-area equation

Substitute the differential from the previous slide into the angle integral.

$$(H/\omega_s)\int u(\delta_0)^{u(\delta_m)} du = \int_{\delta_0}^{\delta_m}(P_{m1} - P_e(\delta))d\delta$$

$$(H/\omega_s)[u(\delta_m) - u(\delta_0)] = \int_{\delta_0}^{\delta_m}(P_{m1} - P_e(\delta))d\delta$$

The rotor starts at rest relative to the grid and stops at its first turning angle.

$$\dot{\delta}(\delta_0)=0, \quad \dot{\delta}(\delta_m)=0, \quad u(\delta_0)=u(\delta_m)=0$$

Appendix I: Deriving the equal-area equation

Zero change in squared relative speed gives

$$\int_{\delta_0}^{\delta_m}(P_{m1} - P_e(\delta))d\delta = 0$$

Split at δ_1, where electrical and mechanical powers are equal.

$$\int_{\delta_0}^{\delta_1}(P_{m1} - P_e(\delta))d\delta = \int_{\delta_1}^{\delta_m}(P_e(\delta) - P_{m1})d\delta$$

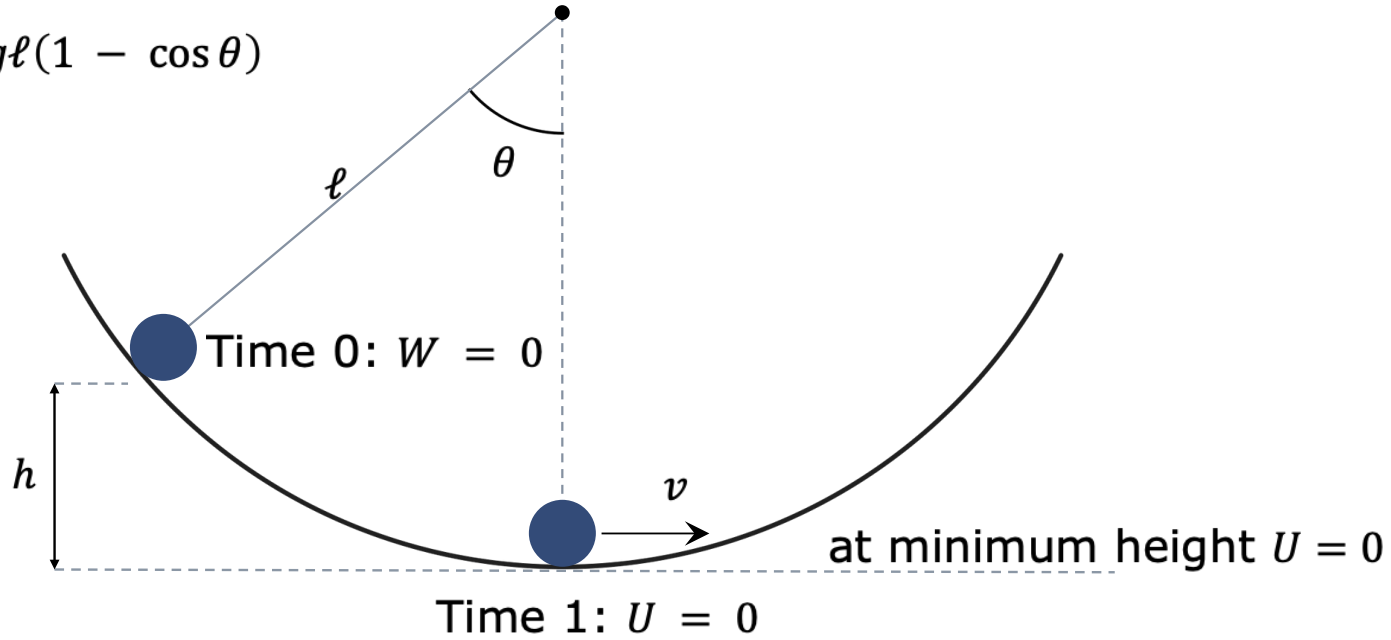
$$A_a = A_D$$

Both areas are positive. The PDF's final printed equation has the opposite sign on the right.

Appendix II: Frictionless marble analogy for the exchange potential and kinetic energy

Potential energy: $U(\theta) = mgh = mg\ell(1 - \cos\theta)$

Kinetic energy: $W = \frac{1}{2}mv^2$



Arc length: $s = \ell\theta$

Force along the bowl: $F = -\frac{\mathrm{d}U}{\mathrm{d}s} = -\frac{1}{\ell}\frac{\mathrm{d}U}{\mathrm{d}\theta} = -mg\sin\theta$

($F = -\frac{\mathrm{d}U}{\mathrm{d}s}$ arises from conservation of energy $U + W = \text{constant}$)

Appendix II: Frictionless marble analogy for the exchange potential and kinetic energy

Why does the potential slope give the force along the bowl?

Conservation of energy: $\frac{\mathrm{d}}{\mathrm{d}t}[U + W] = 0$

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}m\dot{s}^2 + U(s)\right]$$

$$0 = m\dot{s}\ddot{s} + \frac{\mathrm{d}U}{\mathrm{d}s}\dot{s}$$

$$0 = m\ddot{s} + \frac{\mathrm{d}U}{\mathrm{d}s}$$

$$F_\mathrm{t} = m\ddot{s} = -\frac{\mathrm{d}U}{\mathrm{d}s}$$

Appendix II: SMIB system, why does the potential slope give accelerating power?

Inertia equation: $M\ddot{\delta} = P_m - P_e(\delta)$

Conservation of scaled oscillation energy:

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}M\dot{\delta}^2 + U(\delta)\right]$$

$$0 = M\dot{\delta}\ddot{\delta} + \frac{\mathrm{d}U}{\mathrm{d}\delta}\dot{\delta}$$

$$0 = M\ddot{\delta} + \frac{\mathrm{d}U}{\mathrm{d}\delta}$$

$$P_m - P_e(\delta) = -\frac{\mathrm{d}U}{\mathrm{d}\delta}$$