Voltage Collapse

ELEN90060 Power System Analysis

Keith Moffat



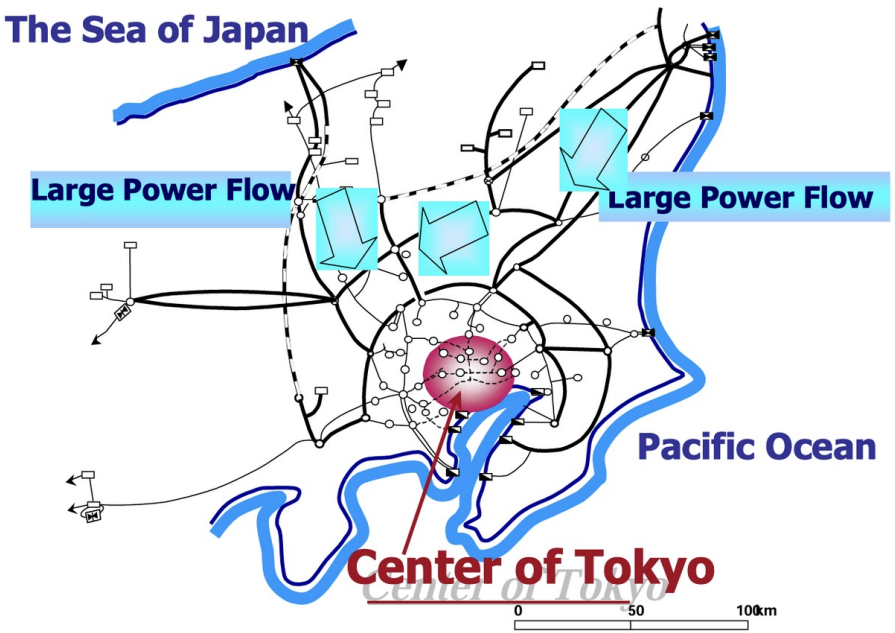
Tokyo, 23 July 1987

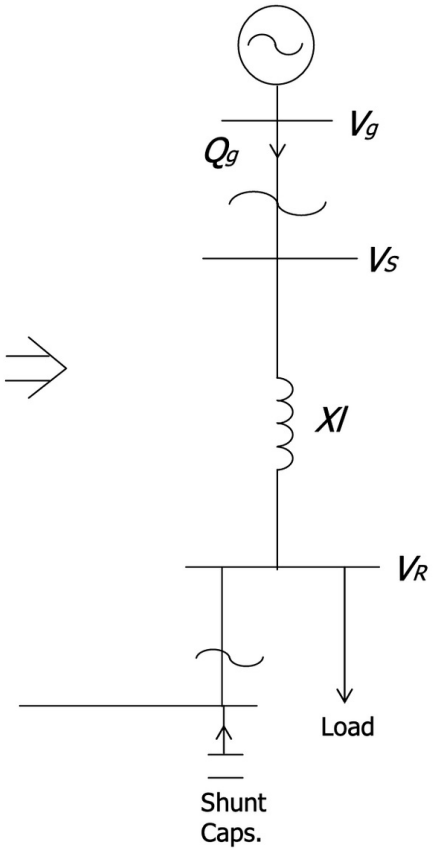
After lunch, demand rose by about 400 MW/min.

13:10: Demand reached 39.3 GW

13:19: “voltages in the Tokyo supply network fell rapidly after sustained demand growth, without an initiating contingency.”

Soon after: Automatic protection sheds 8.2 GW, avoiding a system-wide blackout.





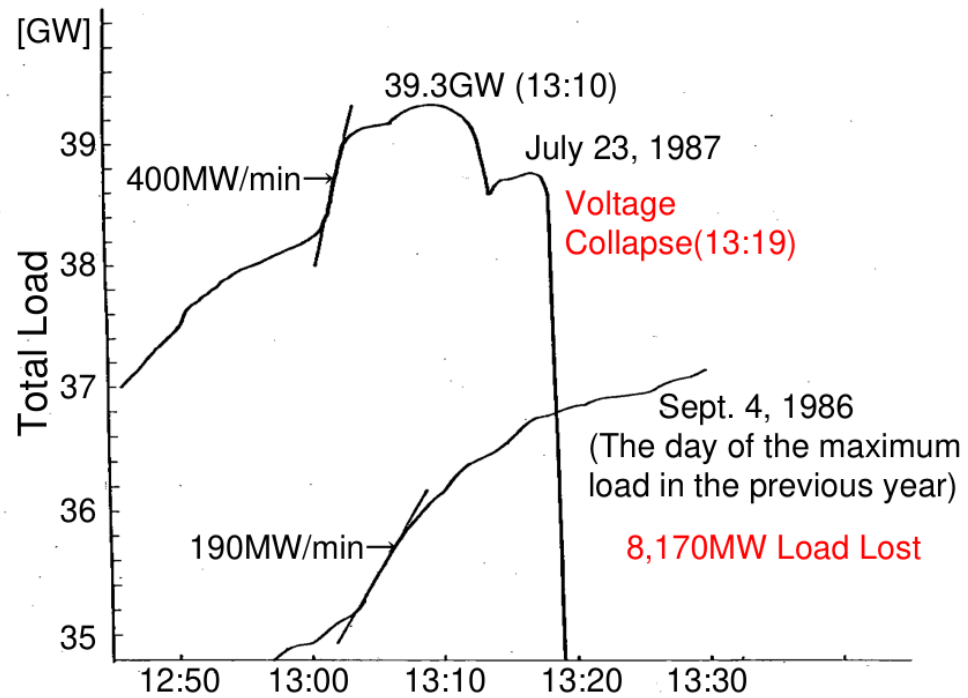
TEPCO reports there was *“no power flow solution”* for the unexpected load.

→ *Is that possible? How does this happen?*

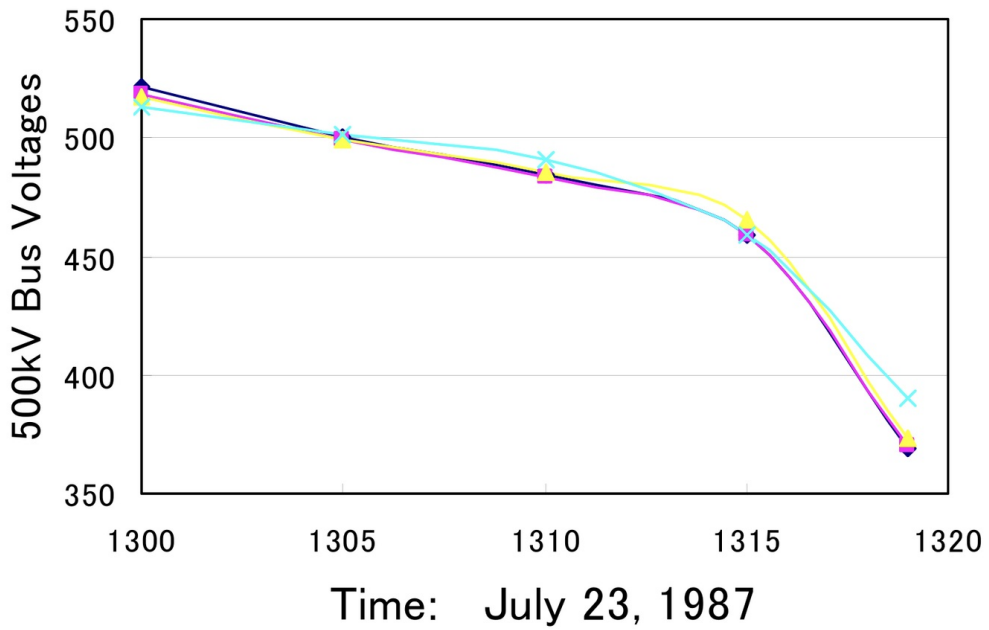
TEPCO (2010), TEPCO (2013)

All available capacitors were connected, but voltage continued to fall.

At 13:19, several 500 kV buses fell to about 370–390 kV.



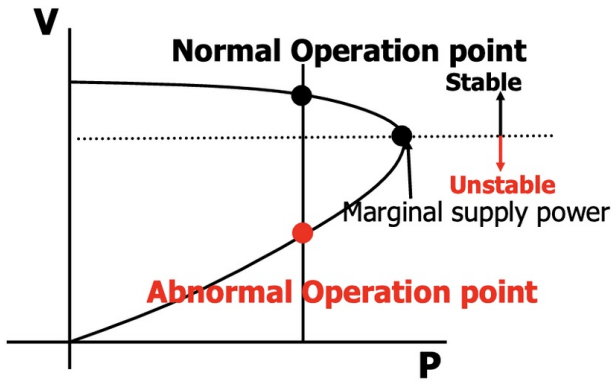
TEPCO (2013)



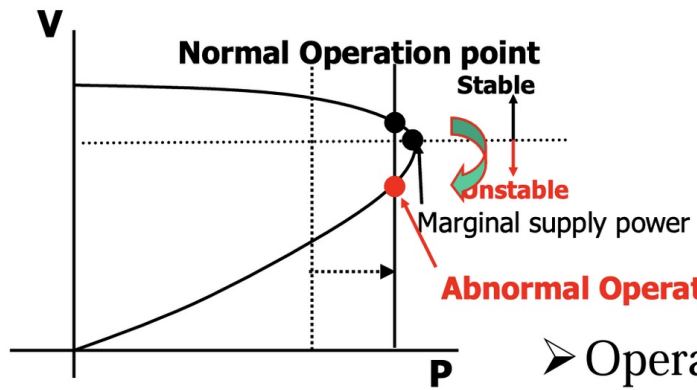
TEPCO (2010)

How the Collapse Happened in 1987 (1/3)

➢ **Conceptual explanation by P-V Curve**



➢ Increasing Demand



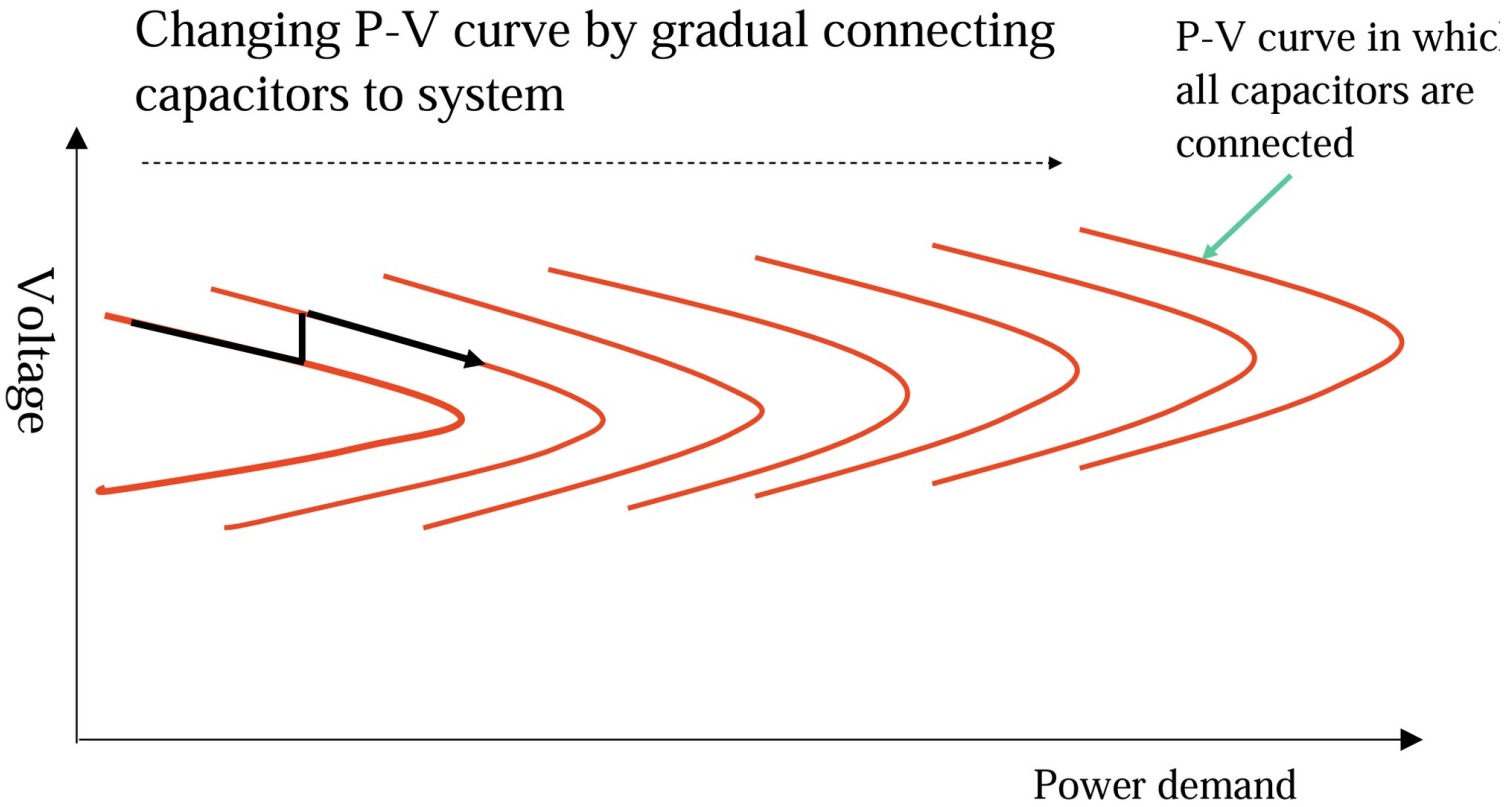
➢ Operation point moved to lower part of the curve

$P_{TEPCO} = -P$

TOKYO ELECTRIC POWER COMPANY

TEPCO (2010)

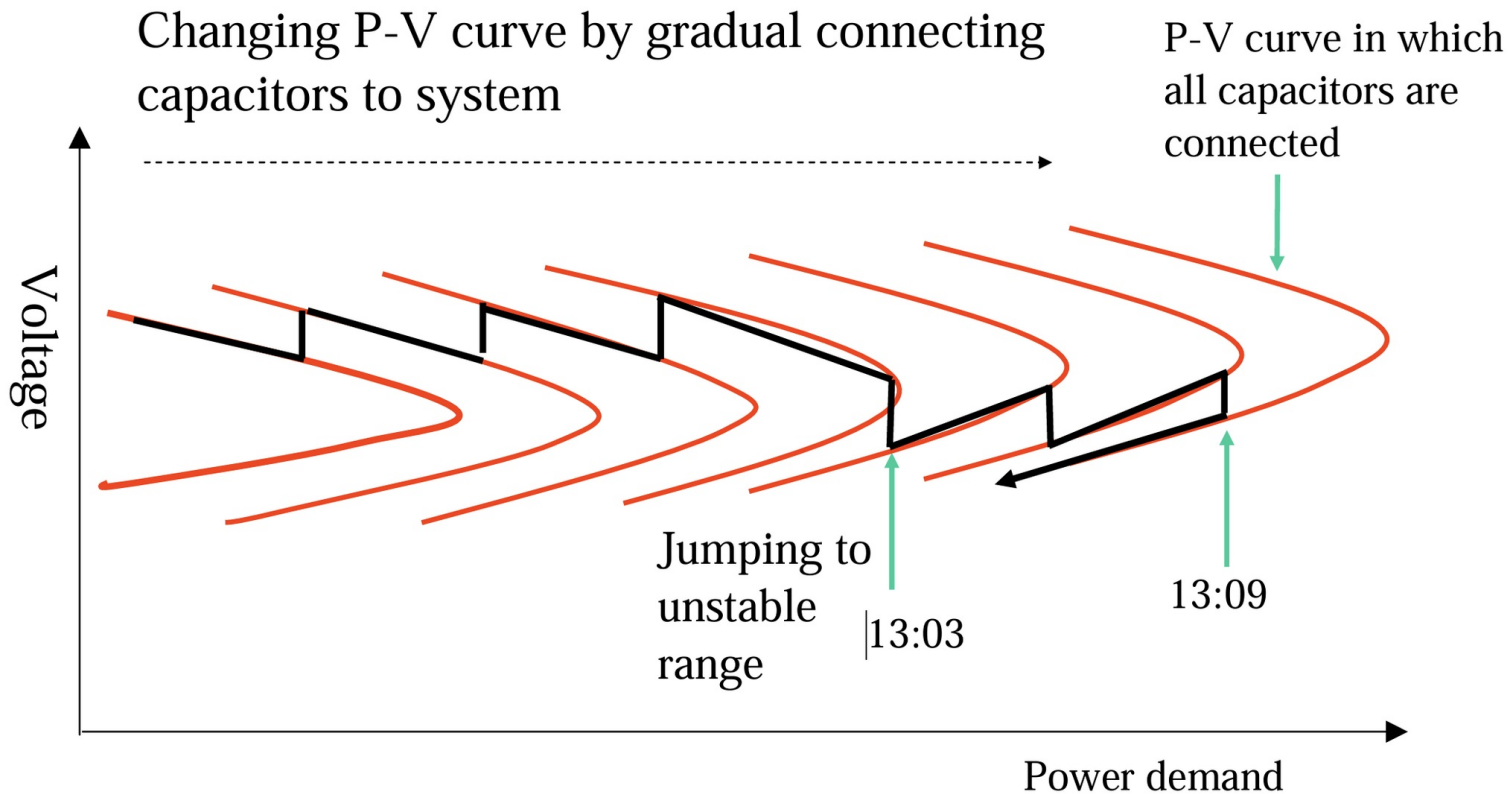
How the Collapse Happened in 1987 (2/3)



TOKYO ELECTRIC POWER COMPANY

TEPCO (2010)

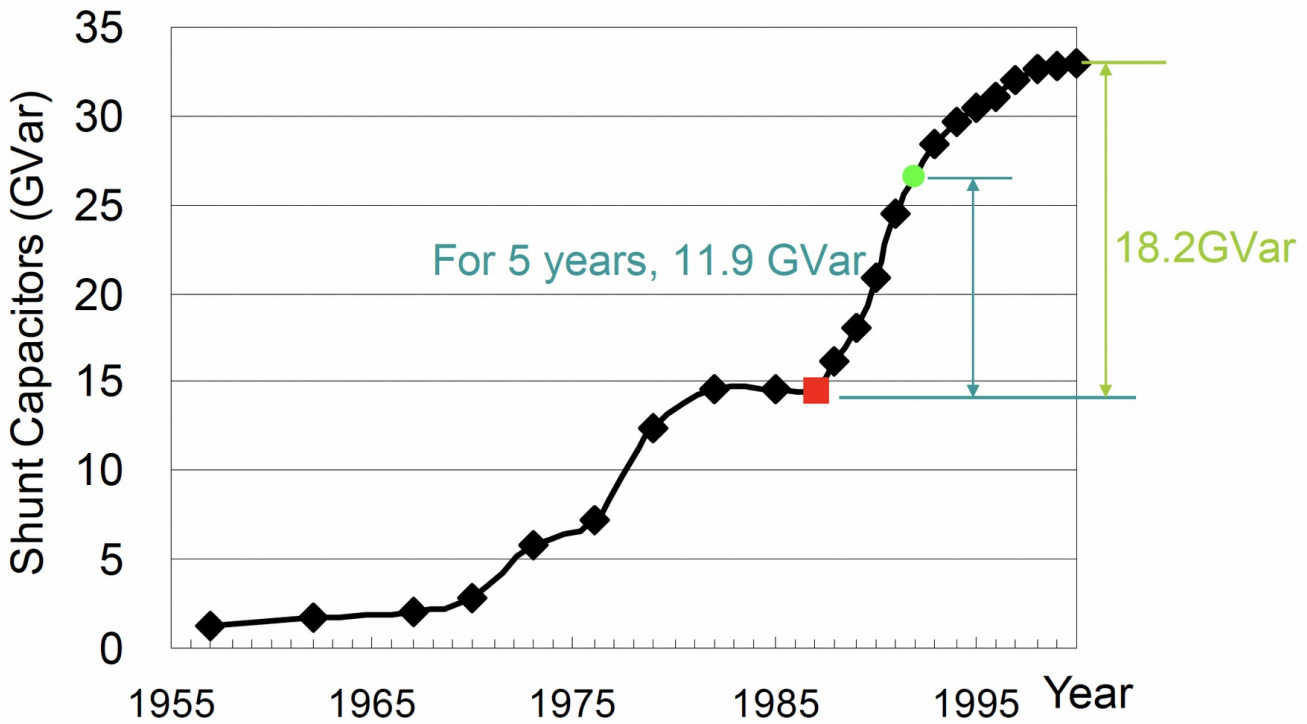
How the Collapse Happened in 1987 (3/3)



TOKYO ELECTRIC POWER COMPANY

TEPCO (2010)

Shunt Capacitors installed in TEPCO power system



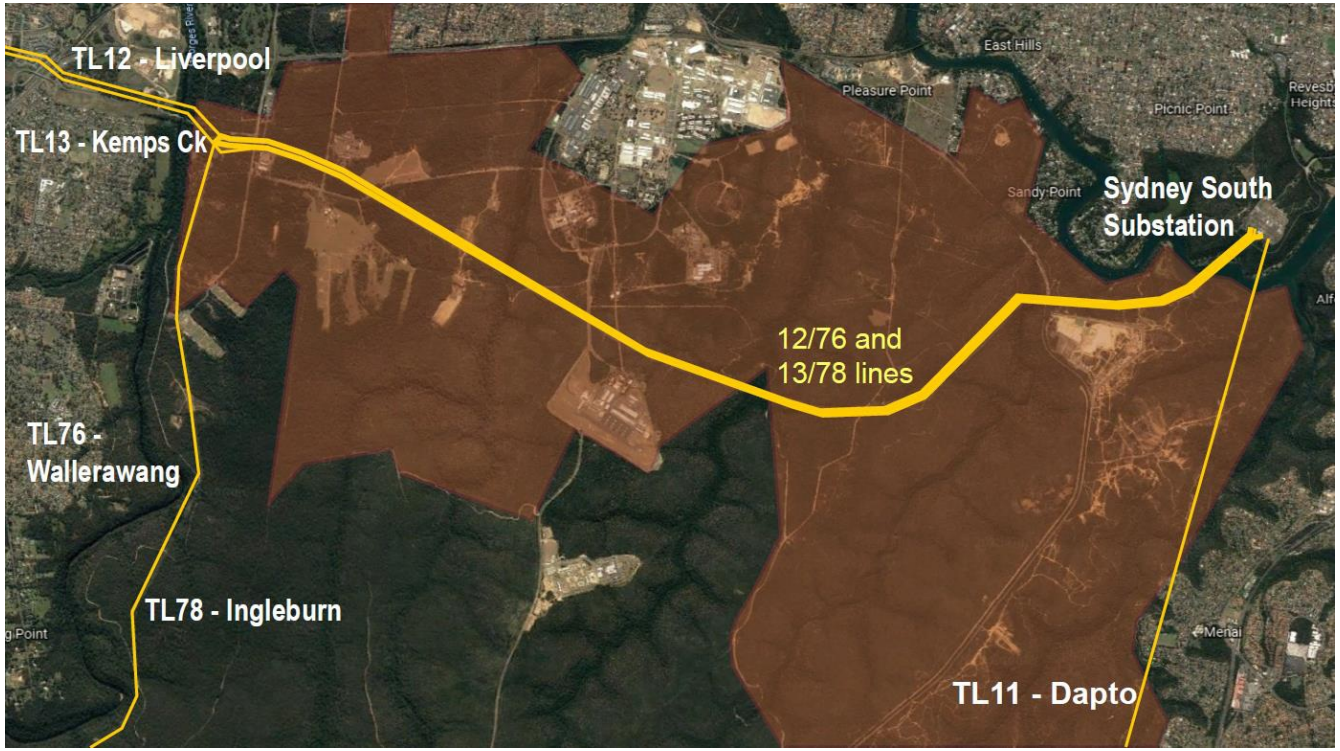
https://openjicareport.jica.go.jp/pdf/12121109_04.pdf#page=40

Sydney, 15 April 2018

Bushfires affected five 330 kV lines supplying Sydney South.

AEMO found that losing all five simultaneously would have caused voltage collapse in the 132 kV network around Sydney South.

At most two were out together. No customer load was lost.



https://www.aemo.com.au/-/media/Files/Electricity/NEM/Market_Notices_and_Events/Power_System_Incident_Reports/2018/NSW-bushfires-on-15-April-2018.pdf#page=12

The *P*–*V* curve: origin, limit and control

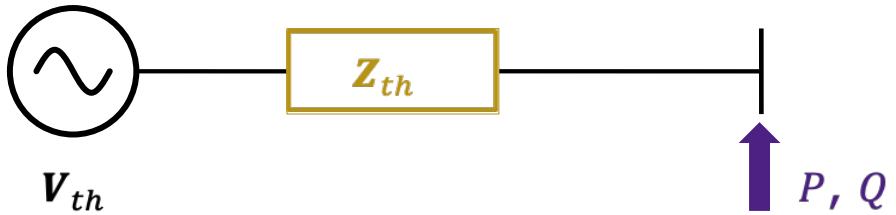
How does voltage collapse happen?

How can an inductive transmission line limit maximum power transfer?

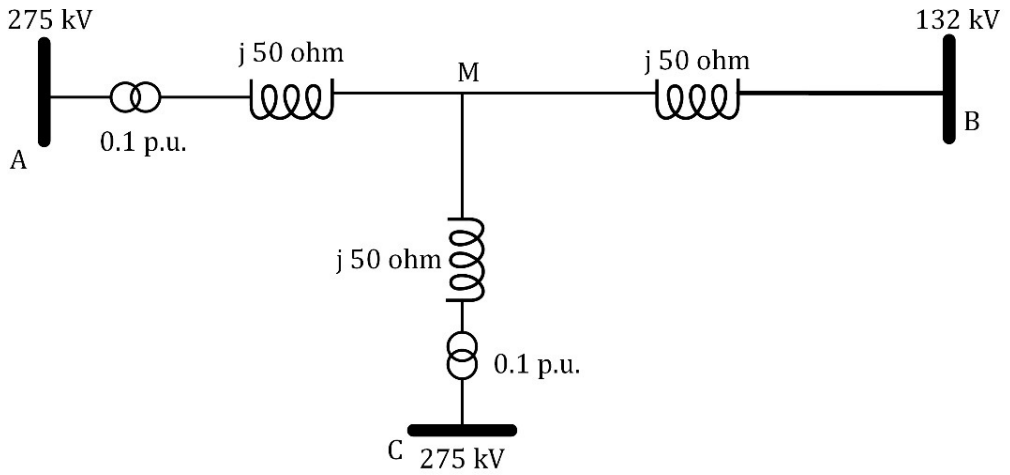
How can reactive support, source voltage, and network reactance change that limt?

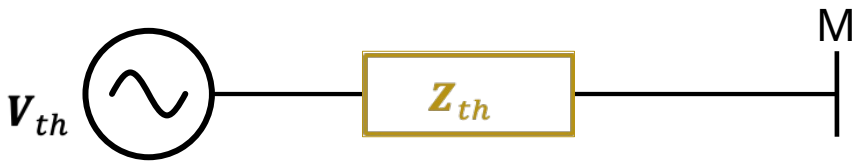
Thevenin-Equivalent Circuit Modelling

At an interconnection point the grid can be approximated with it's Thevenin-Equivalent circuit:



For example, recall from the voltage control lecture:

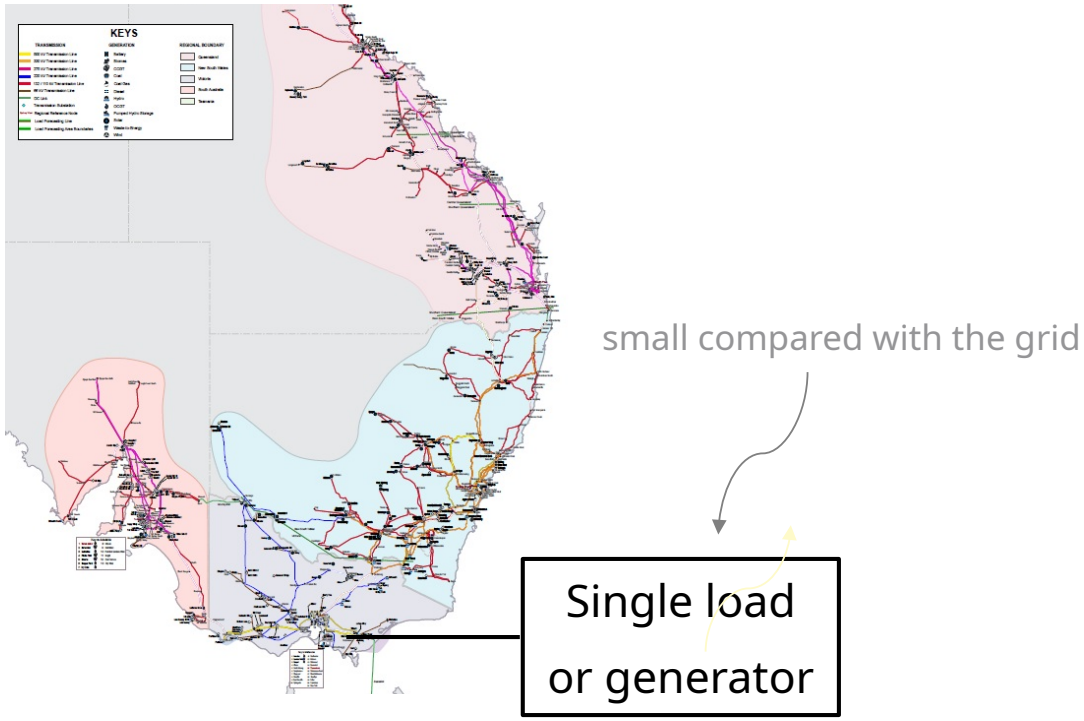


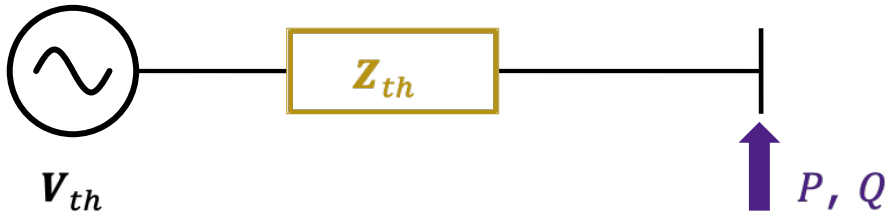


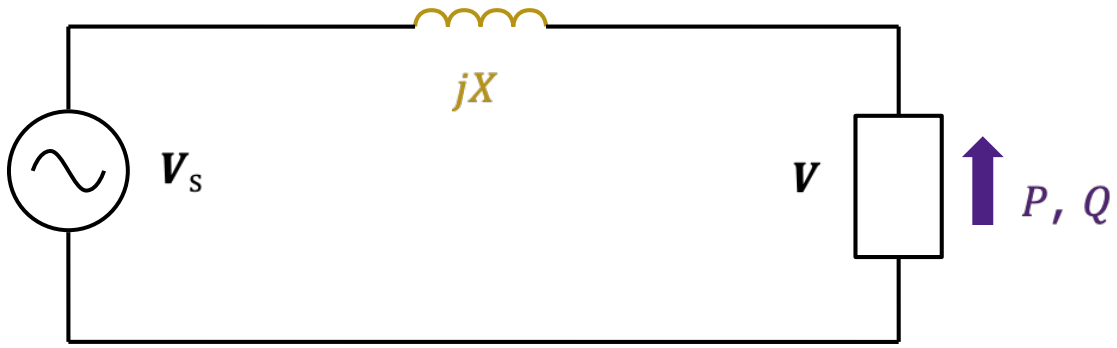
An infinite bus keeps its voltage magnitude and frequency fixed, regardless of the power exchanged.

One load or generator is usually small compared with the interconnected grid, so its power change has little effect on the grid's frequency and regulated voltage.

Thus, the grid can be approximated by a infinite bus/voltage source behind an impedance.



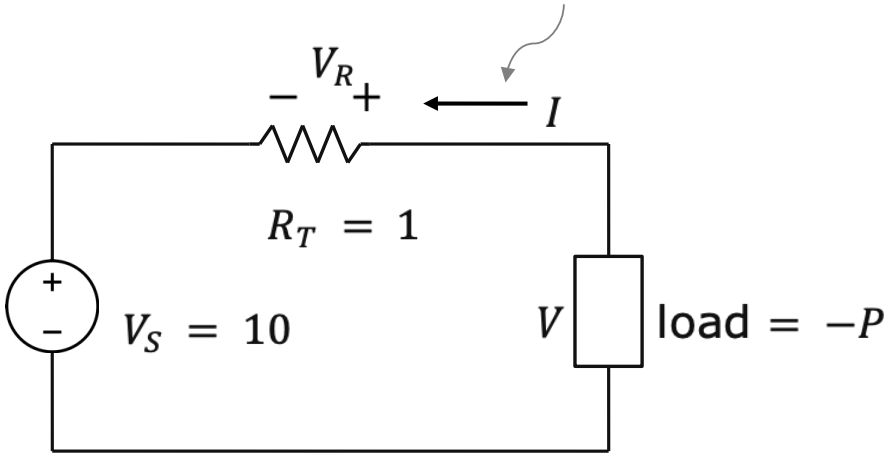




This lecture will focus on two-bus/Thevenin networks, with the understanding that the two-bus network approximates the full power system at any point of interconnection.

Exercise: maximum DC load power

The current arrow can face any direction but it *must* go from + to − across an impedance

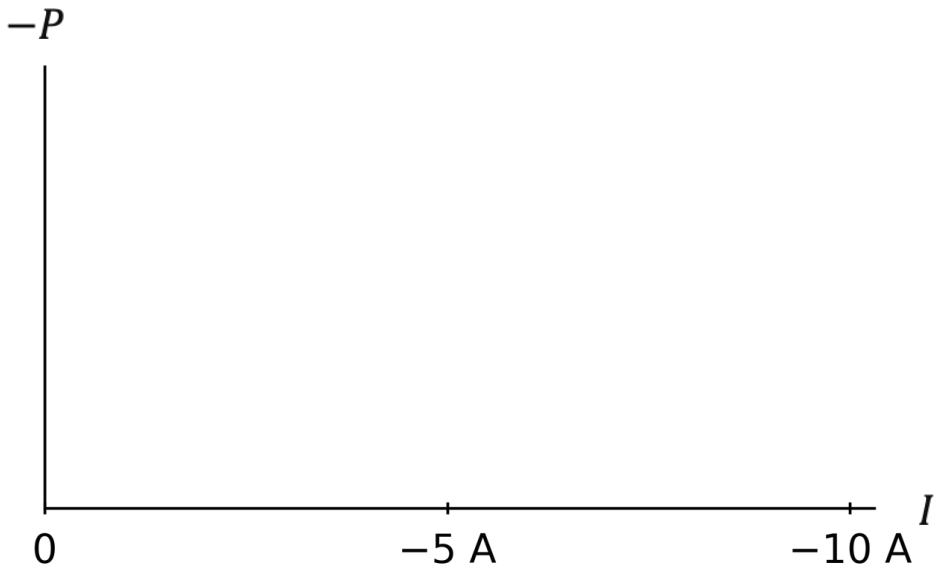


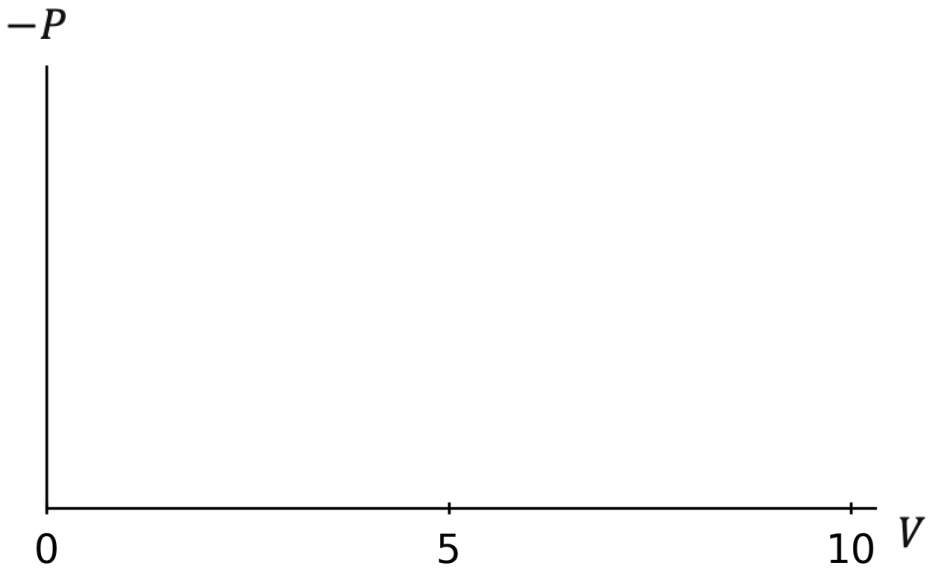
Sketch load power against current and voltage.

What current and voltage maximize load power?

$$V = V_S + IR_T$$

$$P = VI$$





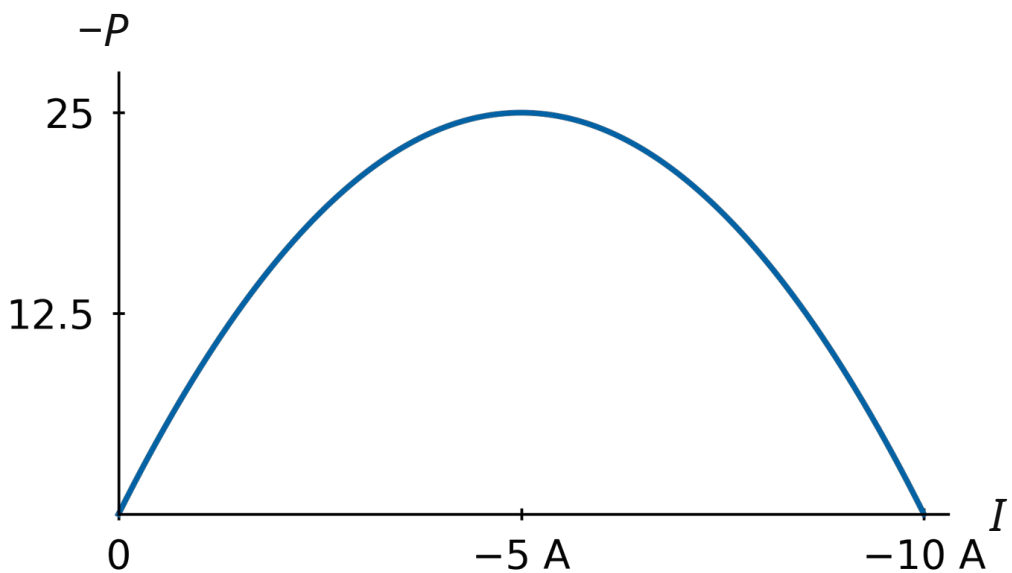
Exercise solution

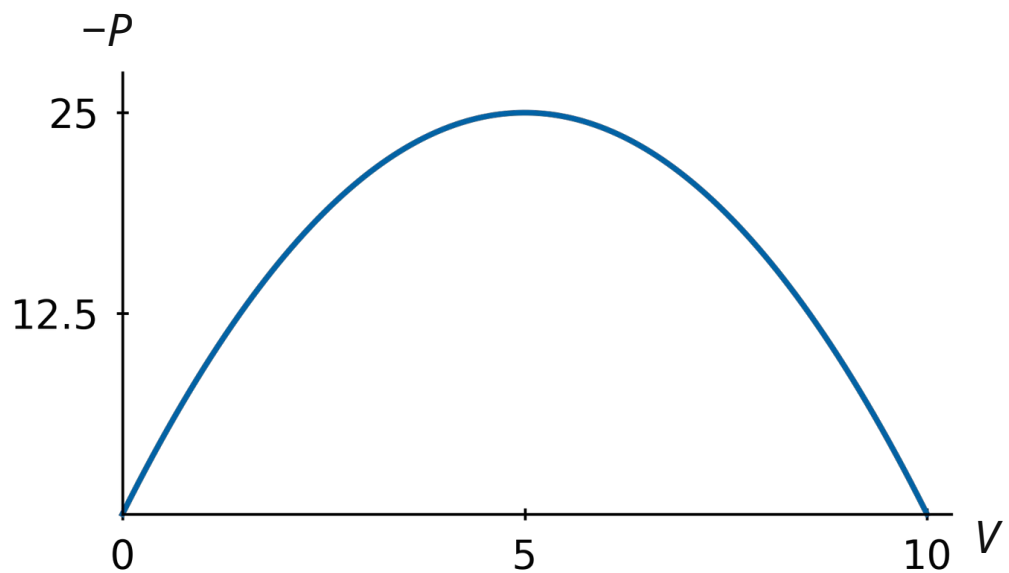
$$P = V_S I + R_T I^2 = 10 I + I^2$$

$$I = -5, V = 5, P = -25$$

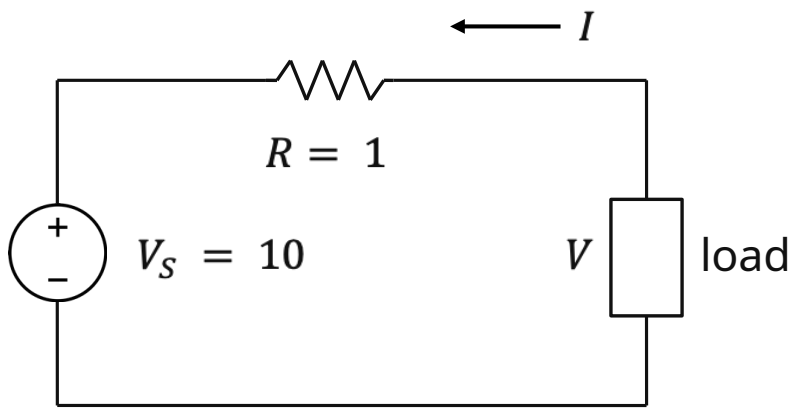
At the max transfer point: $\frac{dP}{dI} = 10 + 2I = 0$

$P = V(V - 10)$





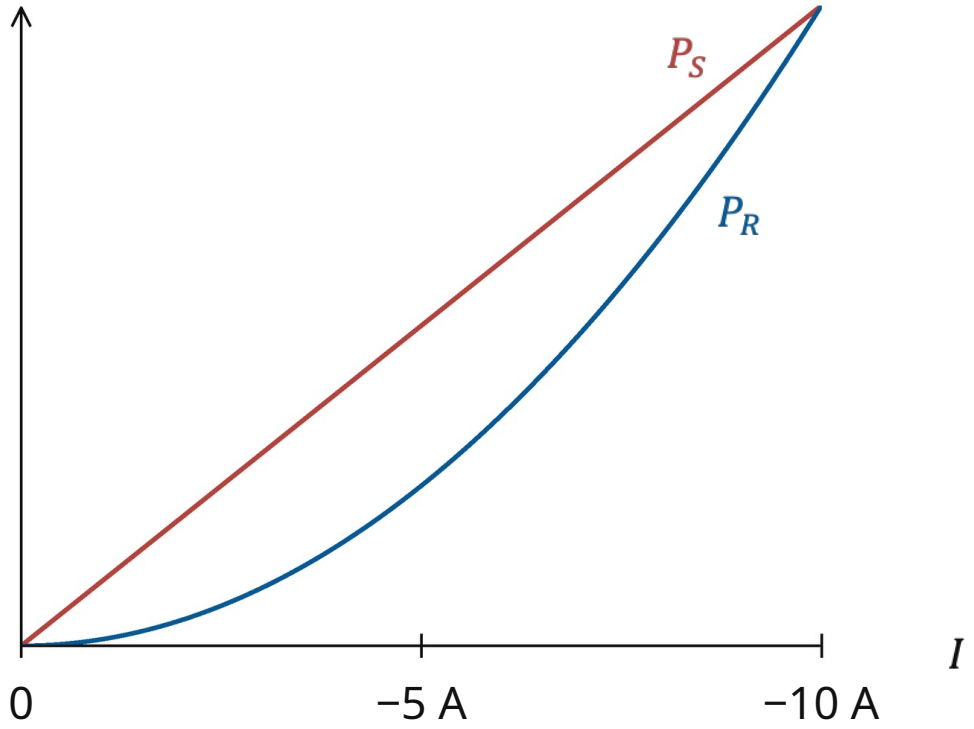
Building intuition: DC circuit maximum power transfer



$$P_S = -IV_S = -10I$$

$$P_R = I^2R = I^2$$

$$P = P_R - P_S$$

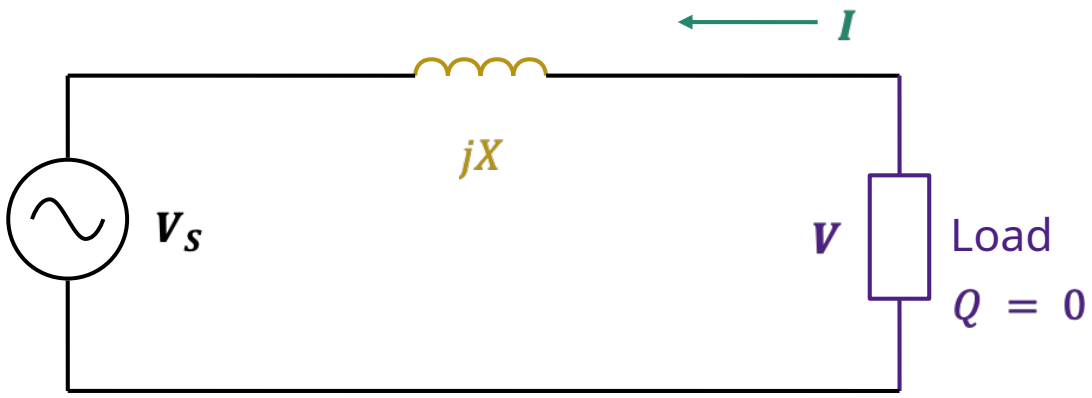


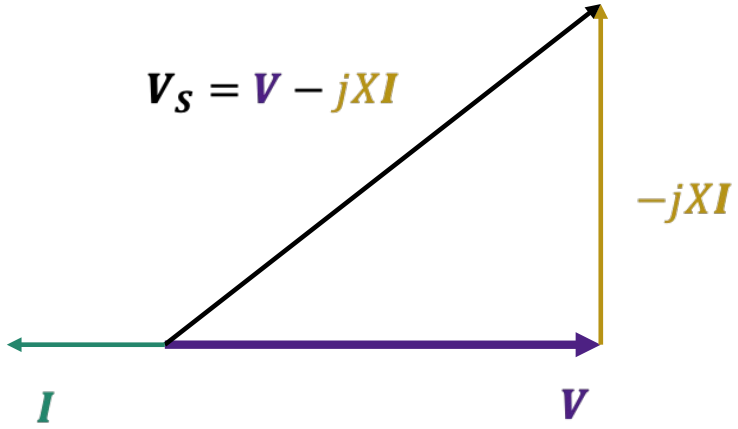
At $\mathrm{I} = -5\,A$, the source sends $50\,W$ ⇒ $25\,W$ reach the load

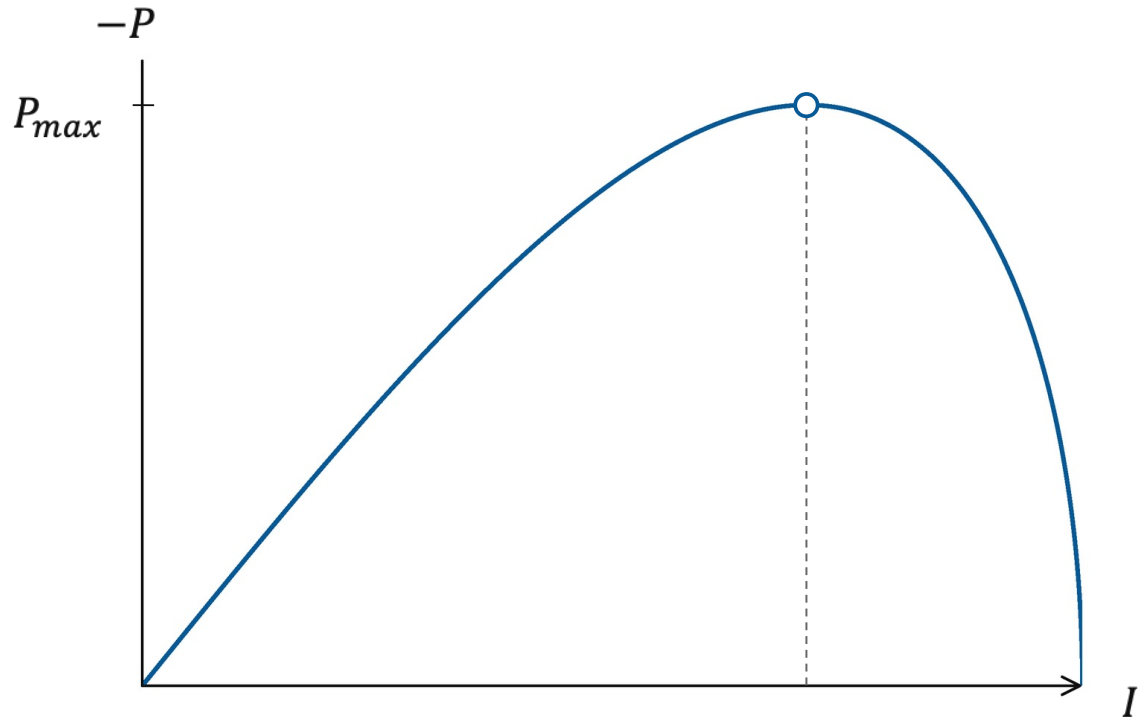
⇒ $25\,W$ dissipate as heat in the resistor

At $\mathrm{I} = -10\,A$, the source sends $100\,W$ but all of it dissipates as heat in the resistor

AC circuit maximum power transfer



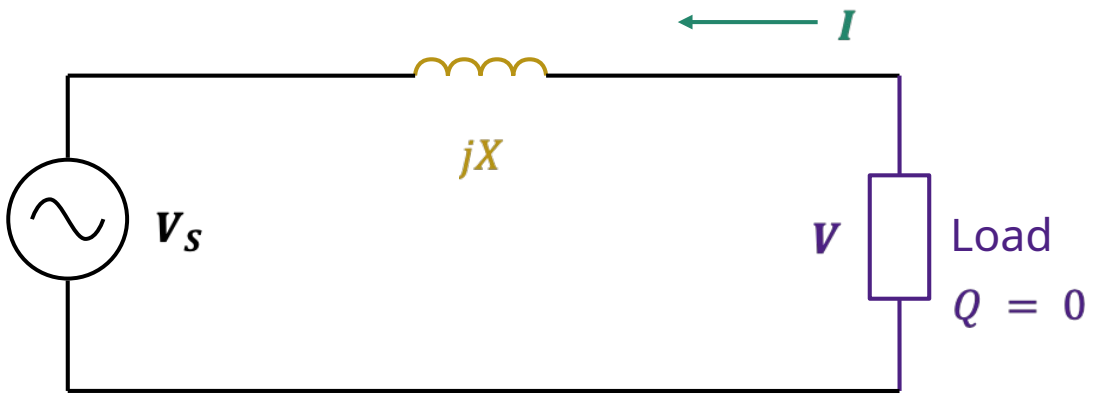


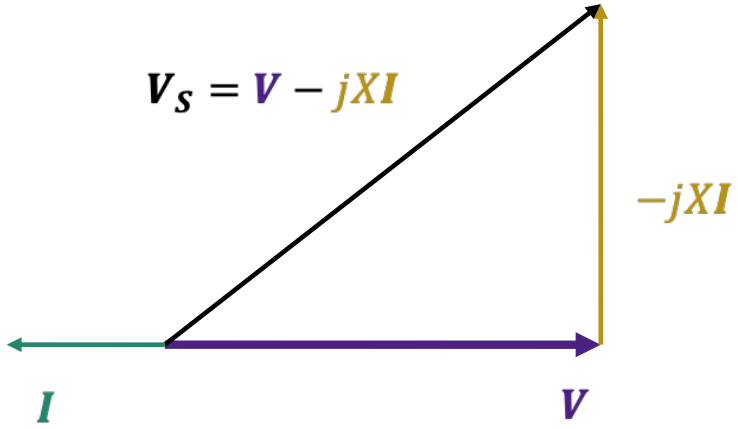


For $Q = 0$, $-P = VI = I\sqrt{V_S^2 - X^2I^2}$

For a given Q injection, AC transmission lines also have max power transfer, even if lossless.

AC circuit maximum power transfer





What if $Q \neq 0$?

From $\boldsymbol{S = V\bar{I}} = \frac{-j}{X}[V_S V(\cos\theta - j\sin\theta) - V^2]$,

$$P = -\frac{V_S V}{X}\sin\theta, \qquad Q = -\frac{V_S V}{X}\cos\theta + \frac{V^2}{X}$$

The power flow manifold

2 Equations: $P = -\frac{V_S V}{X} \sin\theta,\ Q = -\frac{V_S V}{X} \cos\theta + \frac{V^2}{X}$

4 variables: P, Q, V and θ

We cannot plot 4D, so instead we find projections onto 3D or 2D

For example, equating angles in the P and Q equations above gives:

$$V^4 - (V_S{}^2 + 2XQ)V^2 + X^2(P^2 + Q^2) = 0$$

3 unknowns, 1 equation

Alternatively, equating voltage magnitudes in the P and Q equations above gives:

$$V_S^2 Q \sin^2\theta - V_S^2 P \sin\theta \cos\theta - XP^2 = 0$$

3 unknowns, 1 equation

Derivation of the voltage-magnitude equation

Multiply the power equations by X and rearrange:

$$P\,X = -V_S\,V\,\sin\theta$$

$$Q\,X - V^2 = -V_S\,V\,\cos\theta$$

Square both equations and add them, then use $\sin^2\theta + \cos^2\theta = 1$:

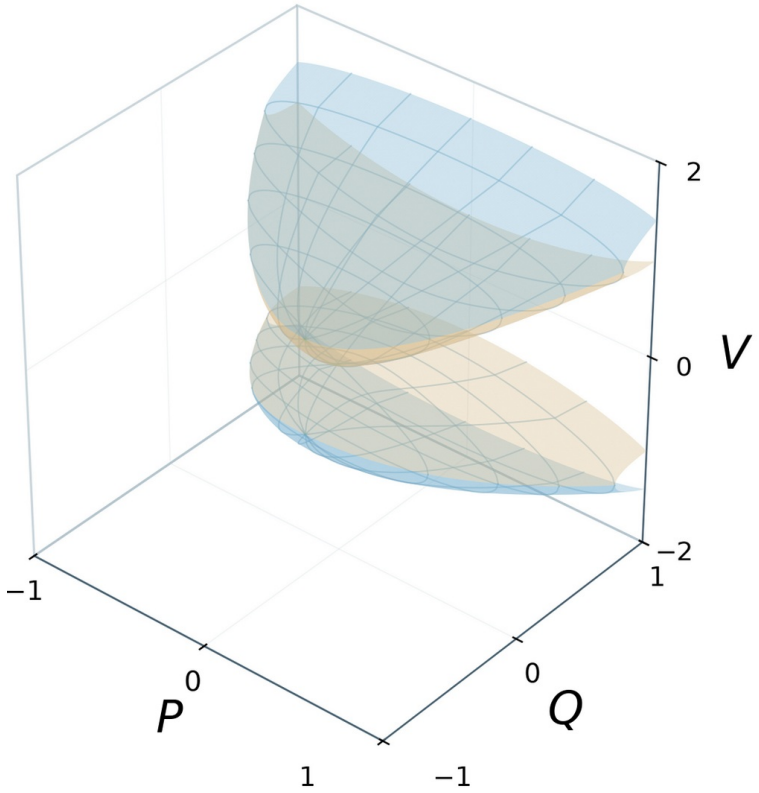
$$(P\,X)^2 + (Q\,X - V^2)^2 = V_S{}^2\,V^2$$

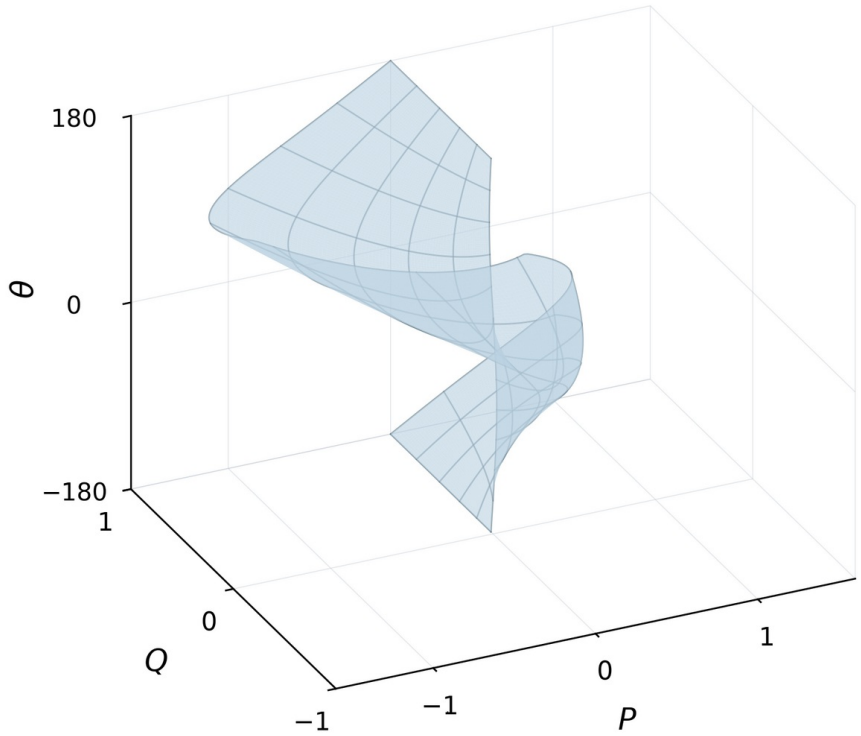
The power flow manifold

The power flow equations define P and Q in terms of V and θ.

Thus, the power flow manifold is a 2D manifold in 4D ambient space.

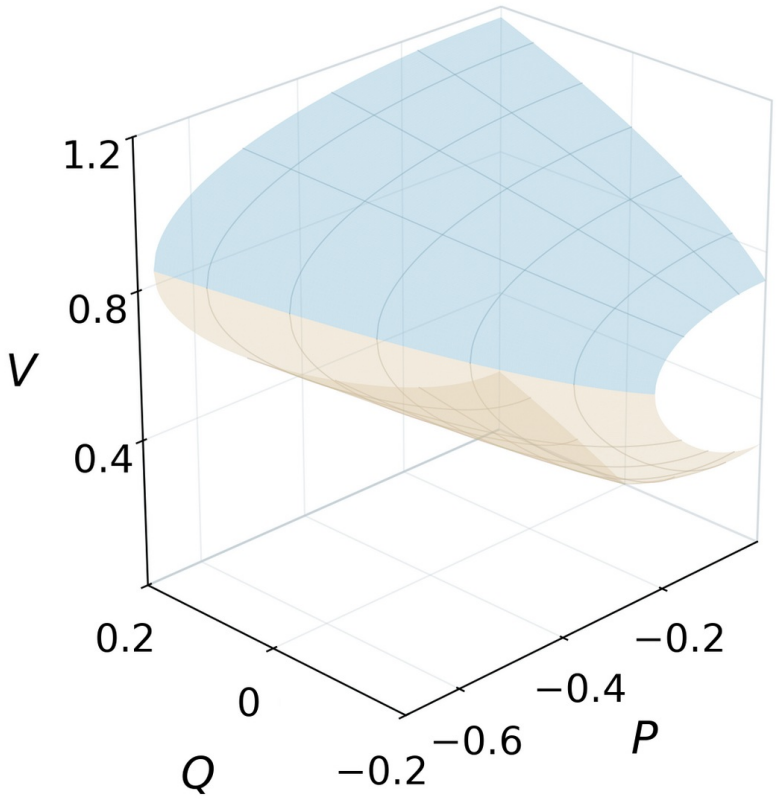
We cannot plot 4D, but we can represent the manifold as two manifolds in 3D:

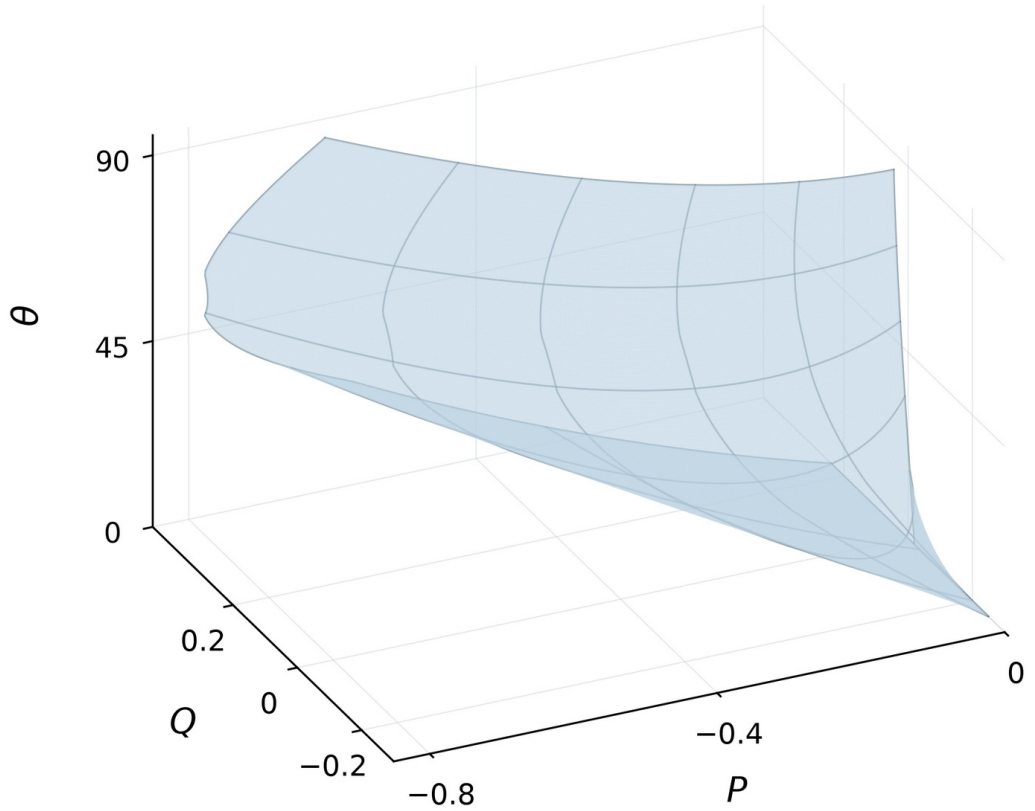




The power flow manifold

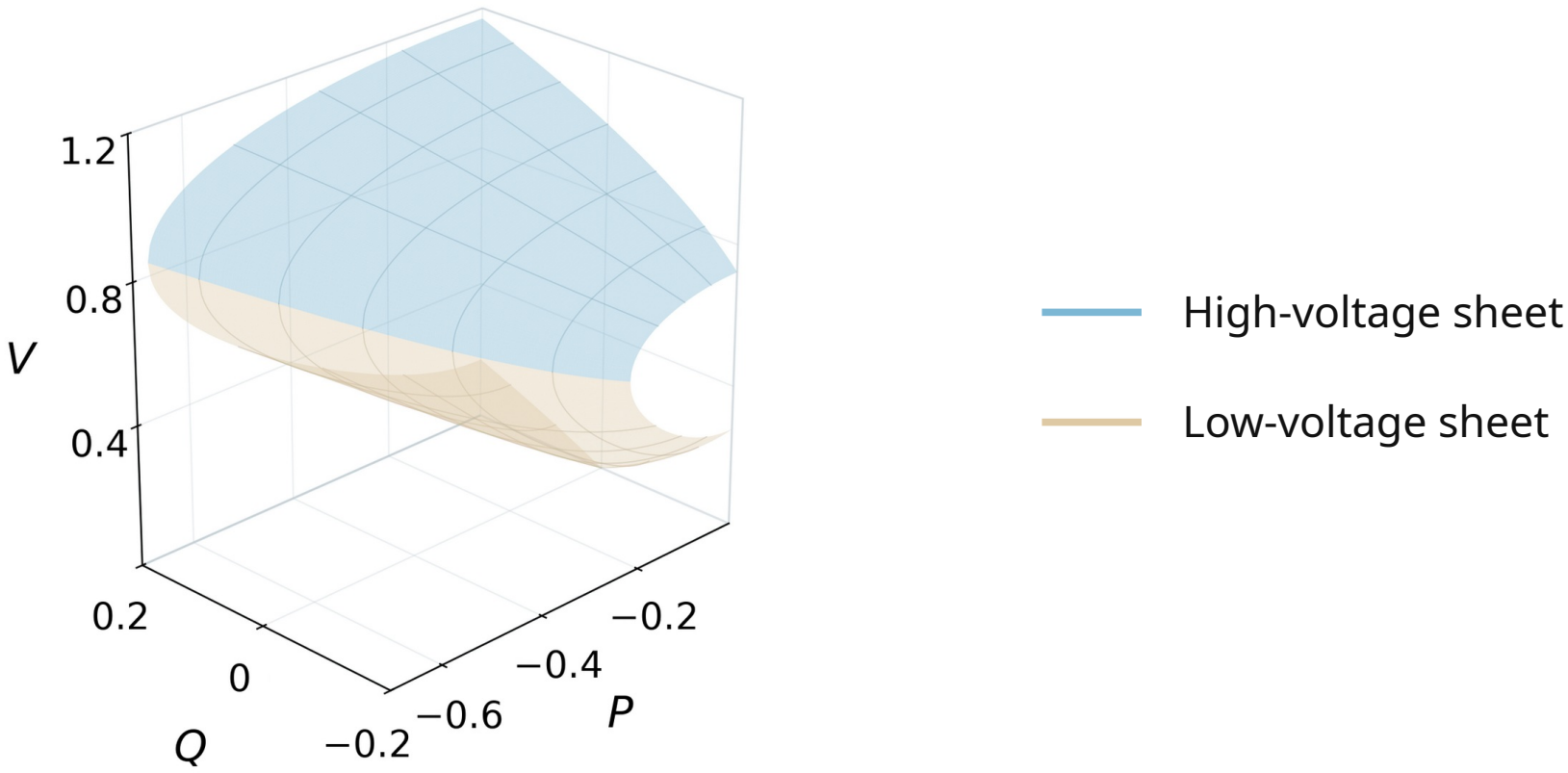
We are interested in only a specific portion of this power flow manifold.





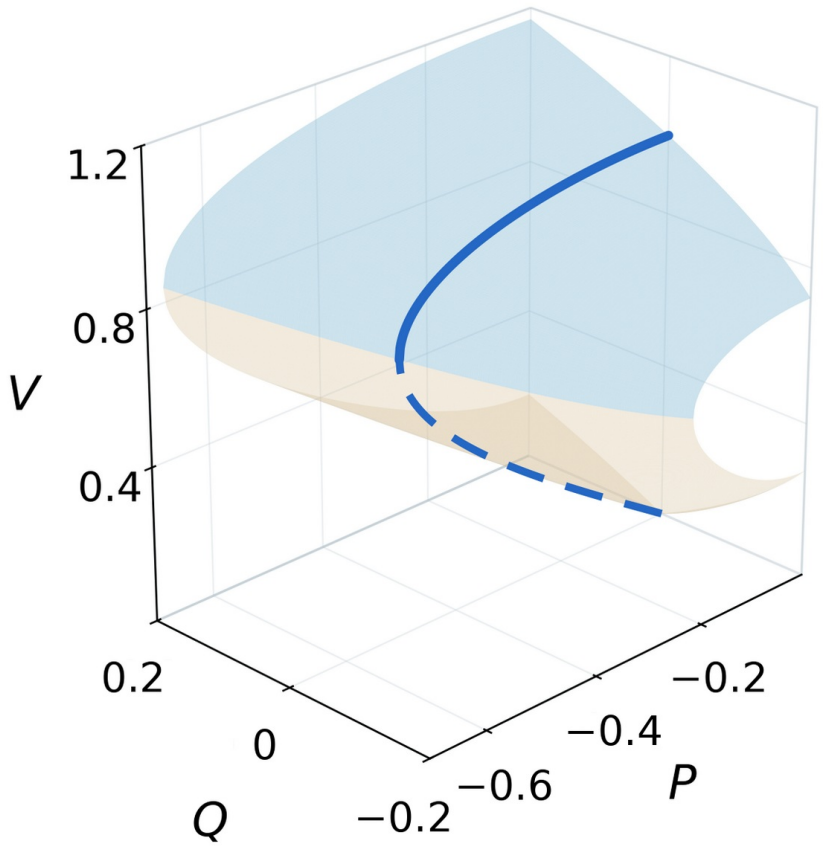
The power flow manifold

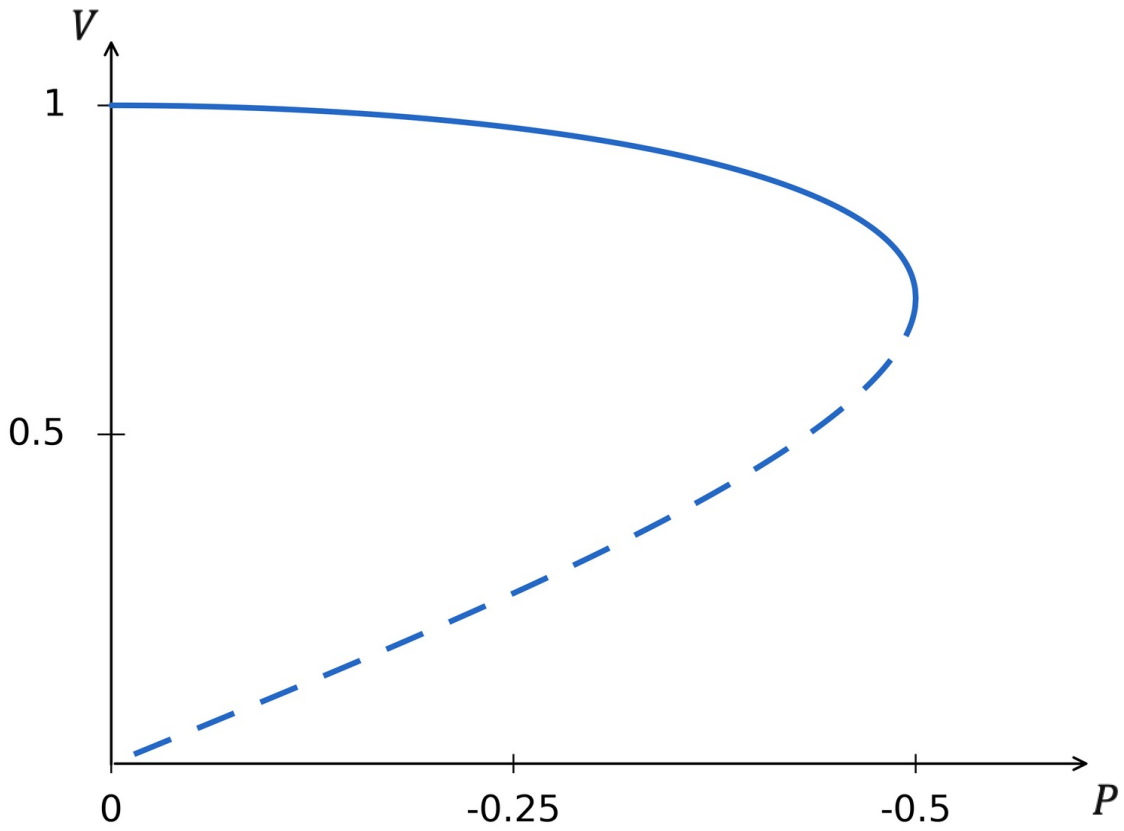
We are interested in only a specific portion of this power flow manifold.



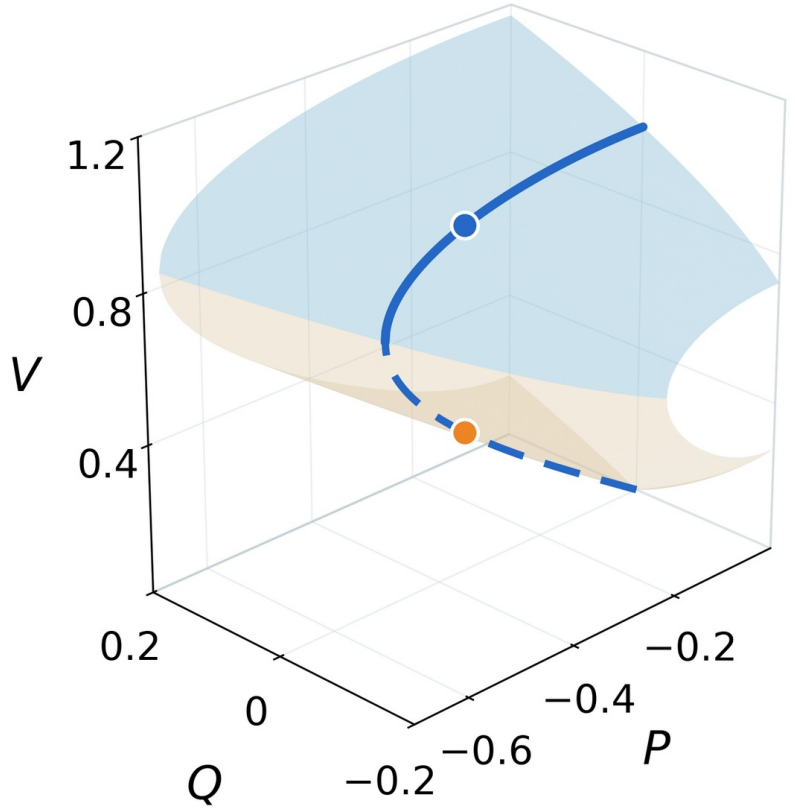
The power flow manifold

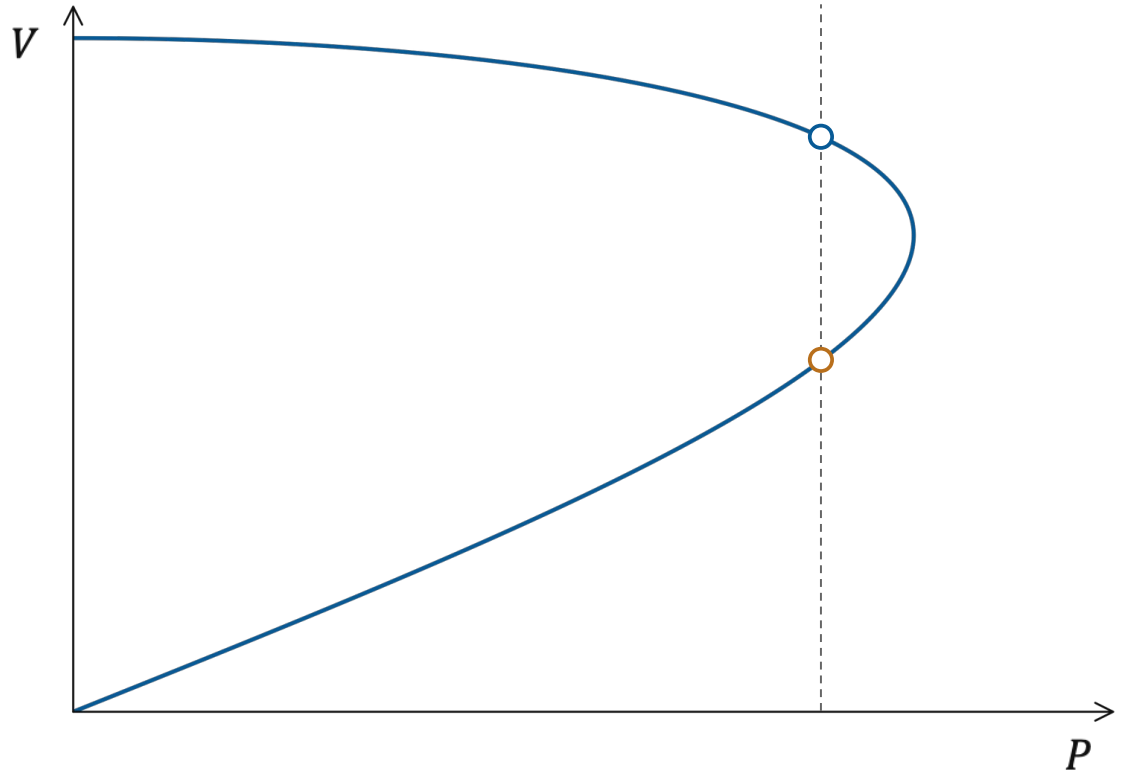
A slice of the manifold can be taken at a specific Q, e.g. $Q = 0$, giving the 2D P-V curve





The power flow manifold





- High-voltage solution
- Low-voltage solution

$$V^4 - (V_s{}^2 + 2XQ)V^2 + X^2(P^2 + Q^2) = 0$$

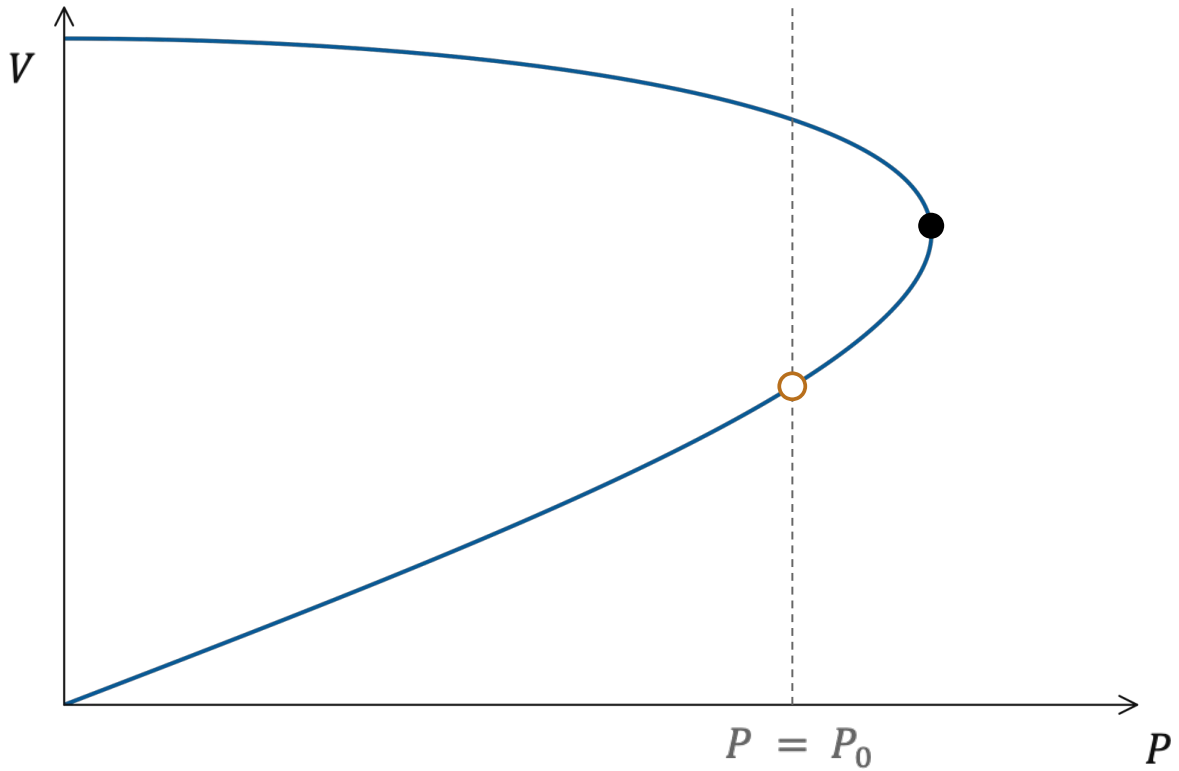
has 2 solutions

Power systems operate on the high-voltage branch.

Exercise: a load on the low-voltage branch

A regulated load sits at the lower-voltage solution for $P = P_0$.

What happens to its voltage and to the real power the line delivers?



see the appendix for a more rigorous answer to this question using transfer functions

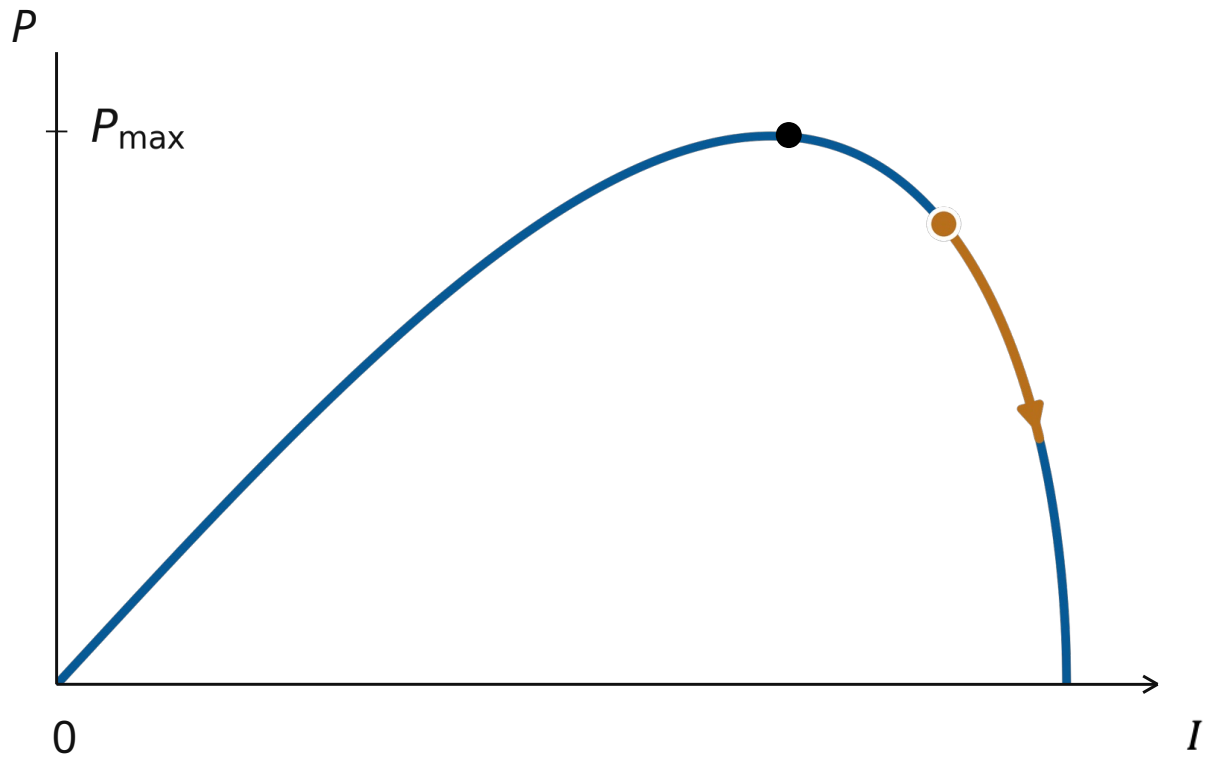
Exercise solution

$$V = \sqrt{V_S^2 - X^2 I^2}$$

$$P = -VI$$

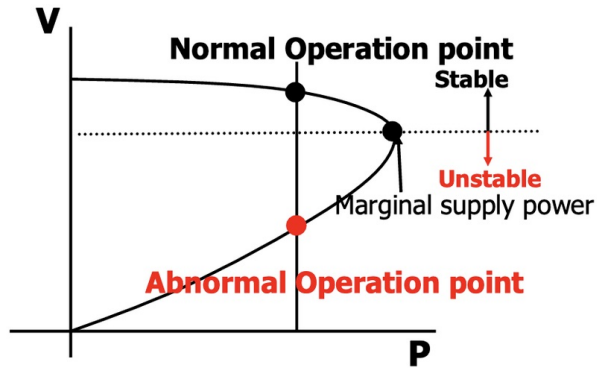
Beyond the maximum transfer point, more I lowers both V and the real power delivered.

The feedback continues until the voltage and real power delivered collapse to 0.

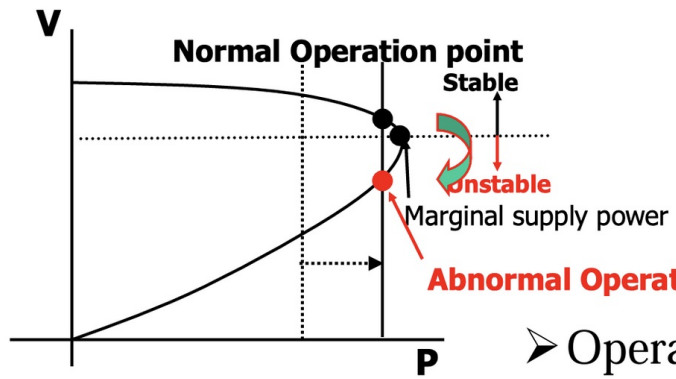


How the Collapse Happened in 1987 (1/3)

➢ **Conceptual explanation by P-V Curve**



➢Increasing Demand



➢Operation point moved to lower part of the curve

TOKYO ELECTRIC POWER COMPANY

TEPCO (2010)

Voltage stability depend on the load

Constant power

=

"Stiff" load

(e.g., induction motors)

$$P(V) = P_0 \left(\frac{V}{V_0}\right)^{n_P}$$

$$Q(V) = Q_0 \left(\frac{V}{V_0}\right)^{n_Q}$$

Constant impedance

=

"Soft" load

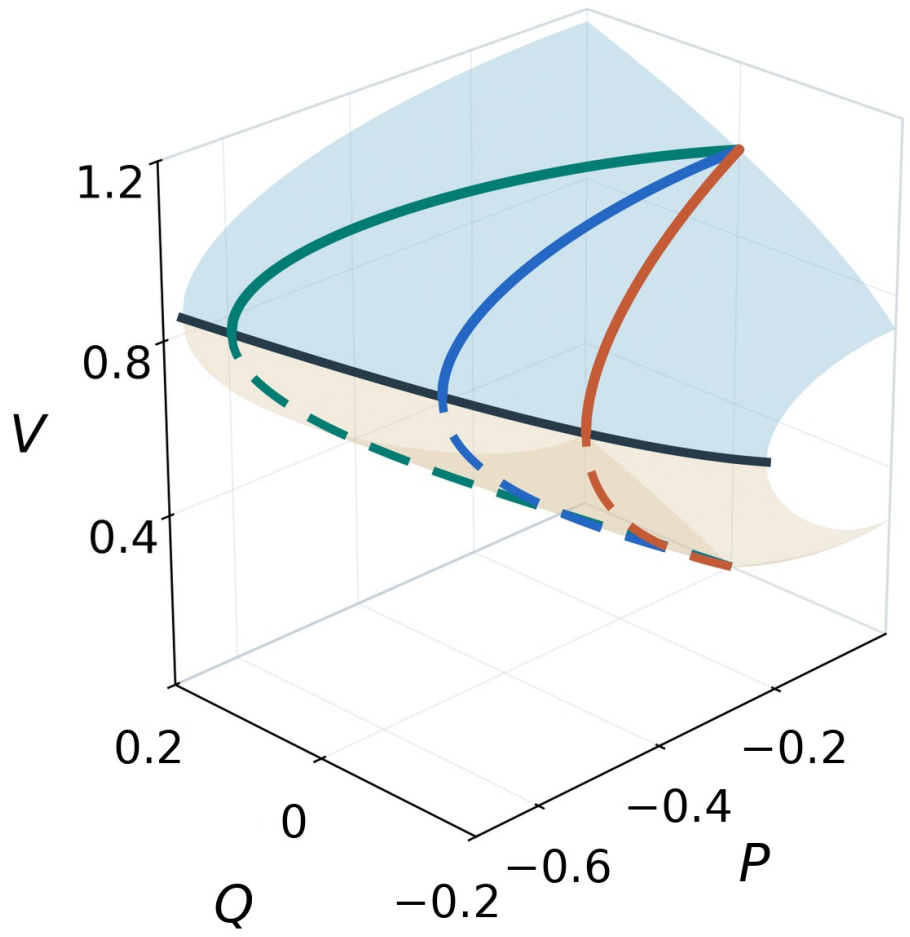
(e.g., resistive heating)

What are the exponents for constant power and constant impedance model?

*Which is worse from a **voltage collapse** perspective?*

An explanation of voltage collapse using load dynamics is given in the appendix slides.

The power flow manifold



High-voltage sheet

Low-voltage sheet

Voltage collapse edge

Capacitive load

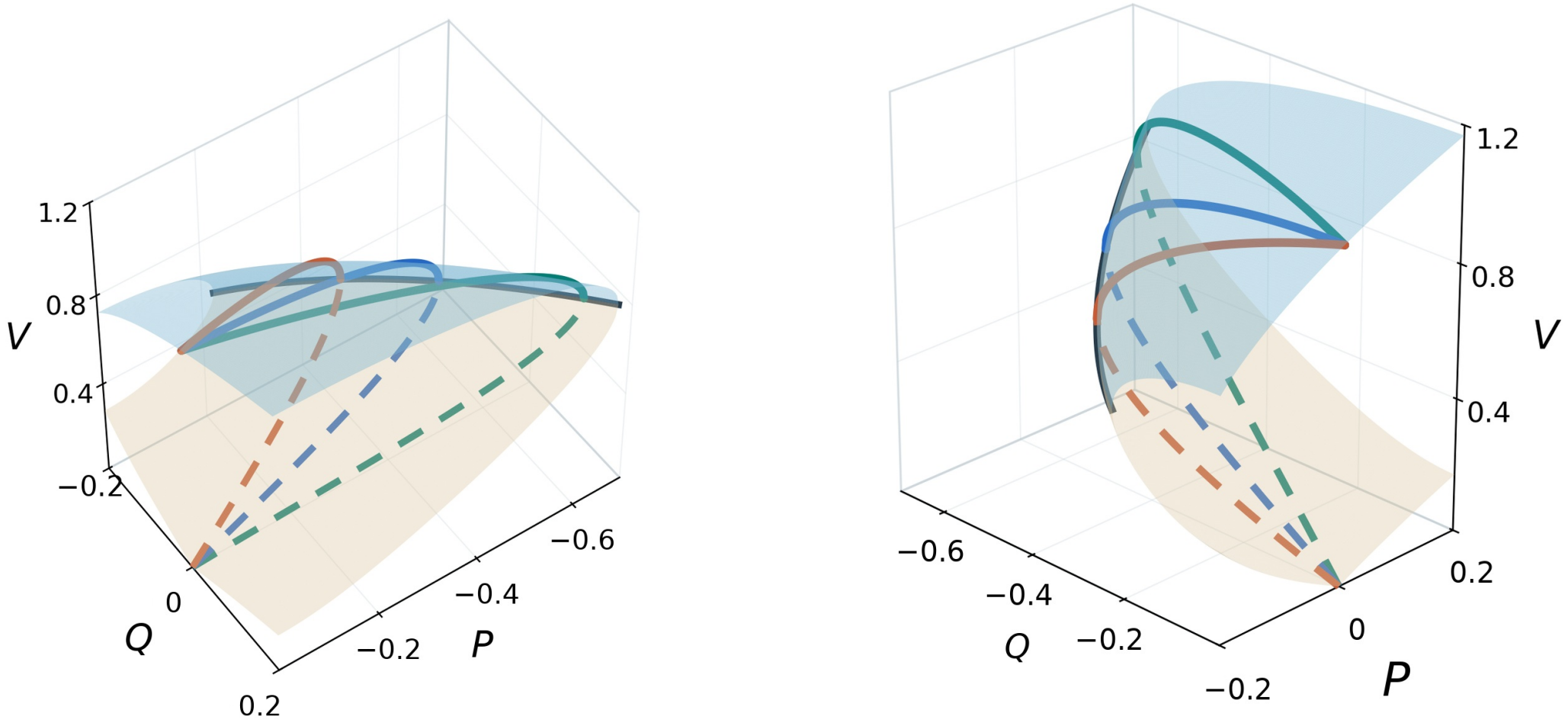
Unity PF load

Inductive load

Solid: high voltage

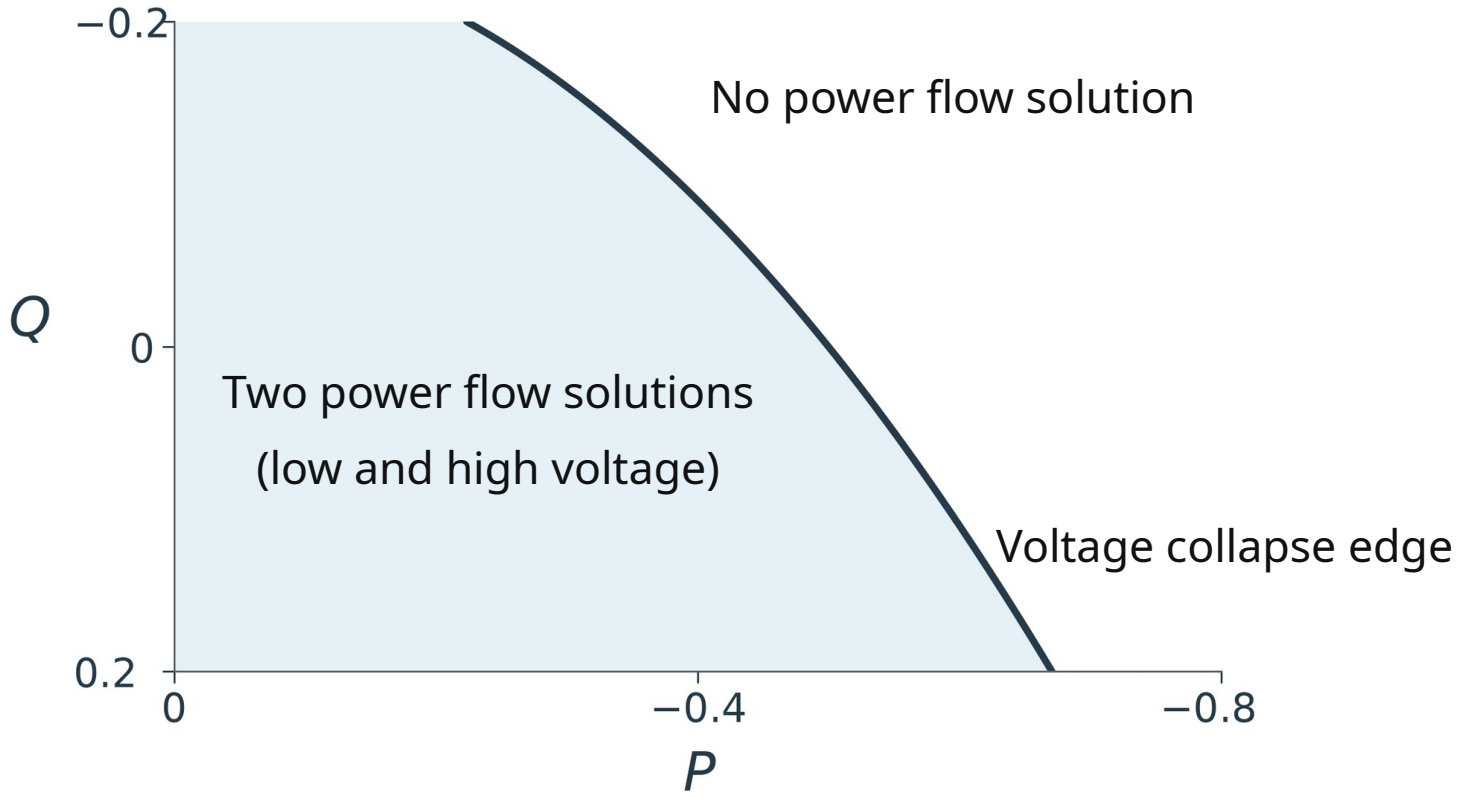
Dashed: low voltage

The power flow manifold



The power flow manifold

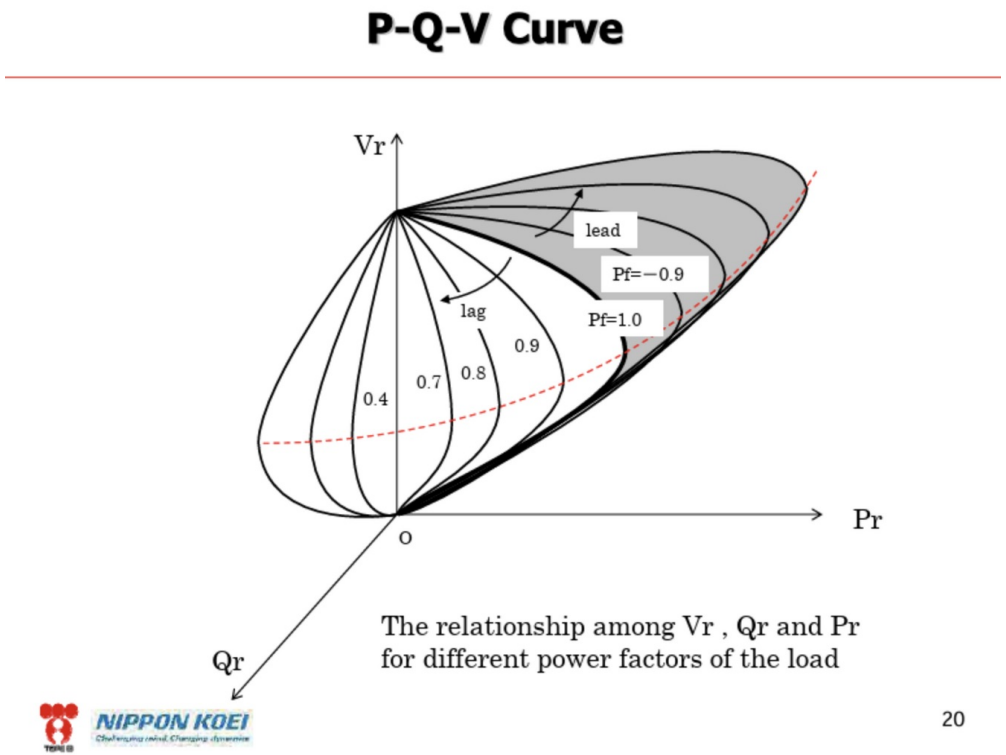
The P-Q-V Curve/3D manifold can be projected onto the P-Q plane:

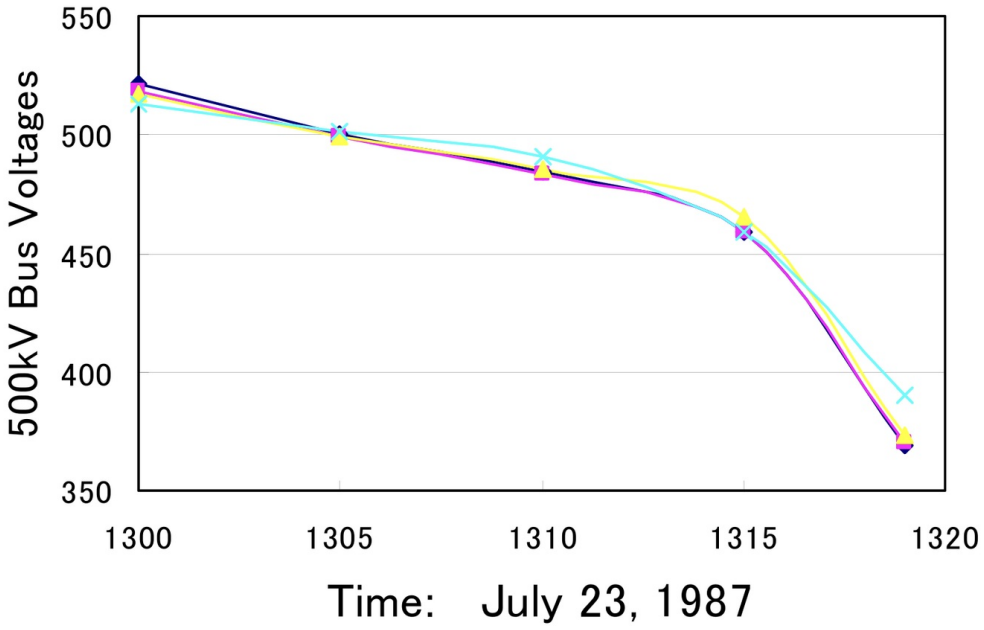


This projection makes it clear that increasing Q increases P_{max}

The power flow manifold

The P-Q-V Curve is relevant for real-world grid operation





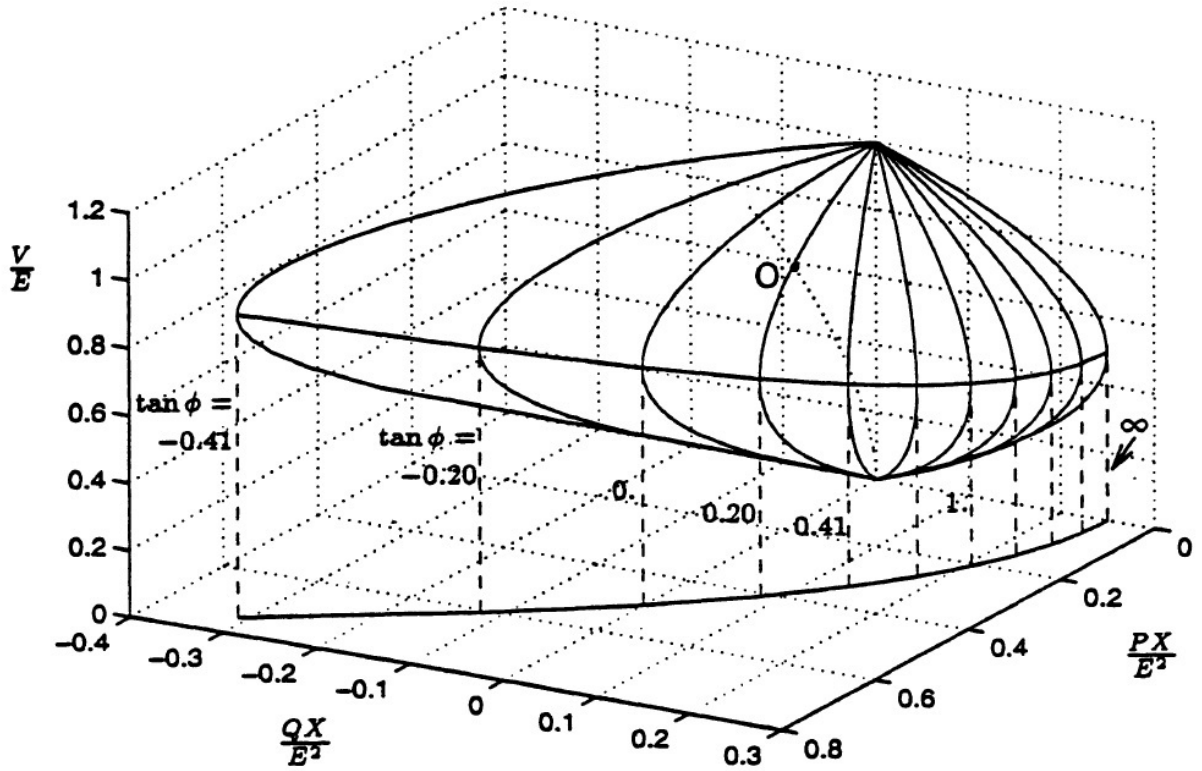
TEPCO (2010)

$$P_r = -P$$
$$Q_r = -Q$$

P-Q-V curves typically use the extraction-positive sign convention. In this class we use injection-positive.

The power flow manifold

Different curves for different power factors, 3D

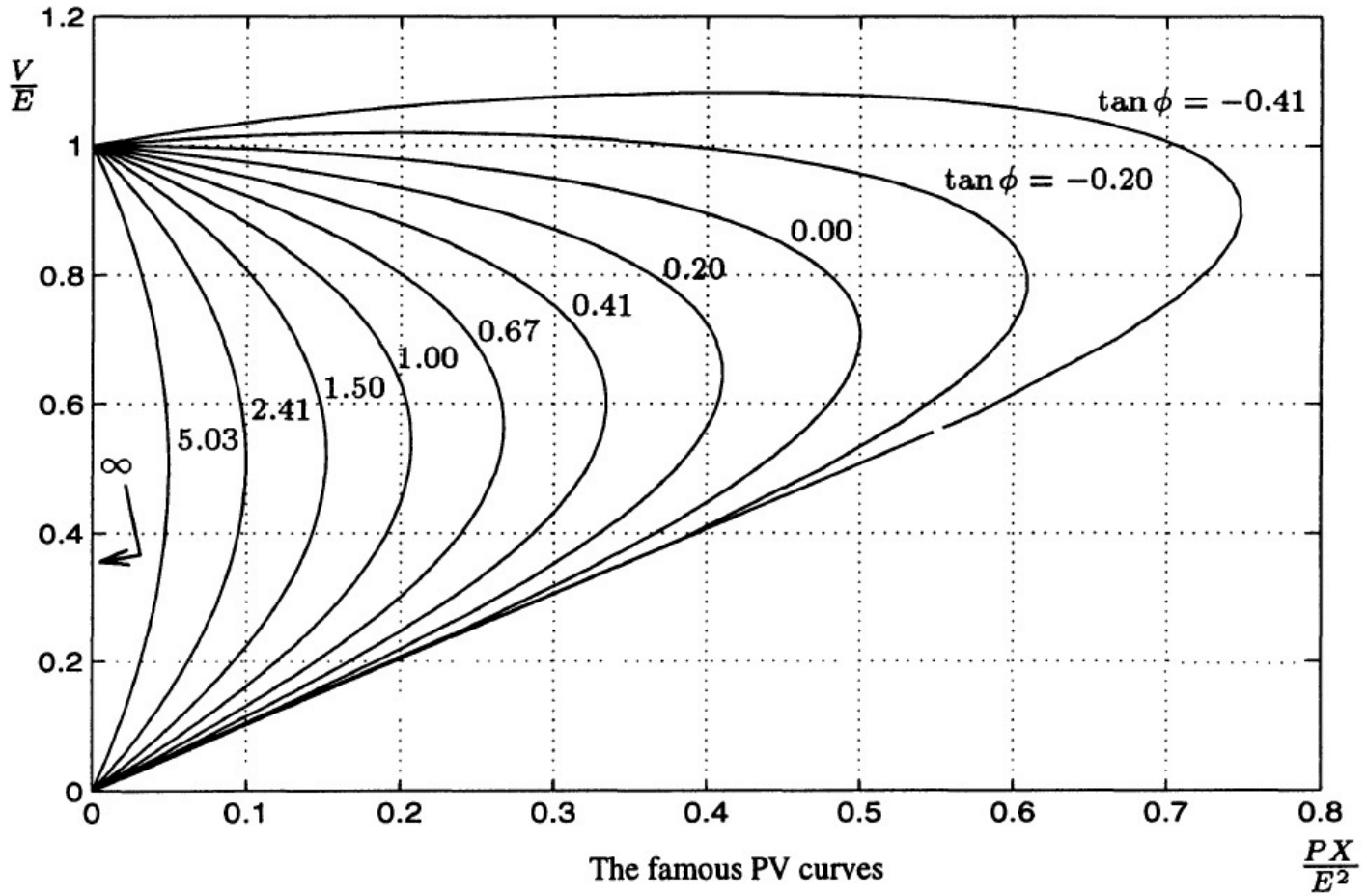


Voltage as a function of load active and reactive powers

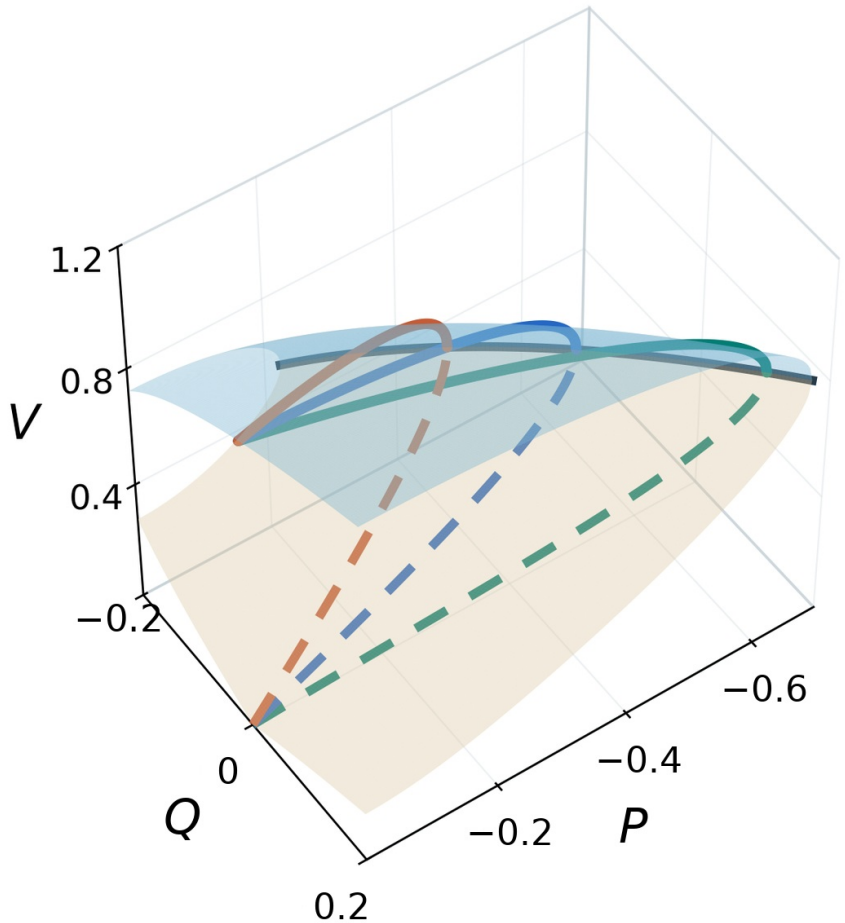
T. Van Cutsem, C. Vournas, " Voltage stability of electric power systems", Springer Science & Business Media, 1998

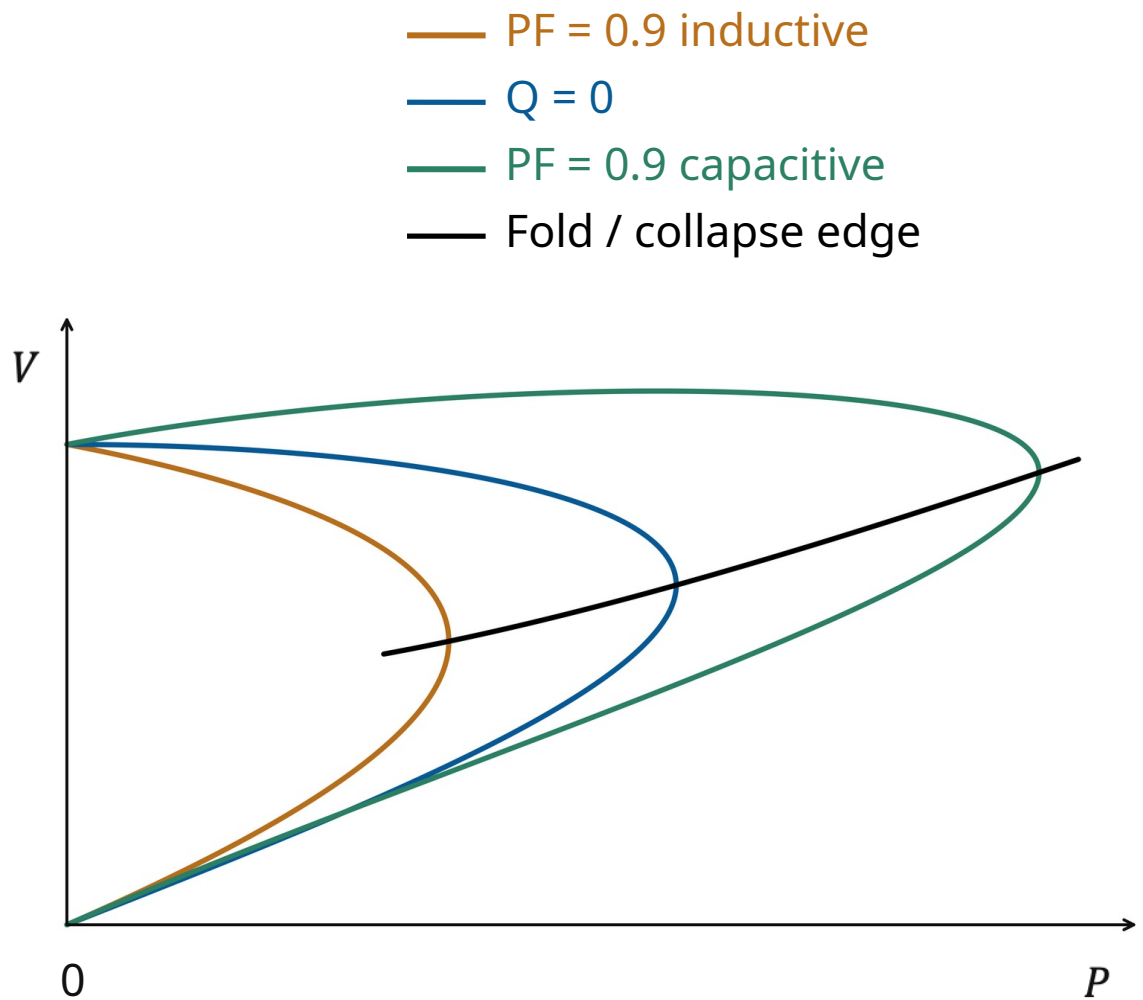
The power flow manifold

Different curves for different power factors, 2D

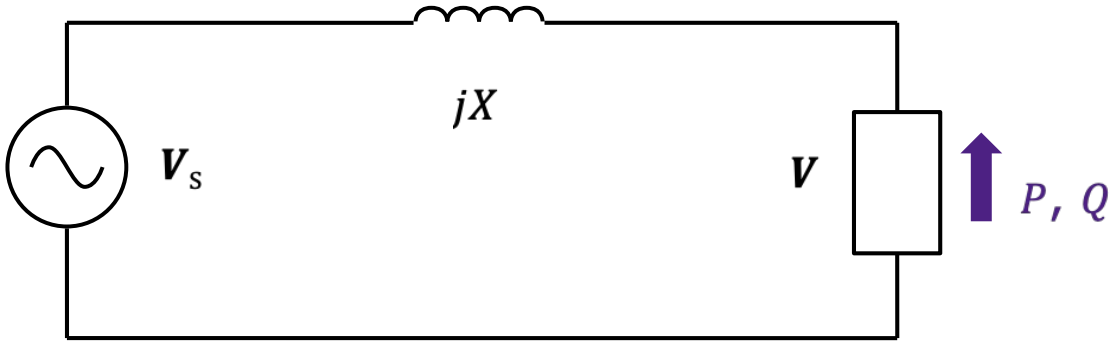


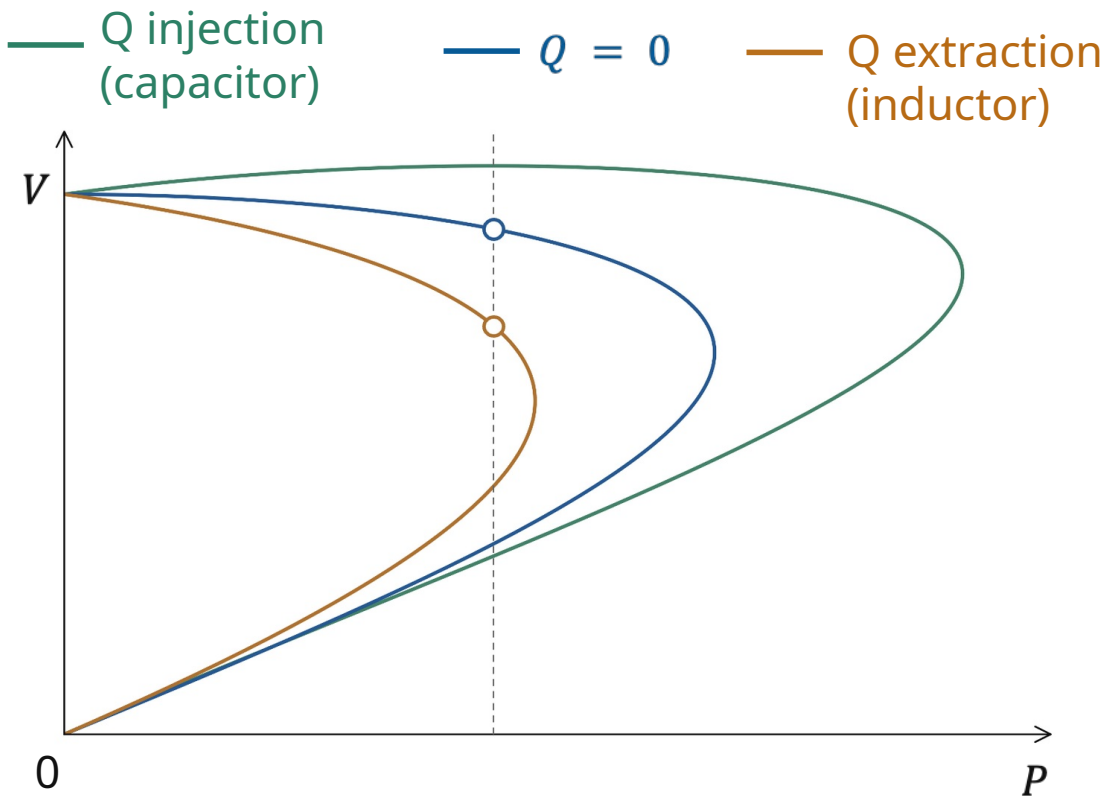
T. Van Cutsem, C. Vournas, " Voltage stability of electric power systems", Springer Science & Business Media, 1998





Different power factors give different P-V curves



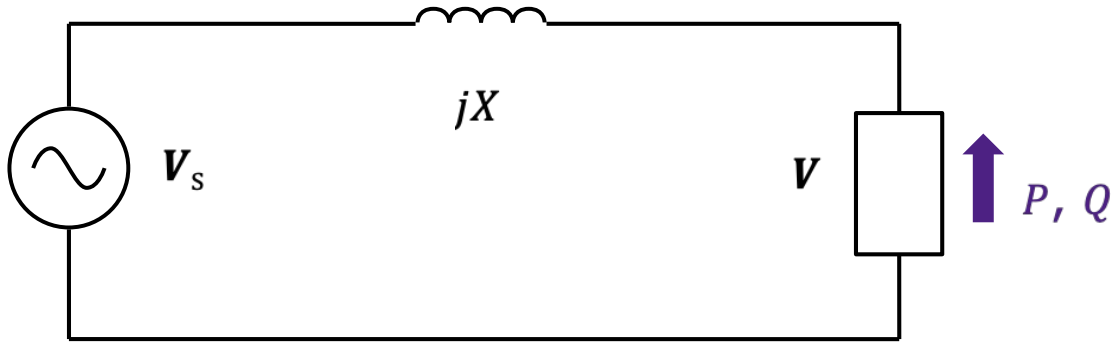


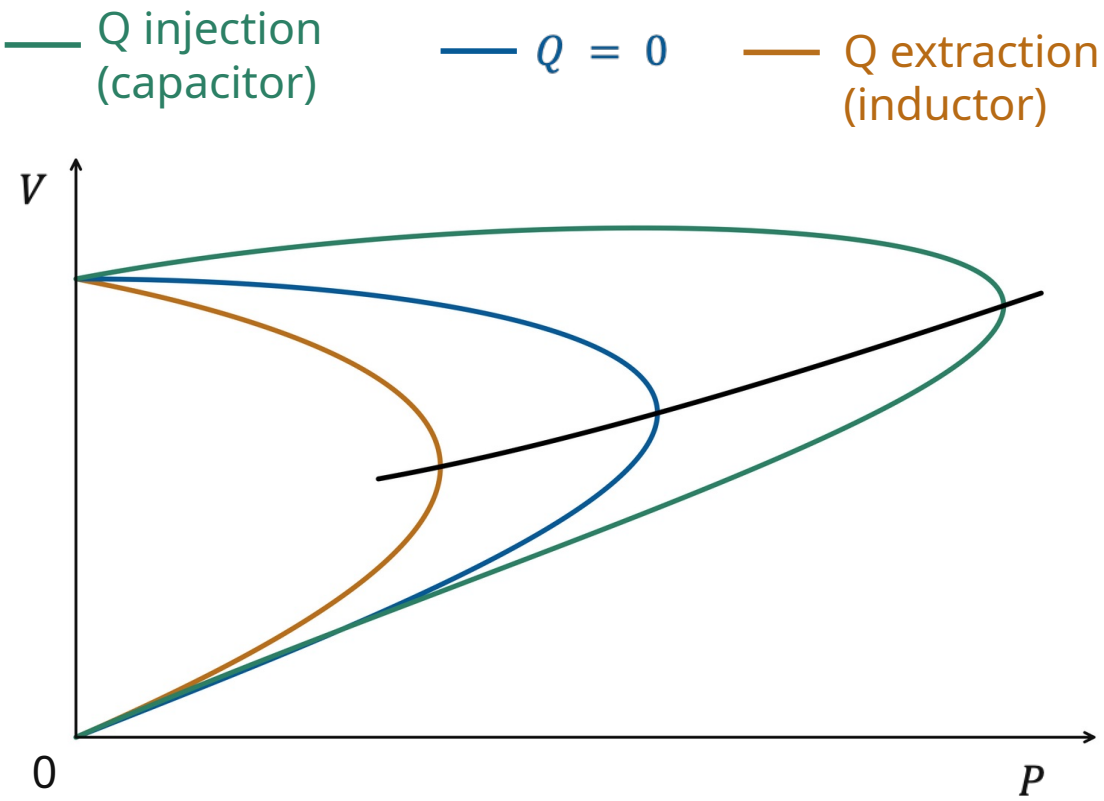
Increasing Q increases the load voltage

The high-voltage solution is $V^2 = \frac{V_s^2}{2} + XQ + \sqrt{\frac{V_s^4}{4} + XV_s^2Q - X^2P^2}$

Increases with Q

Increasing Q increases P_{max}



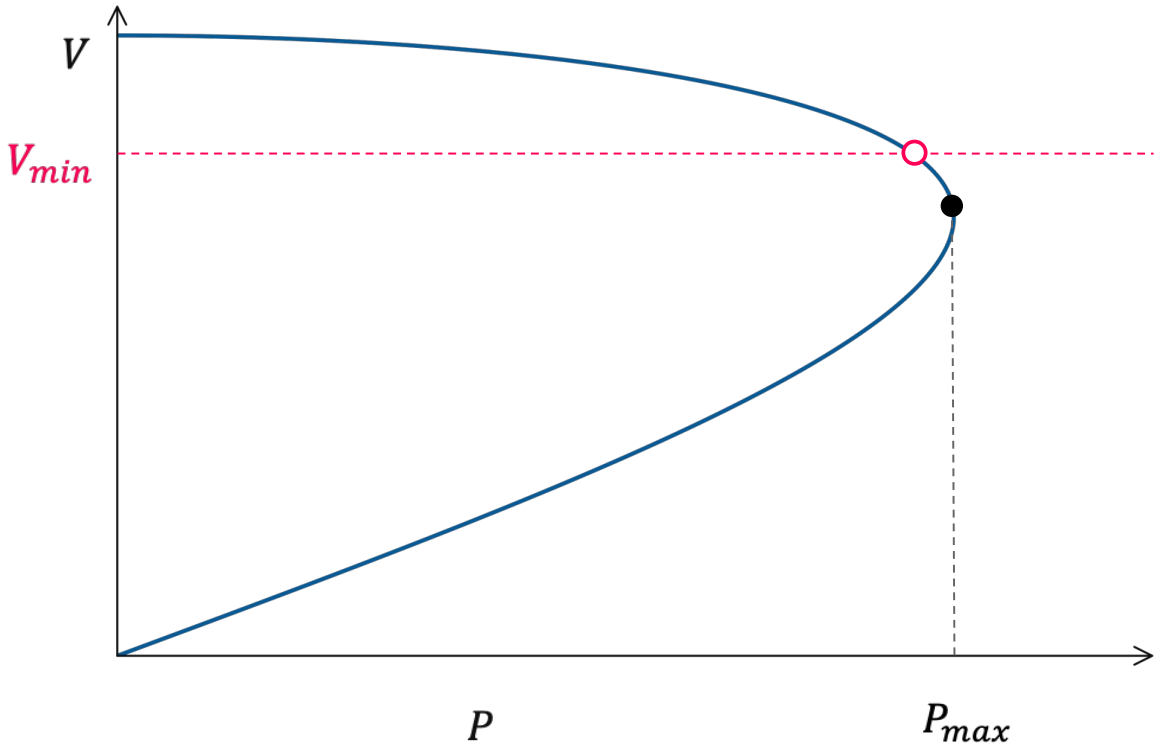


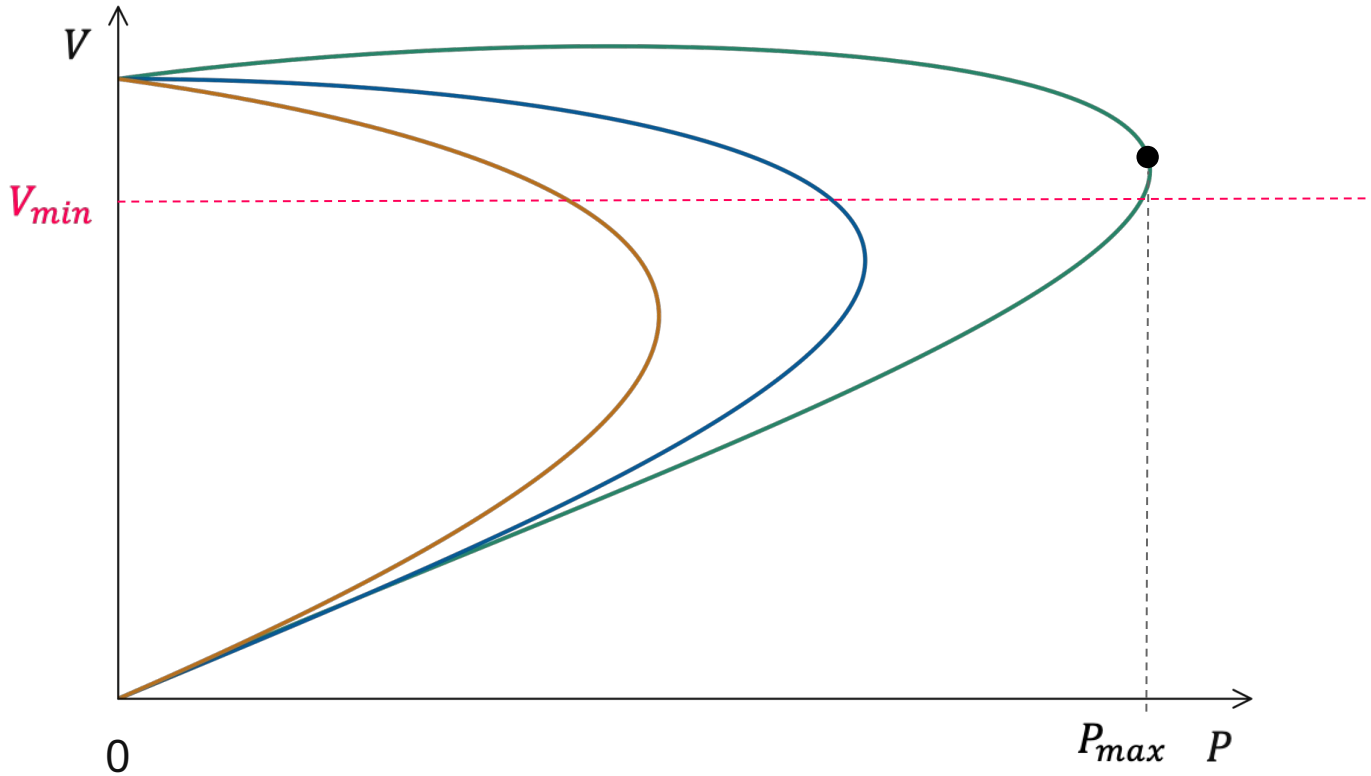
$$P_{\max} = \sqrt{\left(\frac{V_S^2}{2X}\right)^2 + \frac{V_S^2}{X}Q}$$

This equation comes from the requirement for $V^4 - (V_S^2 + 2XQ)V^2 + X^2(P^2 + Q^2) = 0$ to have a real solution

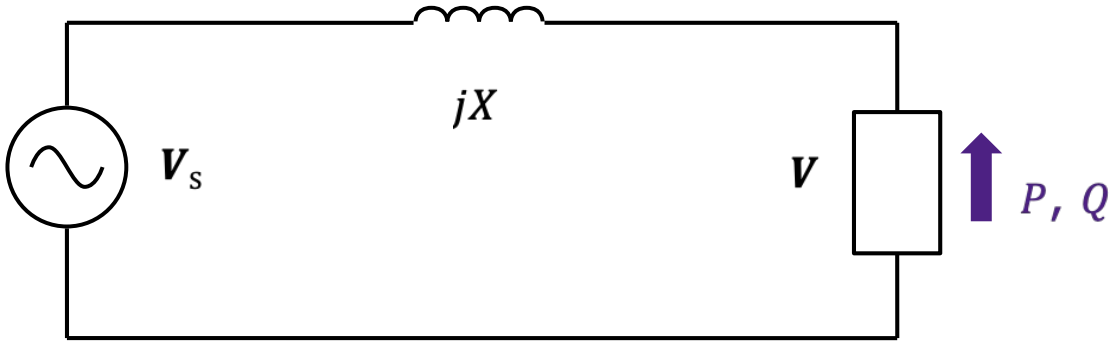
$P_{\max}$ increases with Q

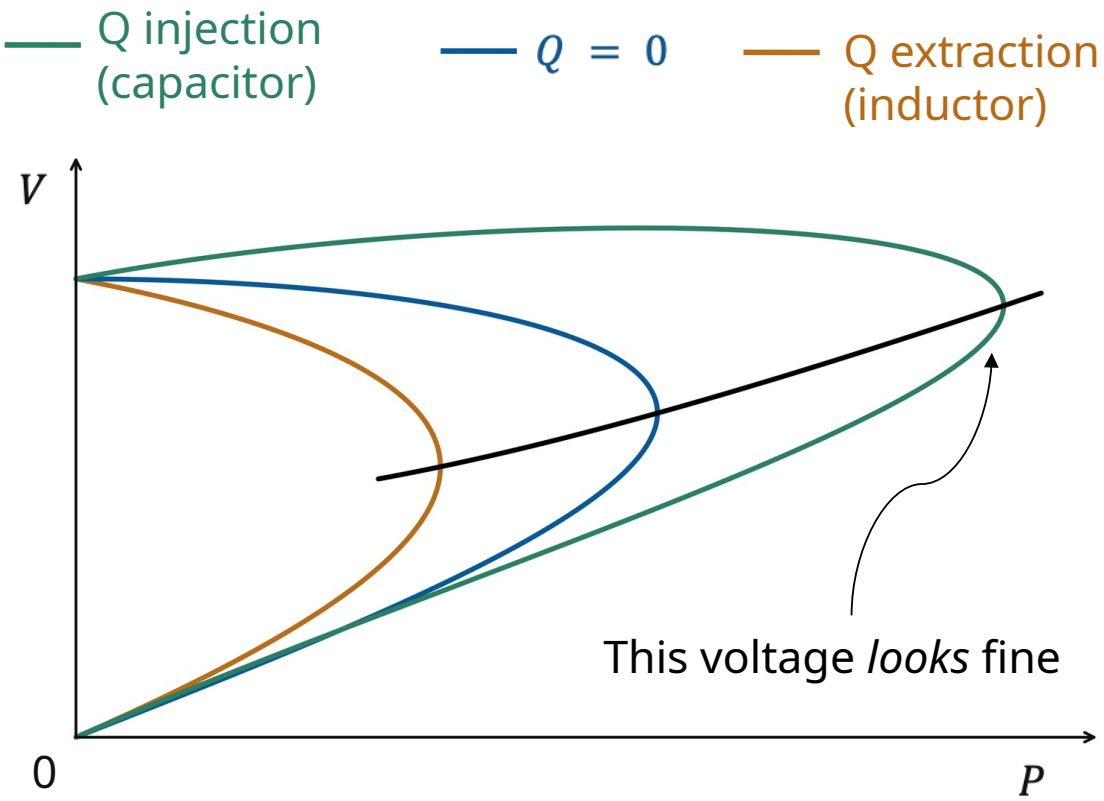
Current/thermal maximums and minimum voltage constraints often bind before the collapse point.





But sometimes, they do not.

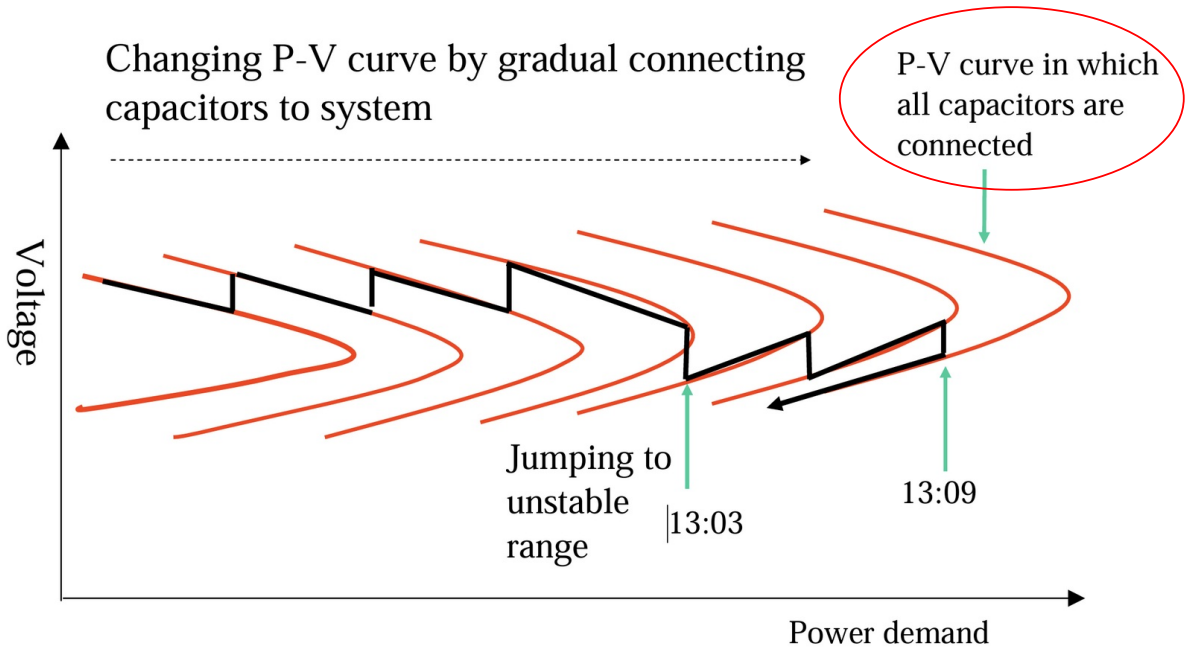




Large Q injections can "hide" the collapse point

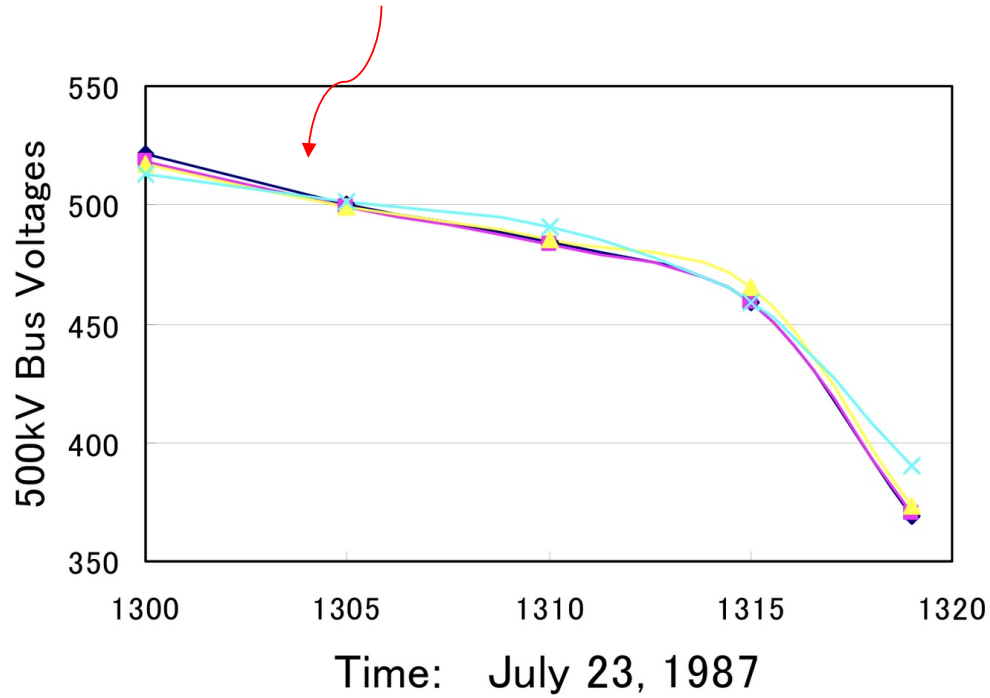
This is not a problem if you can always switch on more reactive power. But it you run out of Q...

How the Collapse Happened in 1987 (3/3)



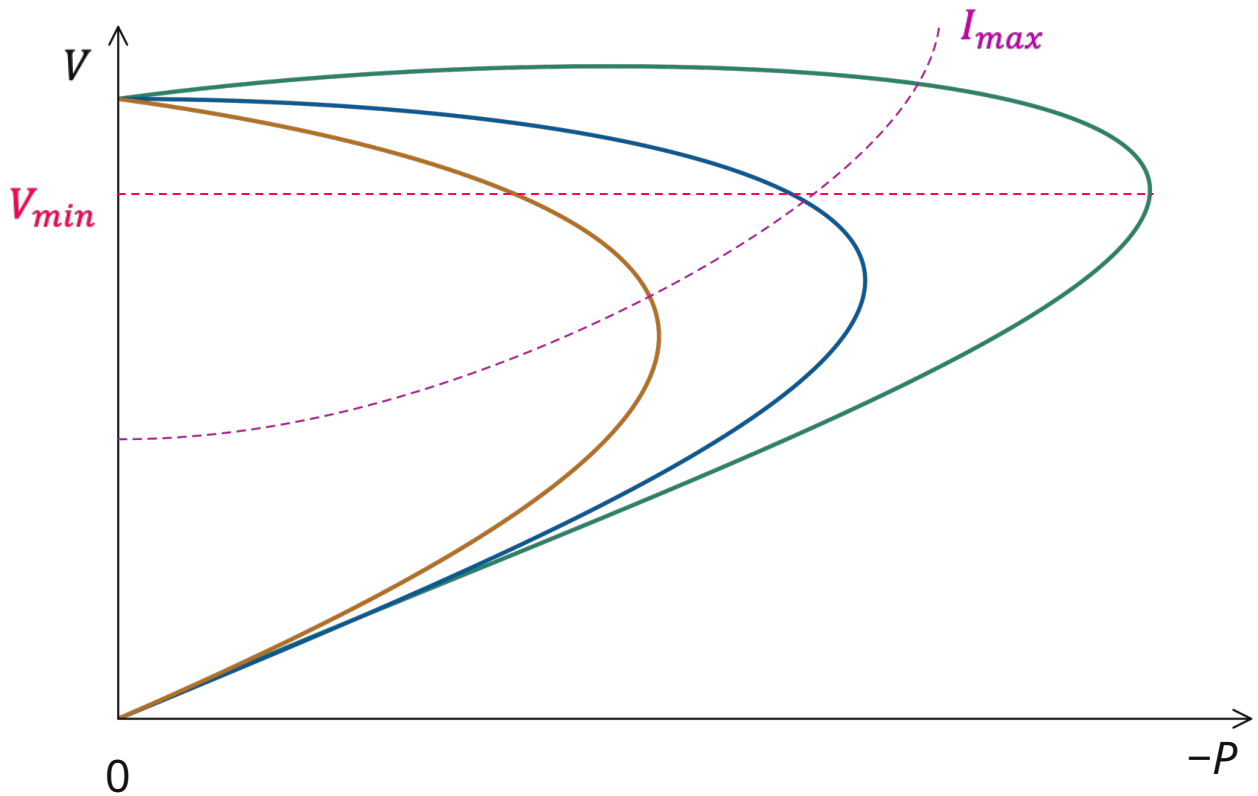
TOKYO ELECTRIC POWER COMPANY

Voltage was high up until collapse

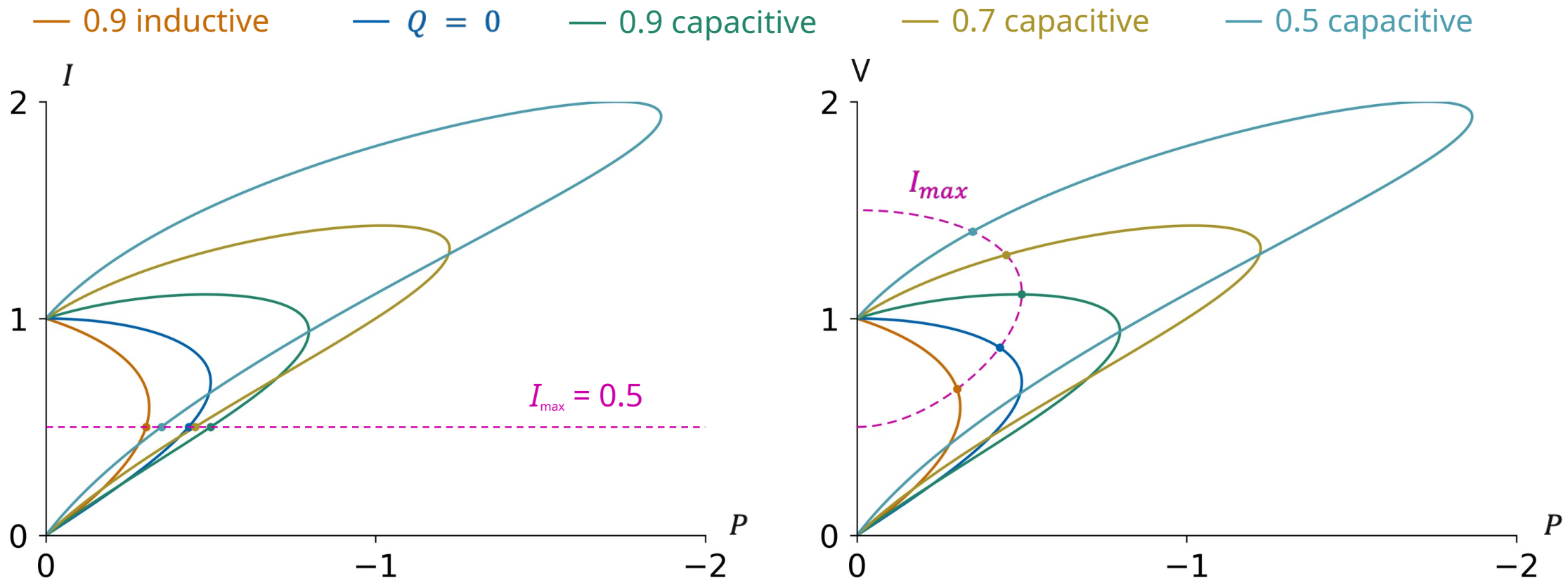


TEPCO (2010)

Even with constraints, Q injections increase P transfer capacity

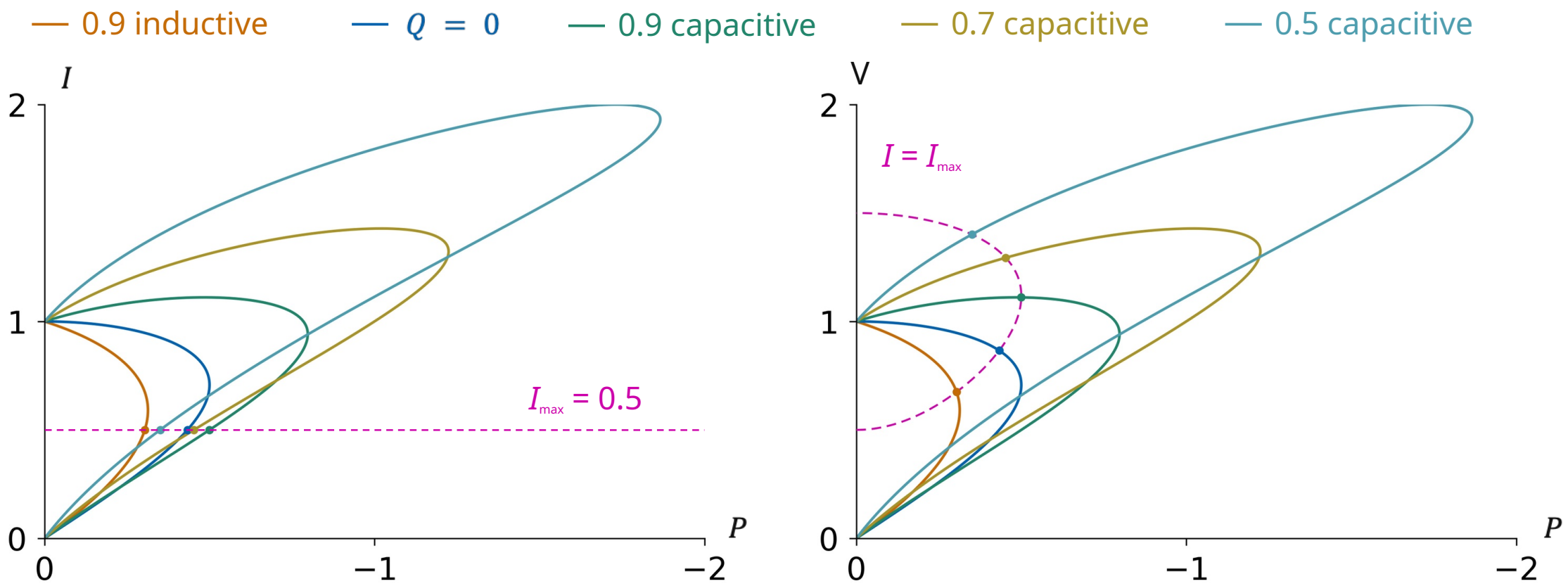


Even with constraints, Q injections increase real power transfer capacity... up to a point.

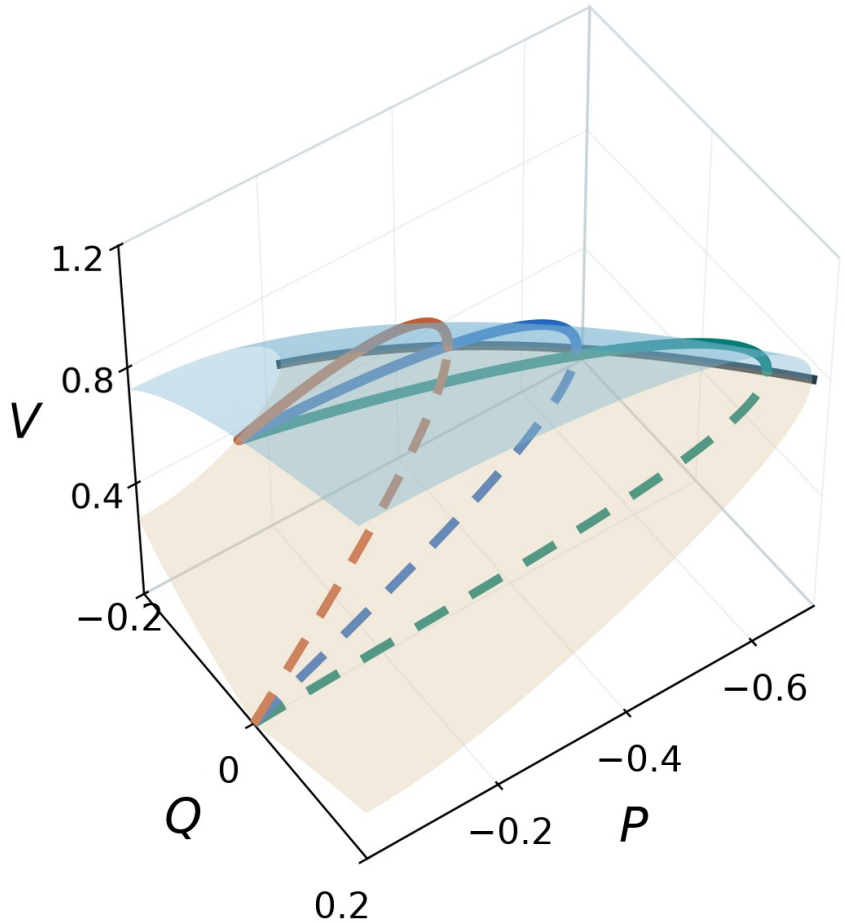


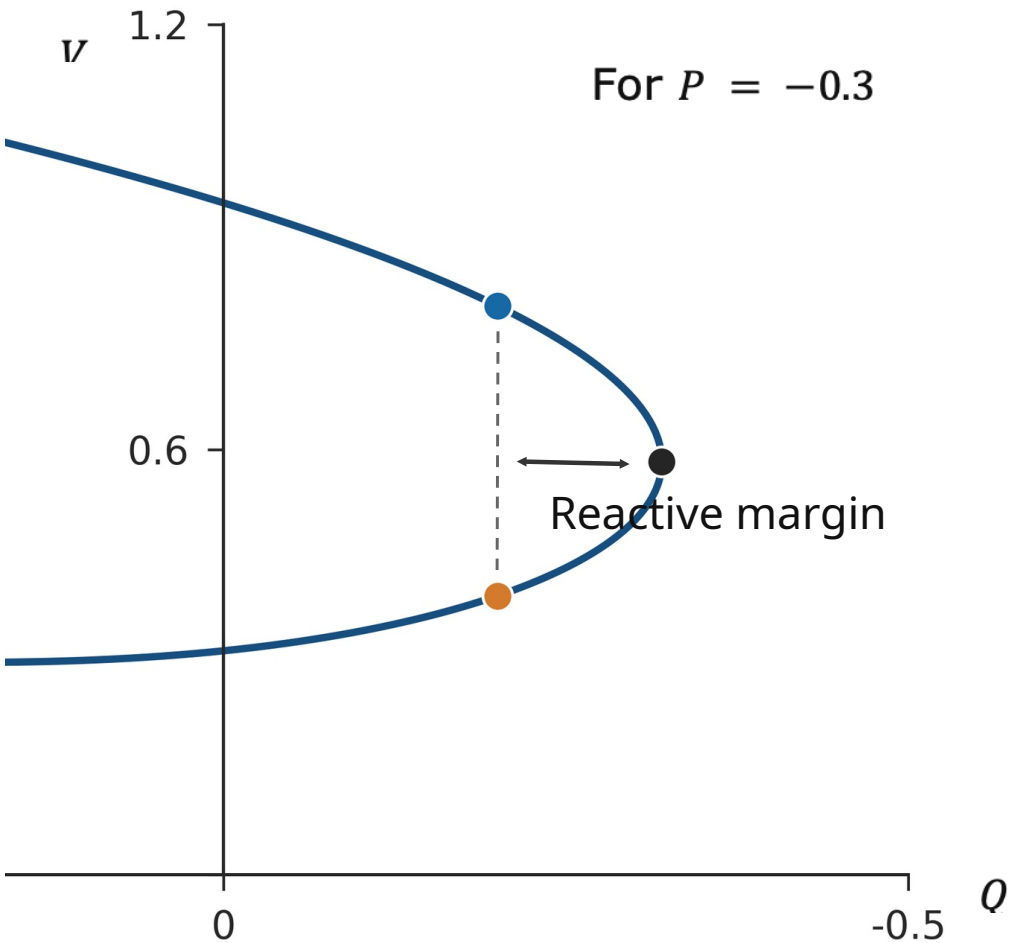
With the current/thermal constraint, a 0.9 capacitive power factor maximizes power transfer.

Exercise: explain the P-V and P-I curves on this slide—why does current get so large?



The power-flow manifold also describes a Q-V curve, which gives the reactive margin for a given P

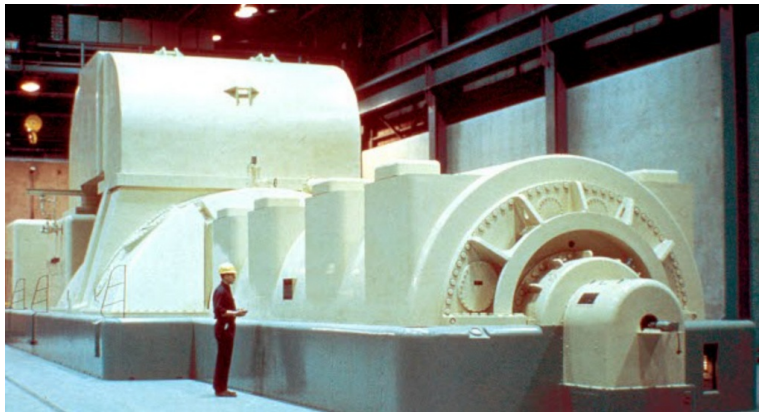




Reactive margin is the increase in Q load until voltage collapse occurs.

Generally, we do not care about sending Q. Reactive margin is useful as a *margin* to voltage collapse.

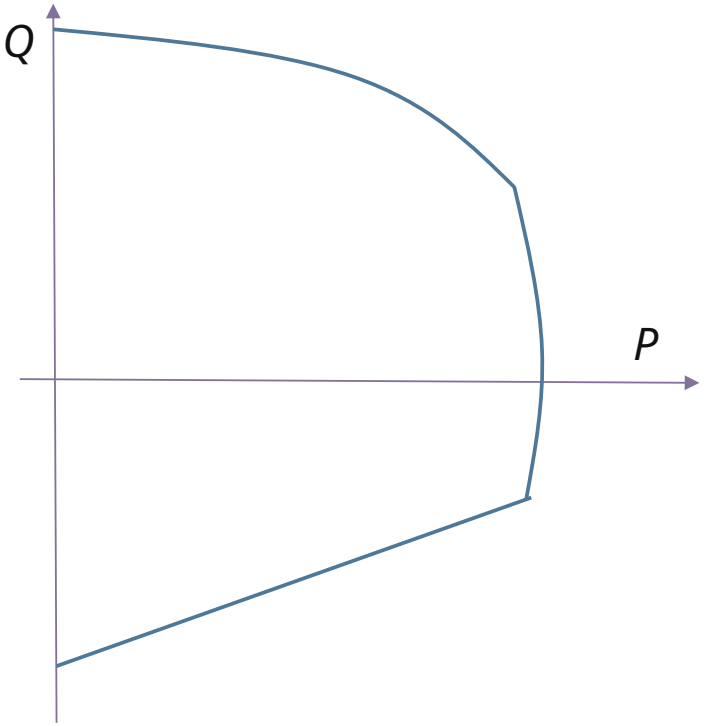
Synchronous generators



The primary source of reactive power and have traditionally been responsible for maintaining the network voltage profile.

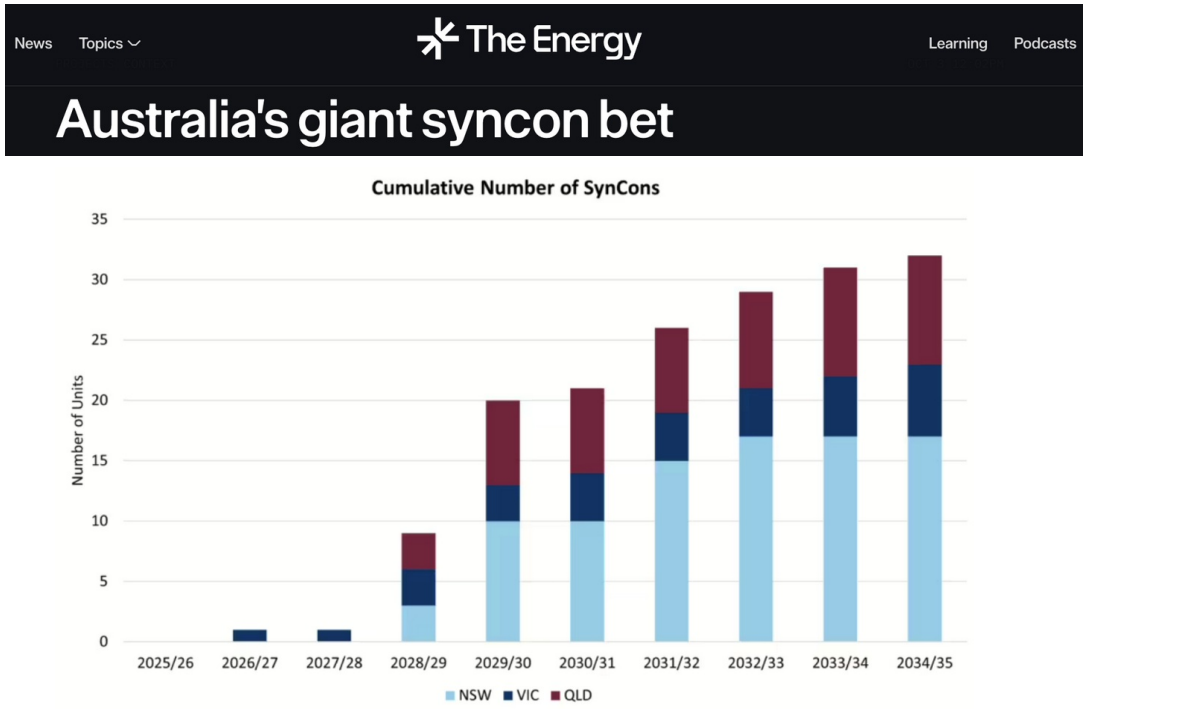
Often equipped with Automatic Voltage Regulators (AVRs), which modulate reactive power injection/excitation to maintain a constant output voltage magnitude.

Reactive power operating limits must be considered, particularly as we rely on fewer machines.



Synchronous condensers

- No active power is produced, very little is consumed
- Downsides: expensive, limited supply
  - Synchronous generators may be repurposed to syncons



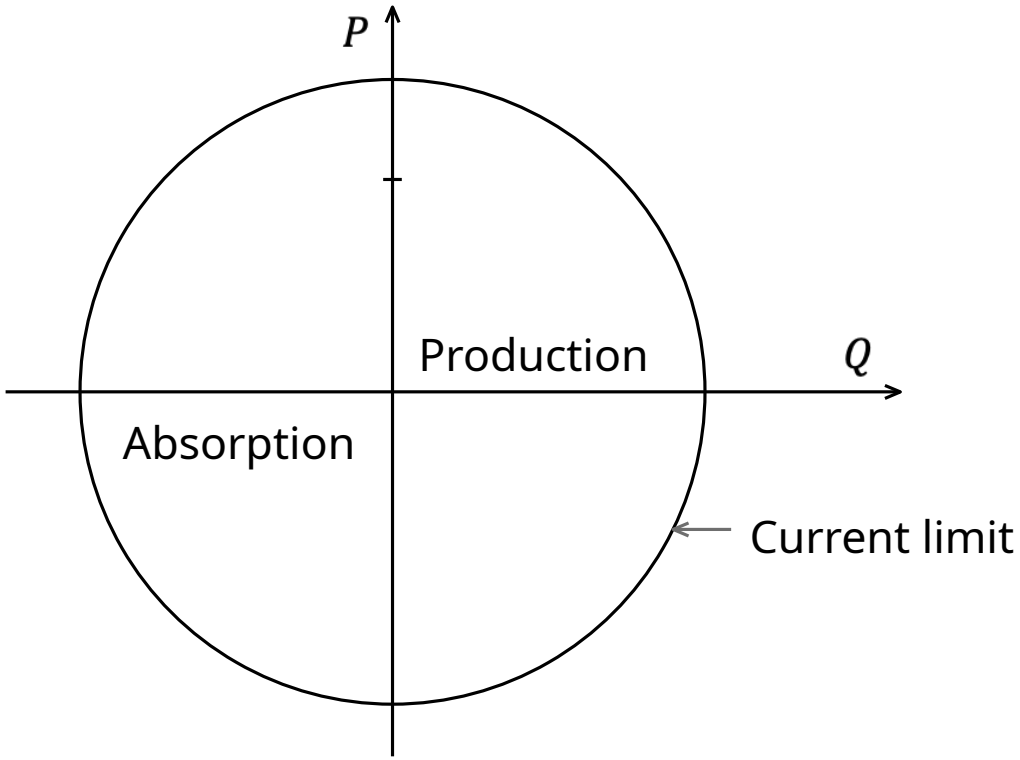


Synchronous condenser installation at Templestowe substation, Melbourne, Victoria, Australia. Built by ASEA in 1966, the unit is hydrogen-cooled and capable of three-phase power at 125 MVA.

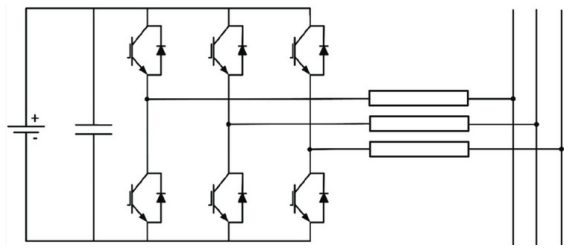
Inverters

Inverters can inject reactive power within their hardware capability limits.

We have under-relied on inverters. Improved inverter integration will lead to more efficient and more secure grid operation.







How operators change the loadability margin

Action	What changes	Effect
Reactive support	$Q_{net} = Q + Q_c$	PV curve moves up and right
AVR voltage setpoints	$V_s \uparrow$ Within reactive limits	PV nose moves up and right
Network switching	$X \downarrow$ Add parallel supply paths	Power limit increases
Generator dispatch and commitment	More local MW; commitment can add reactive support	Reduces imports; may raise the power limit
Load shedding	$P \uparrow$ (less negative) Less consumption	Operating point moves left, away from the nose

NERC, Reactive Power Planning Guideline (2016), pp. 16, 21–22 and 39

Reactive support

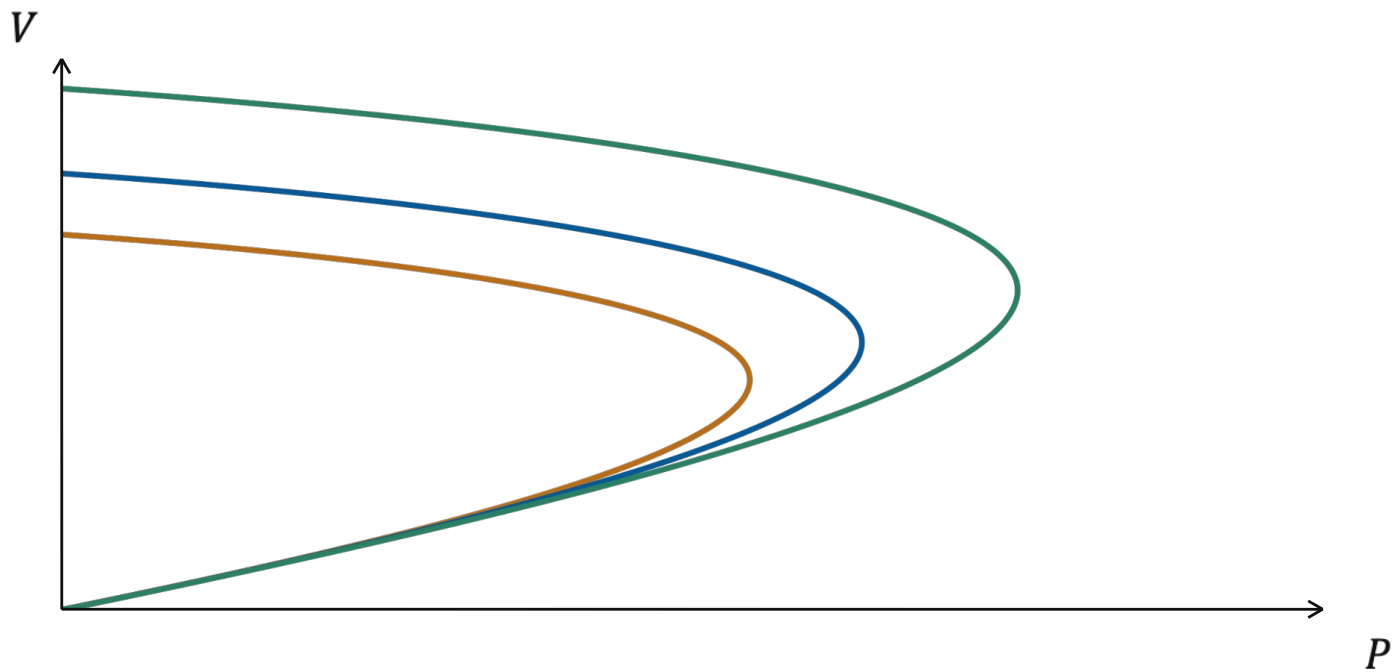
Reactive power injections or “shunt compensation” may come from:

- Mechanically-switched capacitors
- Syncons
- SVCs and STATCOMs
- Inverters

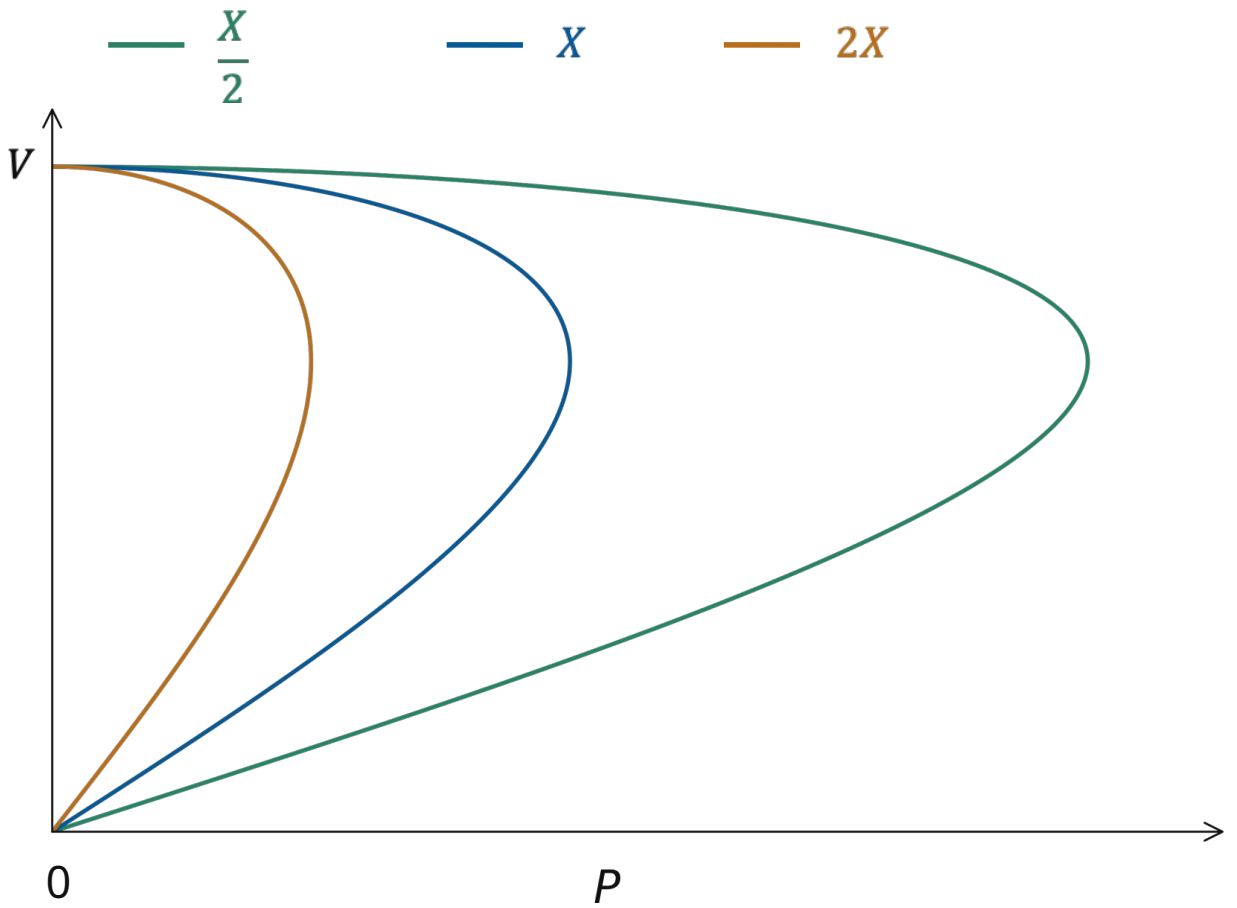
Shunt capacitance reshapes the PV curve, pushing it up and right

Shunt setting

- $B_{CX} = 0$
- $B_{CX} = 0.14$
- $B_{CX} = 0.28$



Changing the line reactance shifts the curve left/right



Derivation:

$-P = VI$ when $Q = 0$

$$-P = V\frac{\sqrt{{V_S}^2 - V^2}}{X}$$

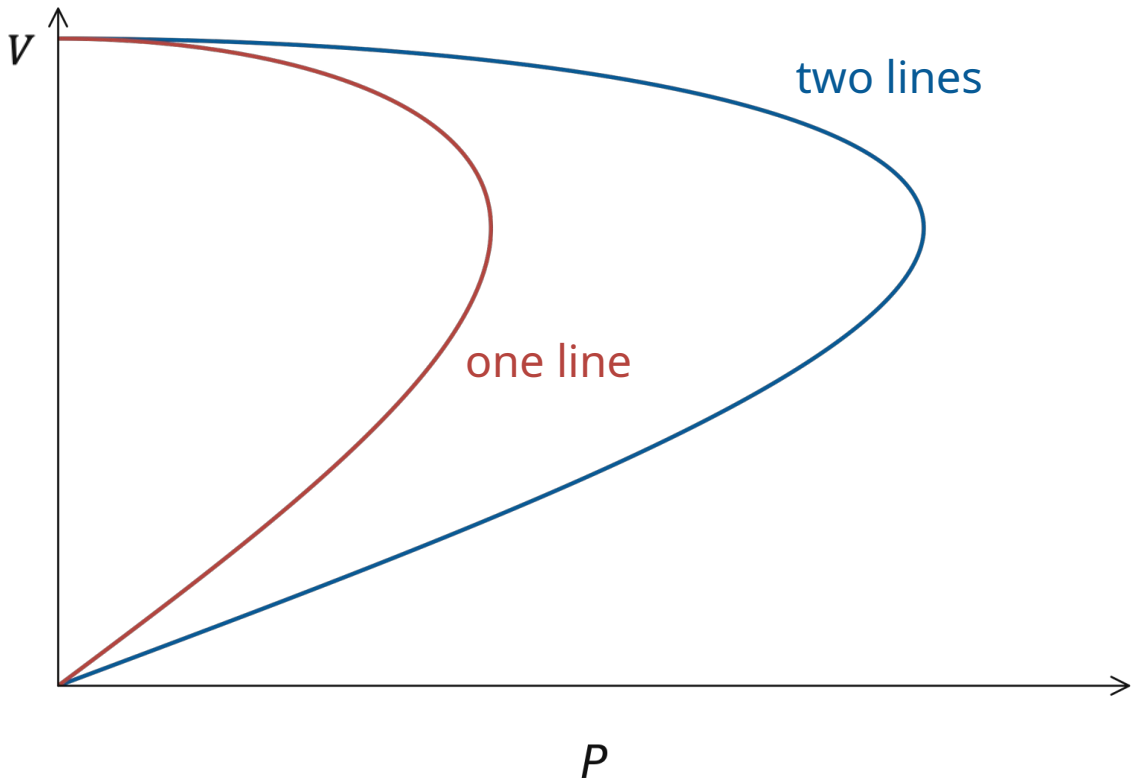
$$P_{max} = \frac{{V_S}^2}{2X}$$

A line outage reduces the transfer limit

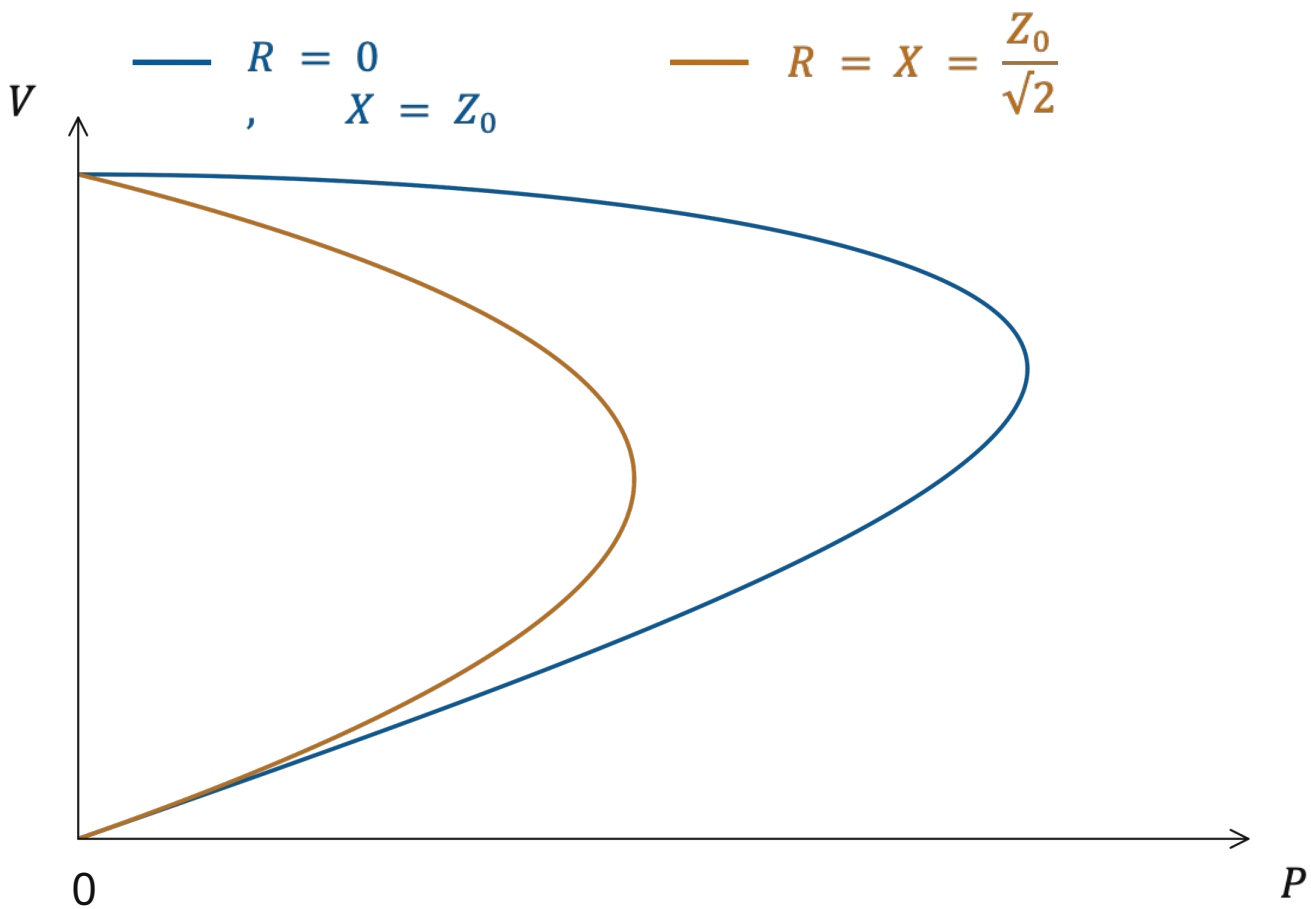
Each circuit has reactance $2X$.

In parallel $X_{eq} = X$.

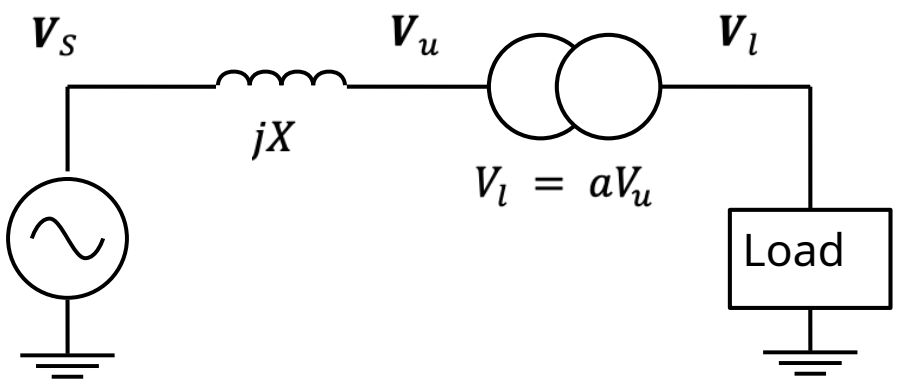
after one trips, $X_{eq} = 2X$.



Resistive + inductive lines also have a loadability curve



Transformers shift the PV curve up

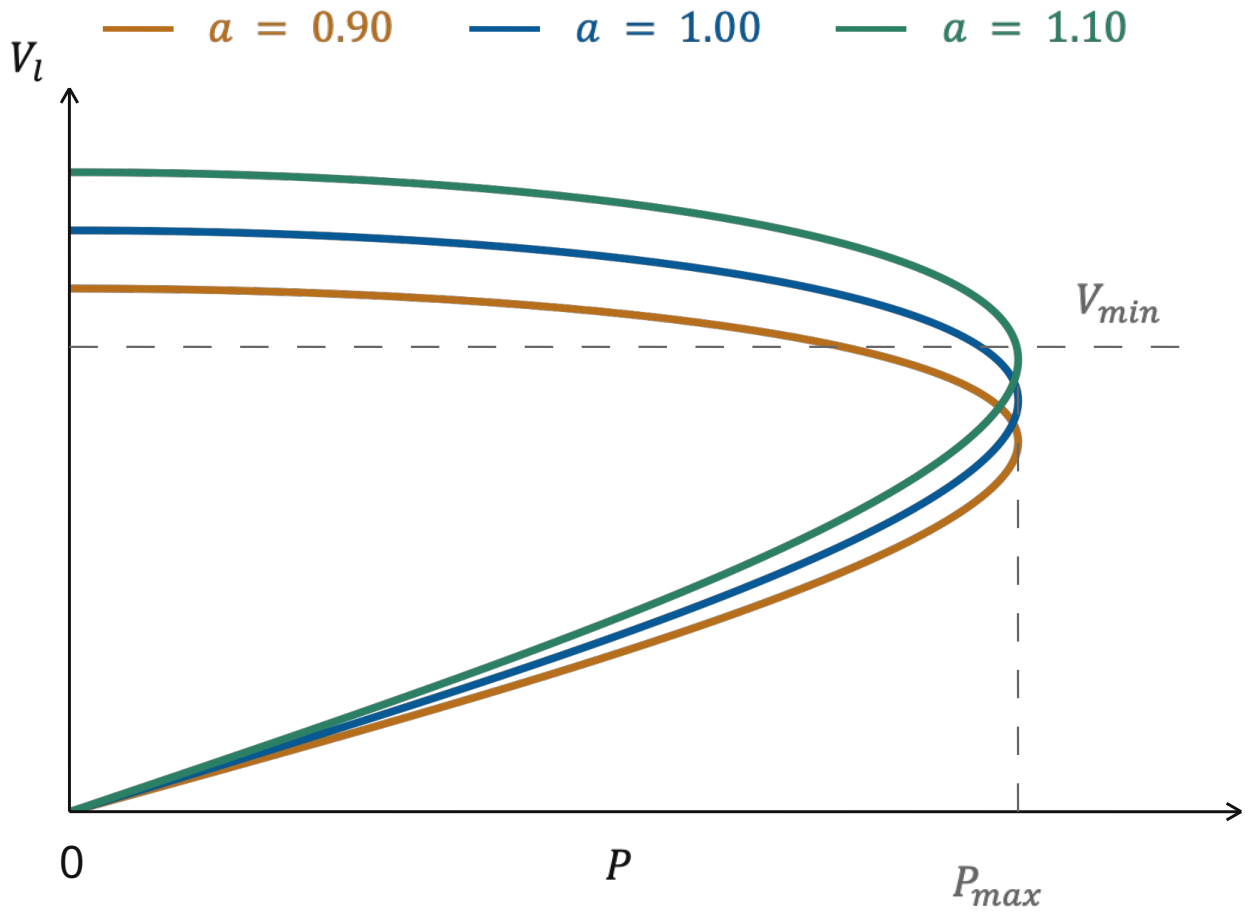


$$P_{max} = \frac{V_S^2}{2X}$$

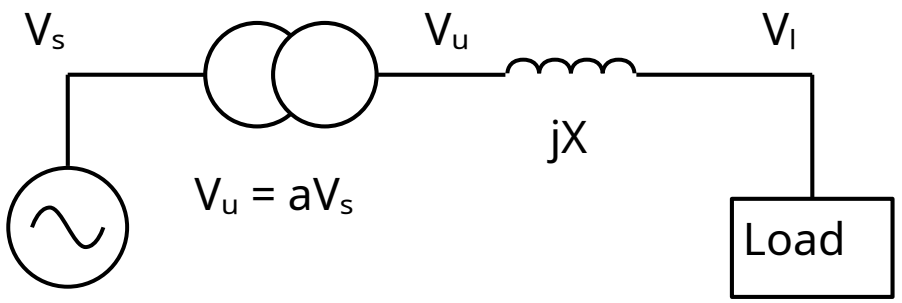
A higher ratio raises load-side voltage.

It can delay reaching the load-voltage limit.

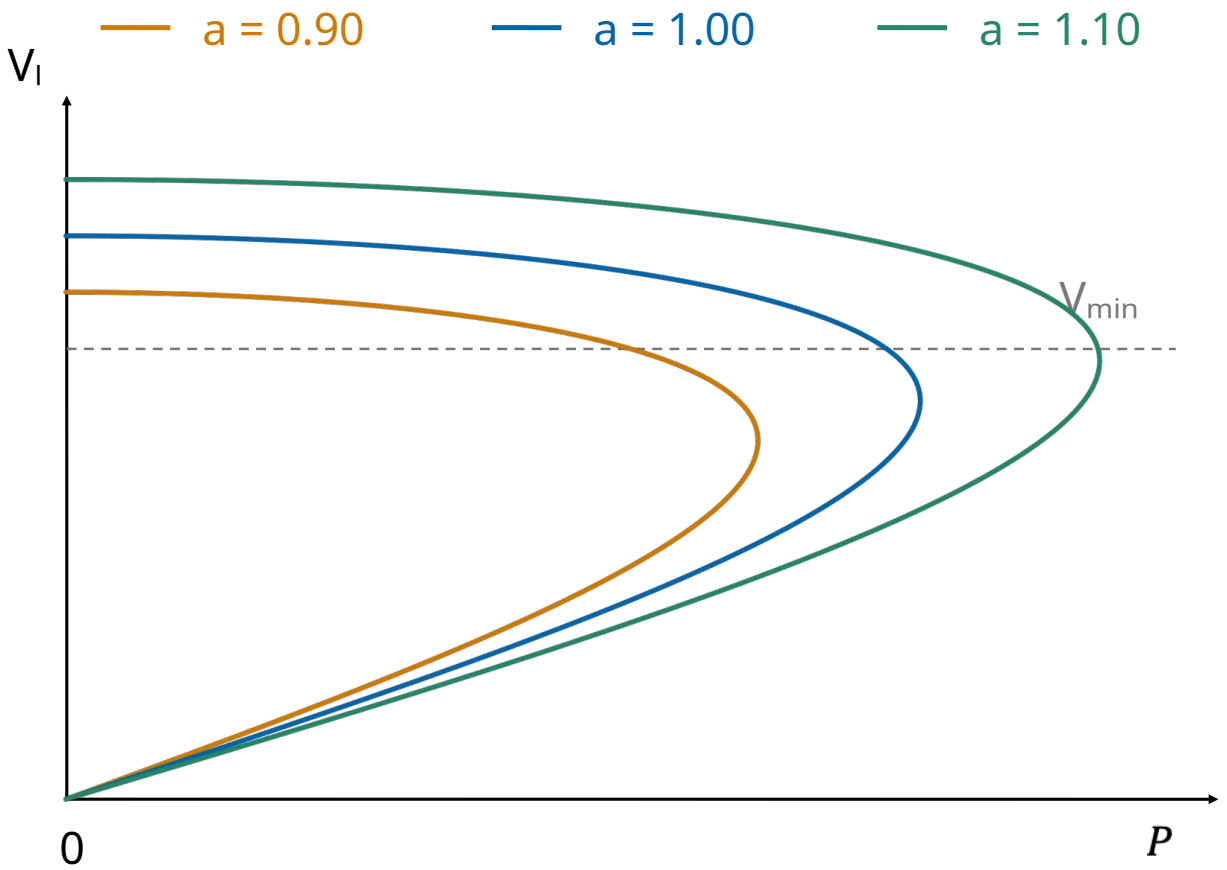
The upstream line still has the same nose.



Tapping up *before* the line increases the power limit



$$P_{max} = \frac{a^2 V_S^{\,2}}{2X}$$



Load voltage rises on the upper branch before the line impedance, increasing P_{max}.

Some commonly used terminology

Short Circuit Ratio (SCR) — a confusing way to talk about Z_{th}

Short-circuit ratio (SCR) compares grid fault level with plant rating.

$$\mathrm{SCR} = \frac{S_{\mathrm{sc}}}{P_{\mathrm{rated}}}$$

The numerator is the grid-side three-phase fault level (MVA), excluding the plant's contribution.

The denominator is the plant's rated active power (MW).

For nominal source voltage behind the grid's Thevenin impedance:

$$I_{\mathrm{sc}} = \frac{V_S}{Z_{\mathrm{th}}}, \quad S_{\mathrm{sc}} = V_S I_{\mathrm{sc}} = \frac{V_S^2}{Z_{\mathrm{th}}}$$

At fixed voltage and plant rating, lower SCR means larger grid impedance/voltage sensitivity.

Low SCR gird regions are sometimes called "weak grid" regions.

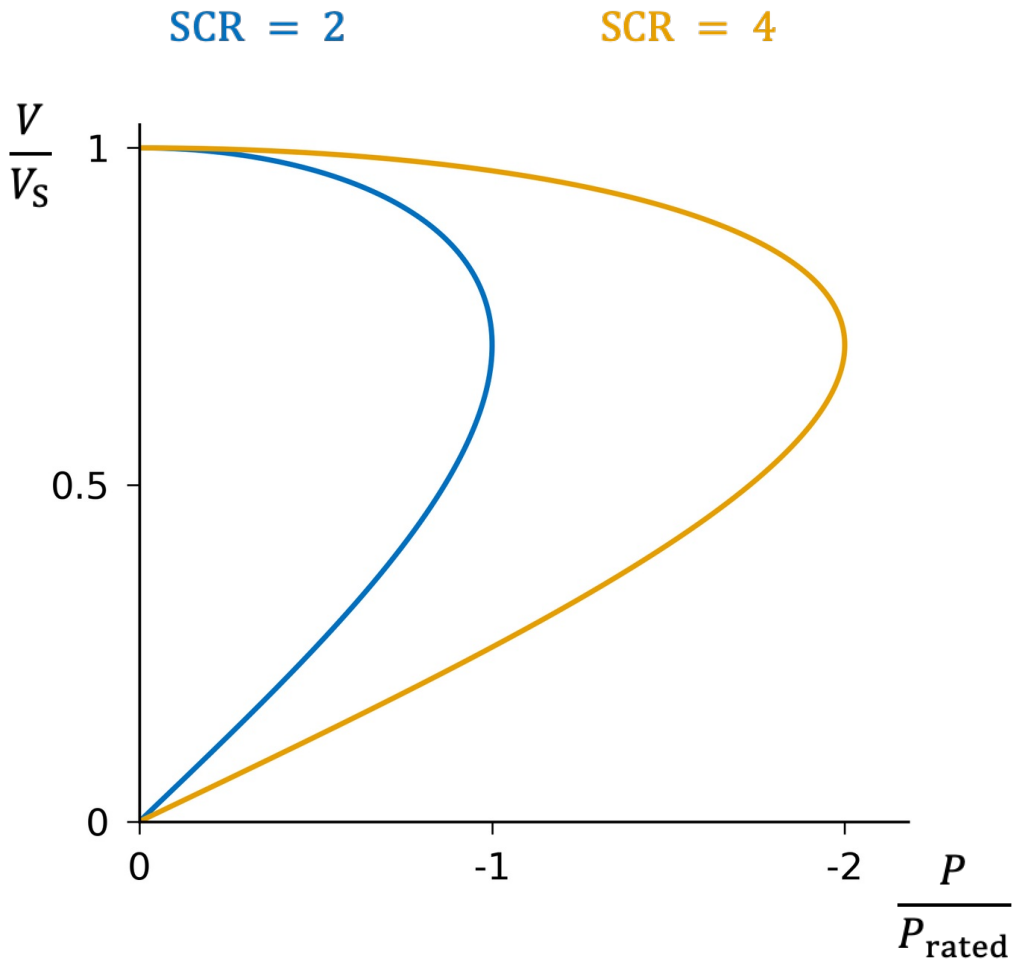
https://www.aemo.com.au/-/media/files/stakeholder_consultation/consultations/nem-consultations/2024/ssiag/system-strength-impact-assessment-guidelines-v22.pdf

Short Circuit Ratio (SCR) — a confusing way to talk about Z_{th}

For a lossless two-bus connection with fixed source voltage and zero reactive injection:

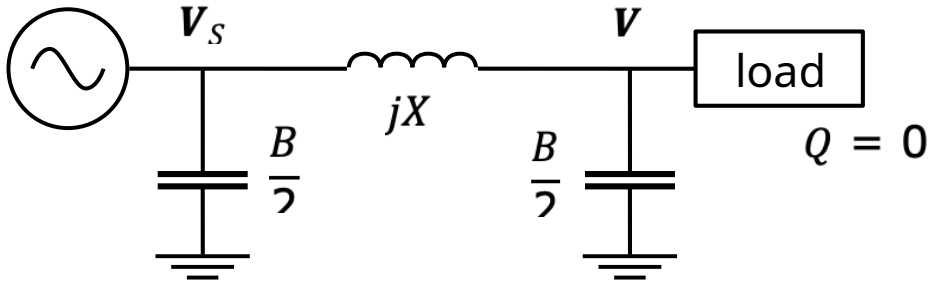
$$\boldsymbol{Z}_{\mathrm{th}} = jX, \qquad Q = 0$$

Lower SCR increases Z_{th} and reduces the maximum load power.



Transmission-line Π model

Π model: series reactance with a shunt capacitance on either end

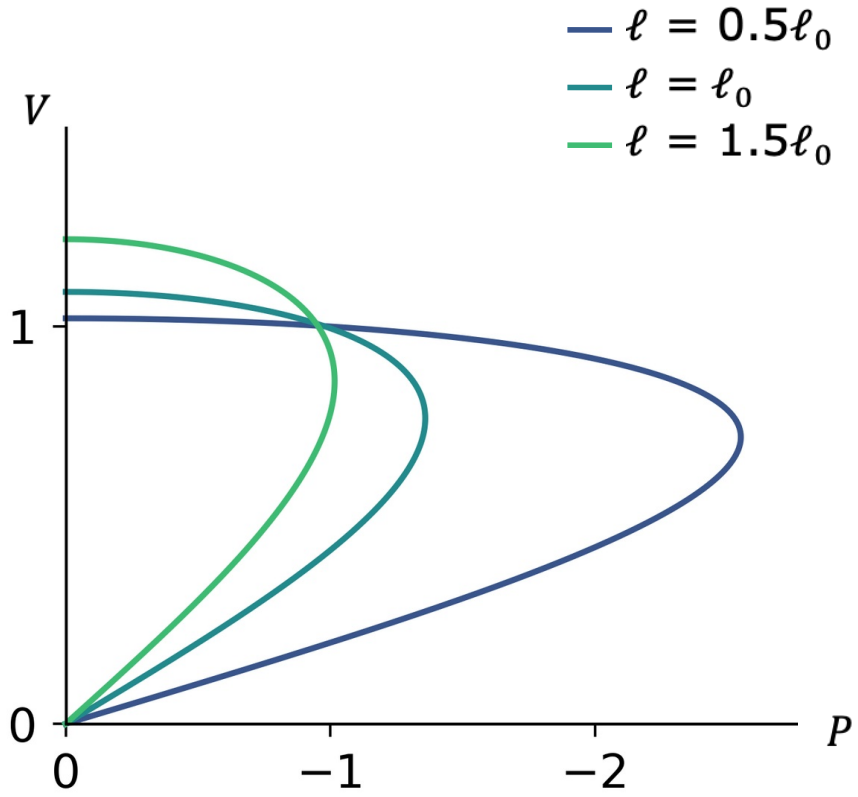


$$X = x\ell, \quad B = b\ell$$

x: series reactance/km

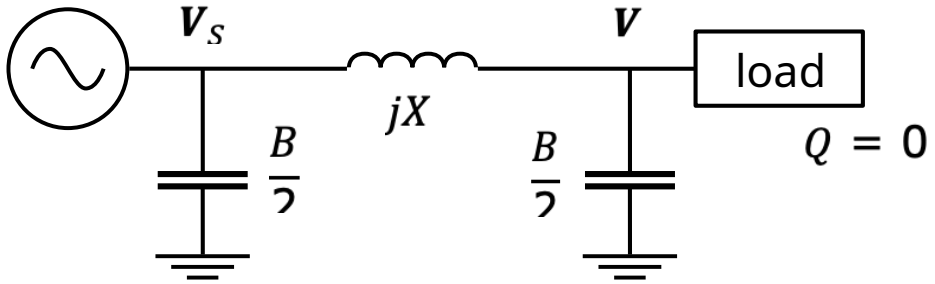
b: shunt/charging susceptance/km

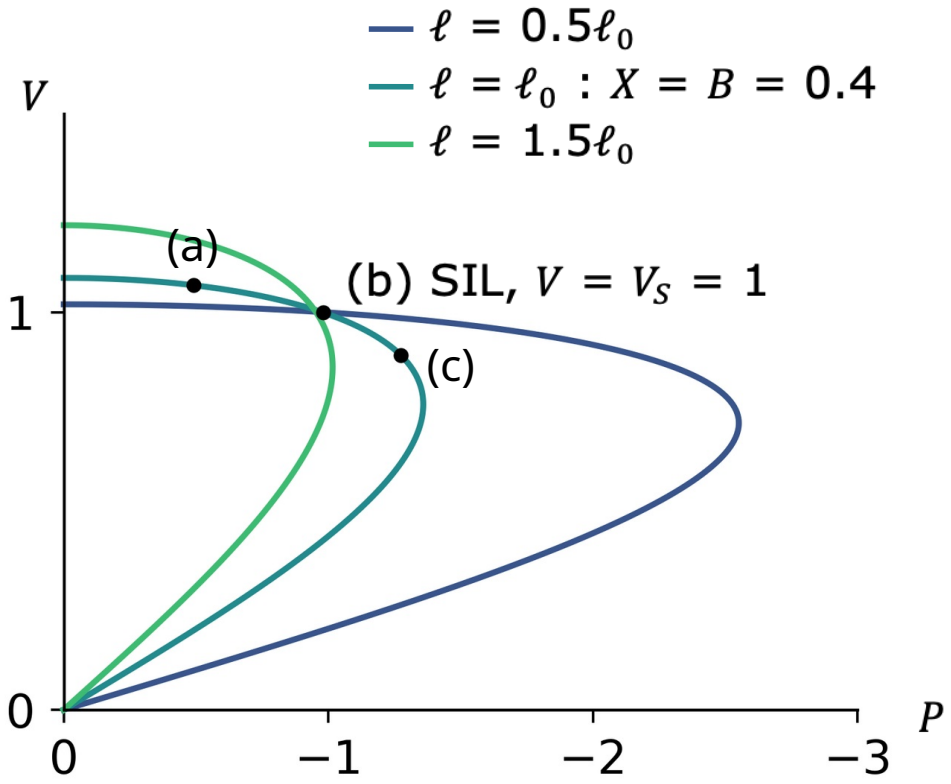
ℓ: line length (km).



Surge Impedance Loading (SIL)

For a lossless line, **SIL** provides a named $PF = 1$ loading level where the sending and receiving voltage magnitudes are equal ($V = V_S$).





(a) Light load

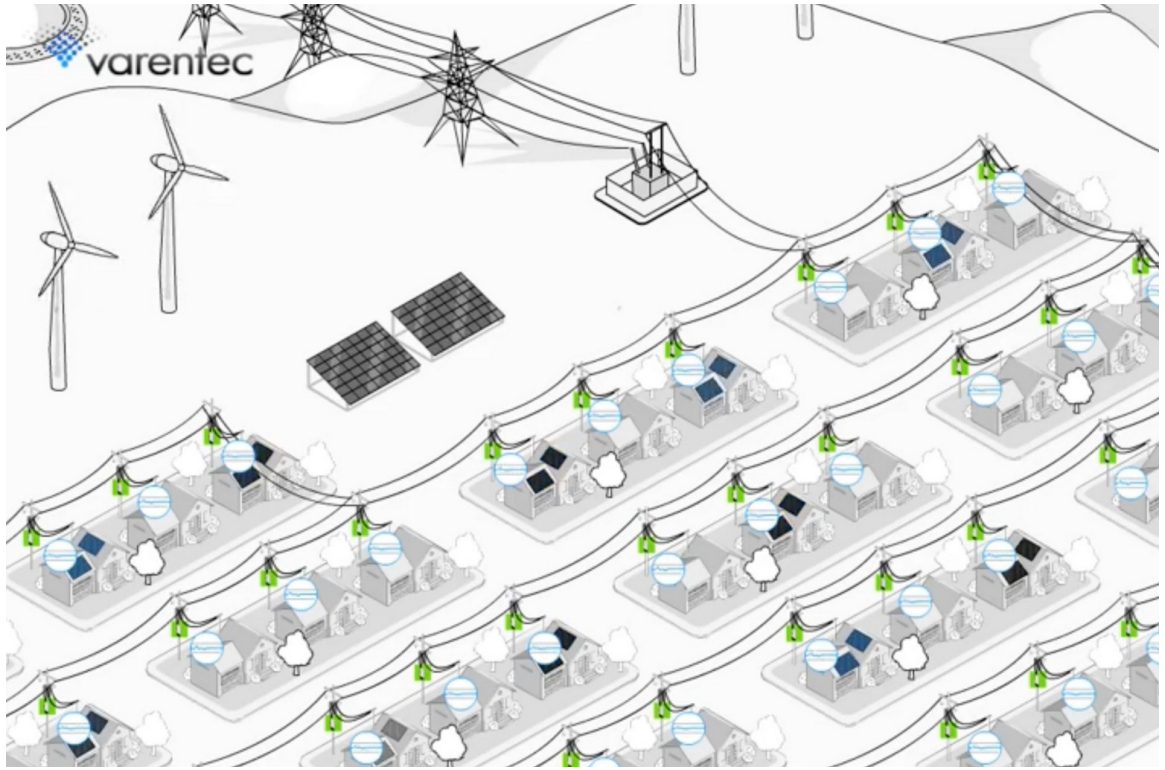
(b) SIL

(c) Heavy load

At load (b), the line's capacitive injection balances its inductive absorption regardless of length.

Exercise: can grid-edge reactive support increase transfer?

Varentec ENGO units inject or absorb reactive power on distribution feeders.





How could local Q injection permit larger load power?

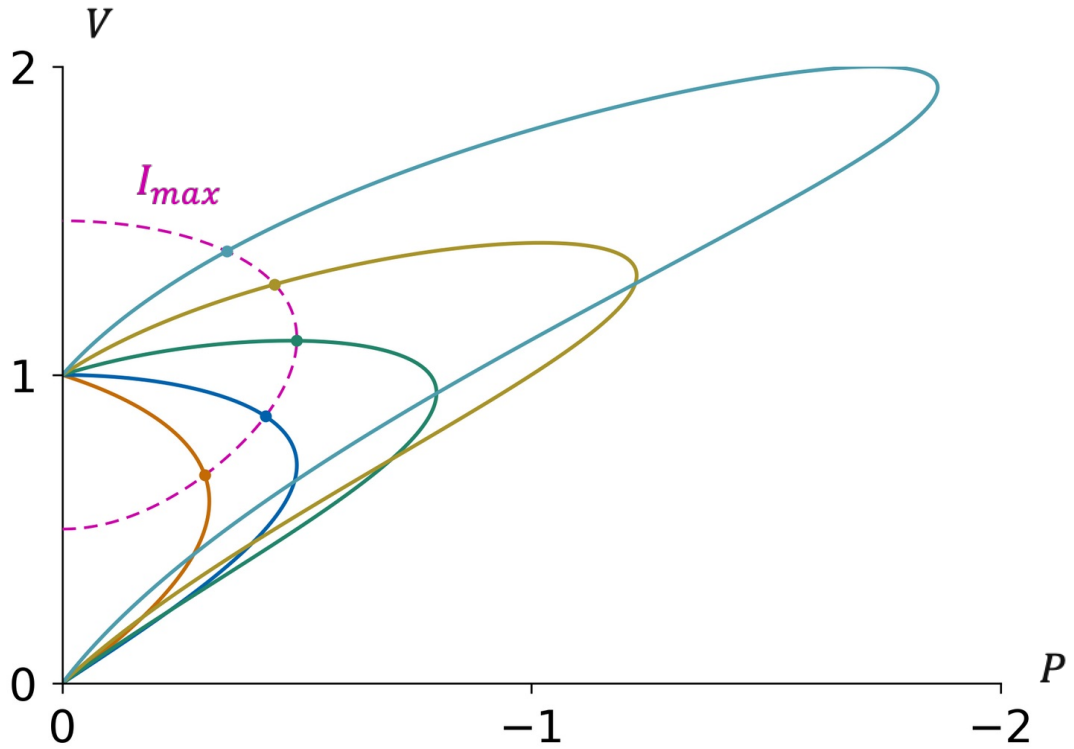
What might still prevent an increase in transfer capability?

If thermal constraints bind, reactive support helps but only to an extent.

Real power flows may also need to change to keep line and transformer currents within their limits.

Batteries: reduce net transfers by charging during surplus generation or discharging near loads.

Dynamic operating envelopes: adjust import and export limits as network conditions change.



Ergon’s batteries support long rural Queensland feeders.

20 units at 13 sites, installed by 2016.

Each unit: 25 kVA / 100 kWh.

They relieve voltage and capacity constraints, avoiding some line upgrades.



Middlemount, Queensland

Source: project engineers, T&D World (2017)

Charleville: close to the PV-curve knee

A static VAR compensator (SVC) supports the long rural 66 kV network.

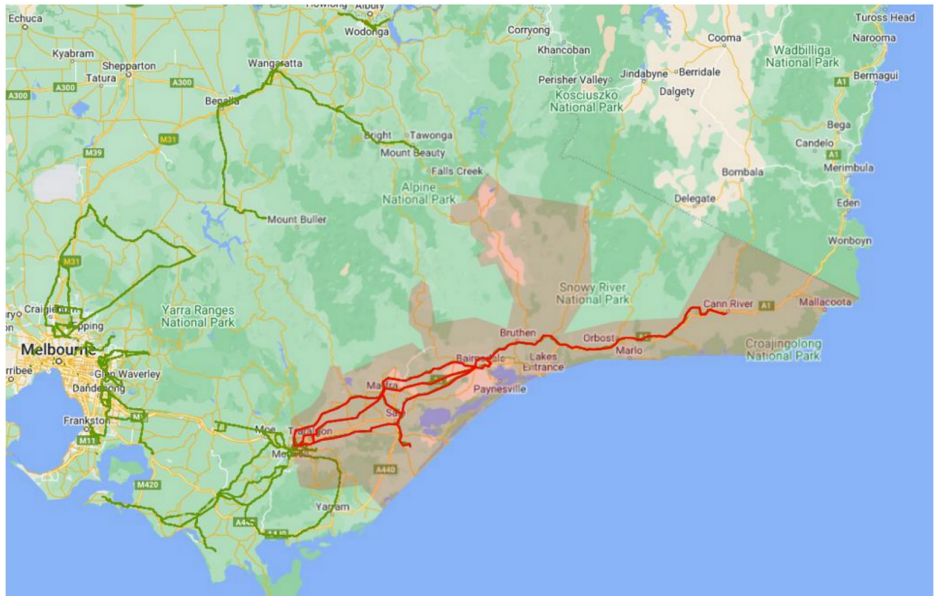
Ergon: “the system operates very close to the voltage collapse knee point.”

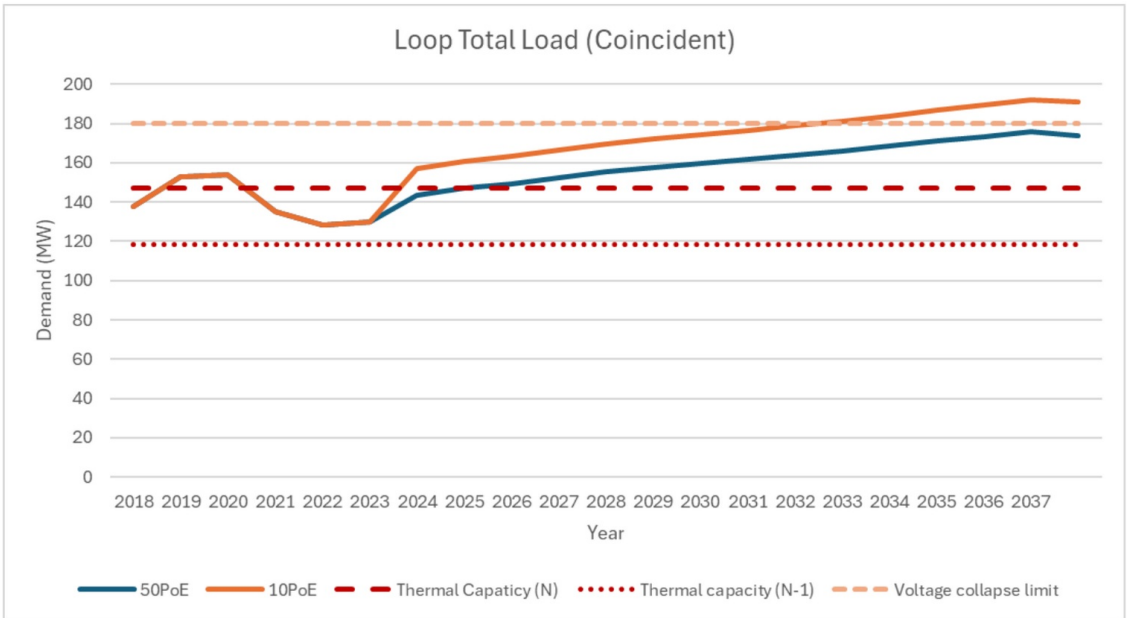


Ergon’s southwest Queensland 66 kV network

Ergon, Charleville Draft Project Assessment Report, p. 5

East Gippsland, Victoria: planning to prevent voltage collapse





2025: AusNet considers a 30 MW / 150 MWh battery at Bairnsdale to reduce line loading and increase the network's voltage-collapse limits.

https://www.aemo.com.au/-/media/files/stakeholder_consultation/consultations/nsp_consultations/2025/osr-ld-tgn-mfa.pdf

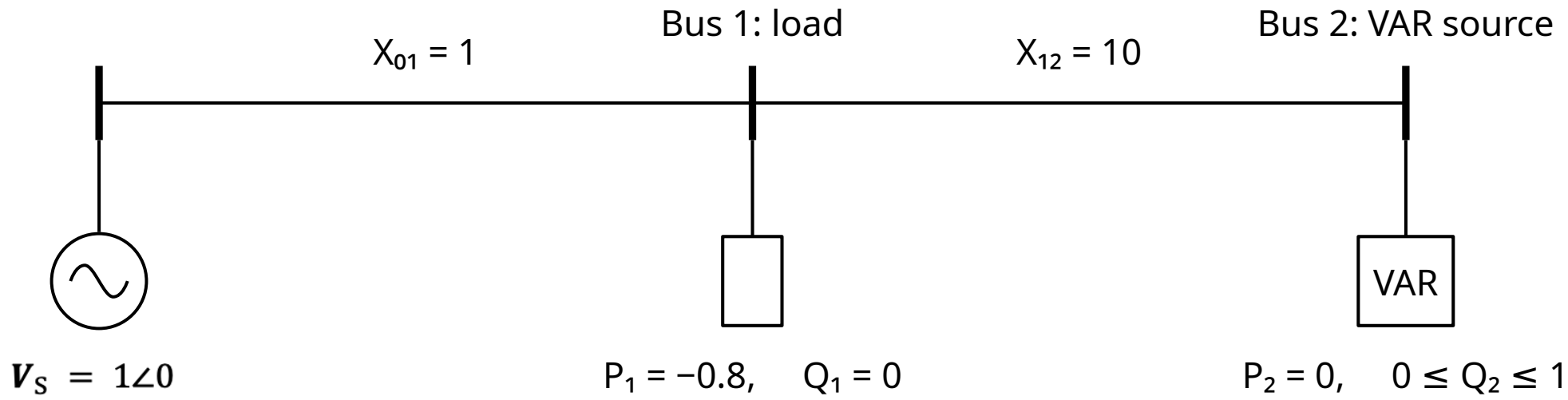


193–205 McKean Street, Fitzroy North

https://www.yef.org.au/community-batteries/fitzroy-north-community-battery

Exercises

Exercise: “VARs don’t travel”



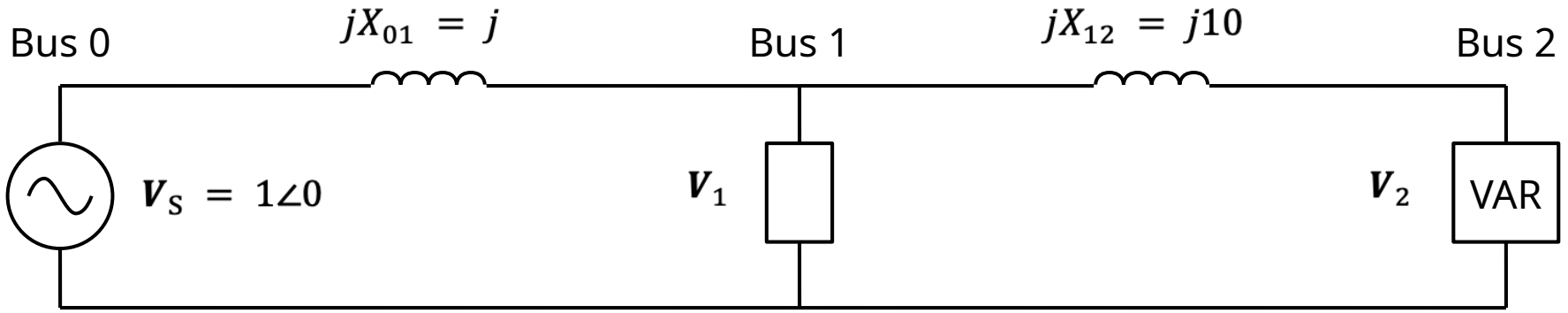
Ignore voltage and current limits.

(a) What is the least Q_2 required to hold $V_1 = 1$? Does bus 2 have enough Q capability?

(b) What fraction of this export does line 2–1 consume?

(c) How much Q is needed if the VAR source moves directly to bus 1?

Exercise solution



We know that $V_1 = V_0 = 1$, $Q_1 = 0$ and $P_2 = 0$. From the power flow equations,

$$P_1 = \frac{V_1 V_0}{X_{01}} \sin\theta_1, \qquad Q_1 = \frac{{V_1}^2 - V_1 V_0 \cos\theta_1}{X_{01}} + \frac{V_1(V_1 - V_2)}{X_{12}}$$

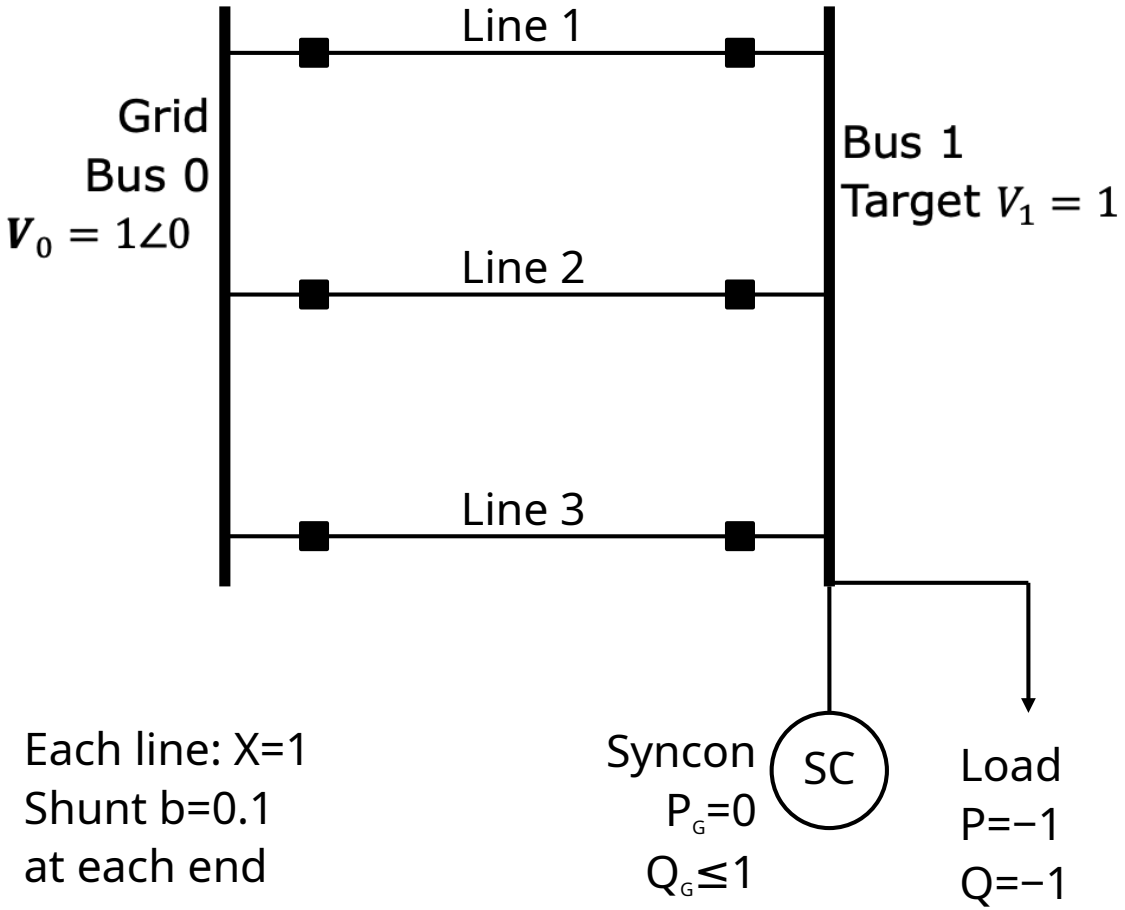
$$P_1 = -0.8 \quad \Rightarrow \quad \sin\theta_1 = -0.8, \qquad \cos\theta_1 = 0.6 \quad \Rightarrow \quad Q_1 = 1 - \cos\theta_1 + \frac{1 - V_2}{10} = 0 \quad \Rightarrow \quad V_2 = 5$$

$$Q_2 = \frac{V_2(V_2 - 1)}{10} = 2 > 1, \qquad Q_{\text{line},12} = \frac{(V_2 - 1)^2}{10} = 1.6 \text{ VAR}$$

Line 2–1 consumes 80% of the VARs export by bus 2. Only 0.4 VAR reaches bus 1.

Bus 2 cannot hold $V_1 = 1$ because $Q_2 = 2 > 1$. Moved to bus 1, the source needs only 0.4 VAR.

Exercise: loss of transmission line



Three identical lossless π lines connect buses 0 and 1.

The load injects $P = -1$ and $Q = -1$.

The synchronous condenser (syncon) supplies only reactive power, with $P_G = 0$ and $0 \leq Q_G \leq 1$.

Can it hold the load voltage at $V_1 = 1$ before and after one line disconnects?

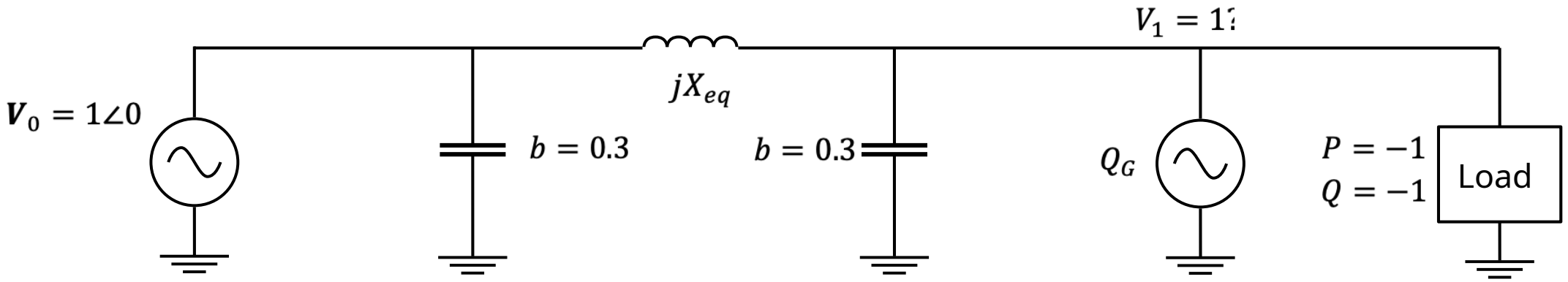
First check with all three lines connected. Assume $V_1 = 1$ and use

$$-1 = 3\ sin\ \theta_1$$

$$Q_G = 1 + 3(1 - cos\ \theta_1) - 0.3$$

from $P = -\frac{V_S V}{x}\ sin\ \theta,\ Q = -\frac{V_S V}{x}\ cos\ \theta + \frac{V^2}{x}$

Exercise solution



Assume $V_1 = 1$ to check the required reactive output.

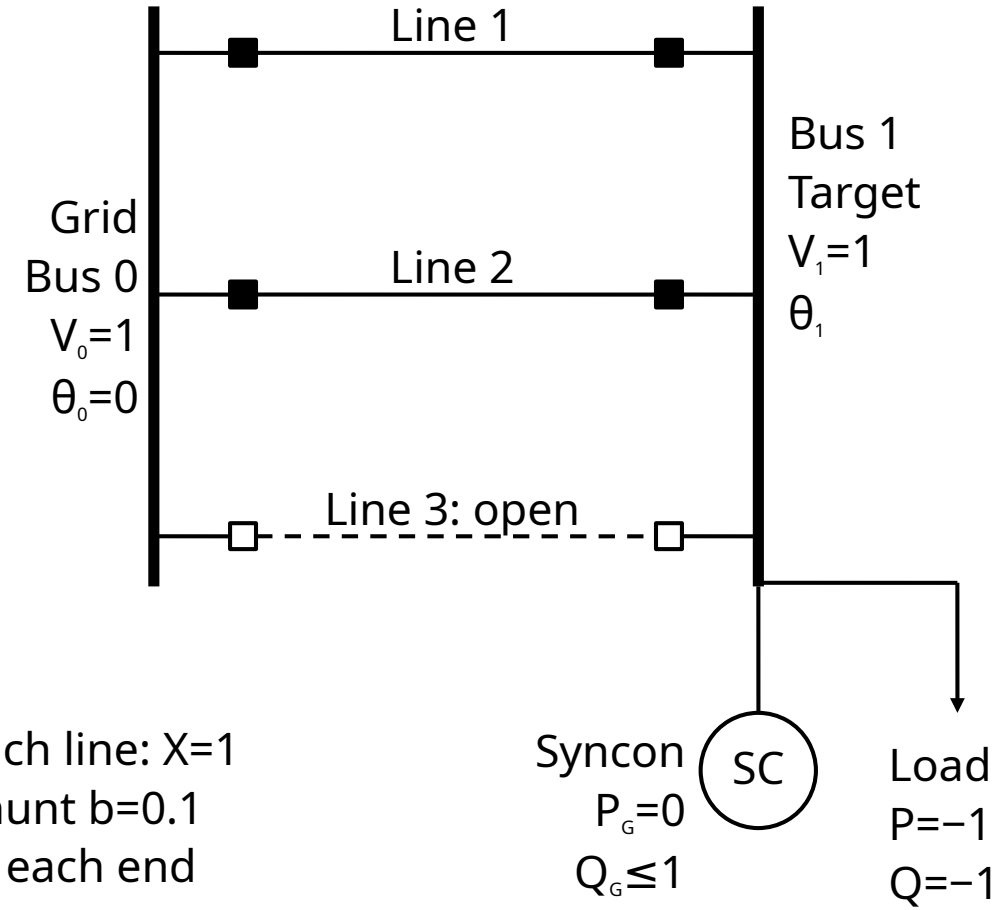
With three lines, $-1 = 3\ sin\ \theta_1$, so $\theta_1 \approx -19.5°$.

Reactive-power balance at bus 1:

$$Q_G = 1 + 3(1 - cos\ \theta_1) - 0.3 \approx 0.9 < 1$$

The syncon can hold the load voltage at $V_1 = 1$.

Exercise: loss of transmission line



One line disconnects completely, including its shunts. Two lines remain.

Assume $V_1 = 1$ and calculate the syncon output needed to hold it.

Can it still hold $V_1 = 1$ with the limit $Q_G \leq 1$?

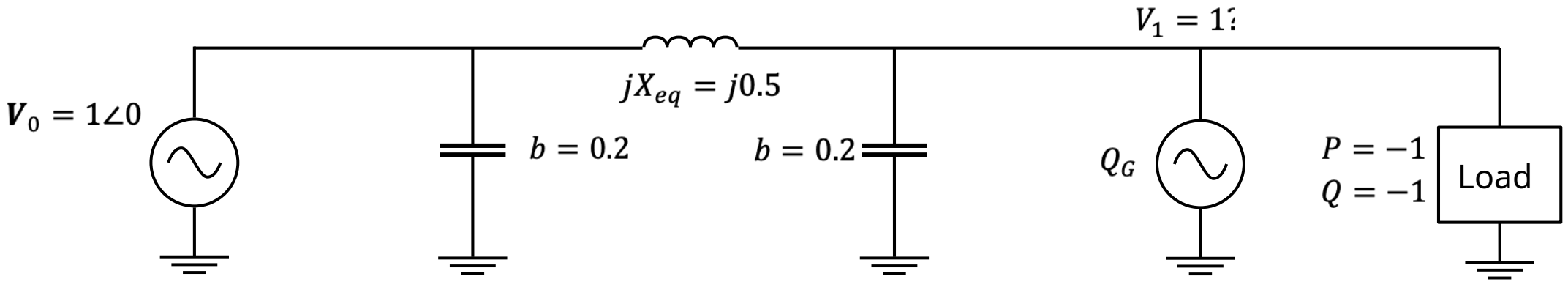
With two lines, use

$$-1 = 2\ sin\ \theta_1$$

$$Q_G = 1 + 2(1 - cos\ \theta_1) - 0.2$$

from $P = -\frac{V_S V}{X}\ sin\ \theta,\ Q = -\frac{V_S V}{X}\ cos\ \theta + \frac{V^2}{X}$

Exercise solution



With two lines, $-1 = 2\ sin\ \theta_1$, so $\theta_1 = -30°$.

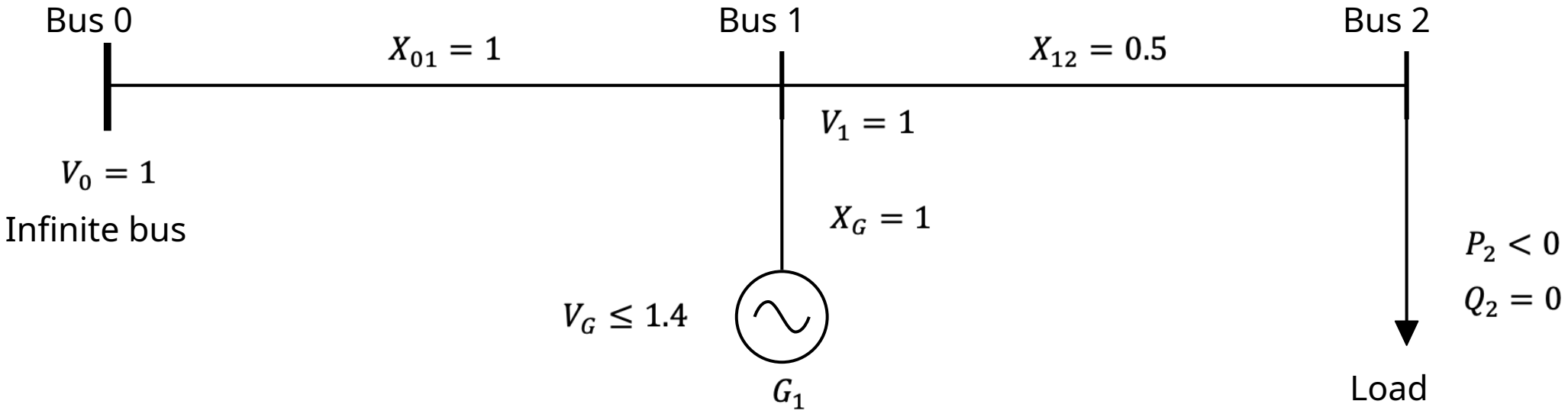
Required syncon output:

$$Q_G = 1 + 2(1 - cos\ 30°) - 0.2 \approx 1.1 > 1$$

The syncon reaches its limit and cannot hold $V_1 = 1$.

Exercise: the effect of AVR limits

The AVR holds $V_1 = 1$ until the generator's internal voltage reaches $V_G = 1.4$.
Assume the generator's internal voltage stays in phase with bus 0.



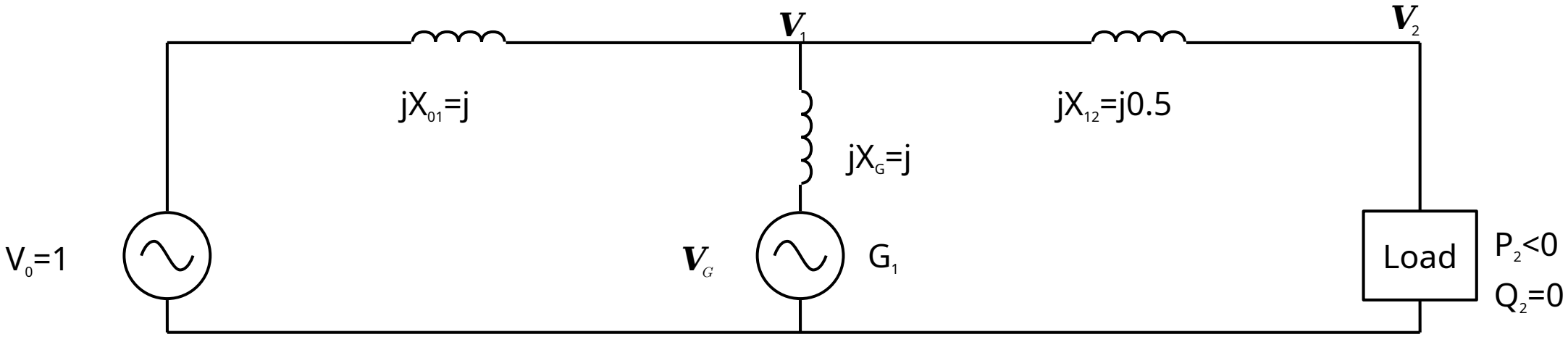
(a) With $V_1 = 1$, find the maximum load $-P_2$. Ignore the excitation limit.

(b) With $V_G = 1.4$ fixed, find the Thevenin equivalent and the new maximum load.

(c) Can $P_2 = -0.8$ be supplied after the excitation limit?

P. Kundur, Power System Stability and Control, McGraw-Hill, 1994, Fig. 14.6, p. 968

Exercise solution



Unlimited excitation, $V_1 = 1$:

$$max(-P_2) = \frac{{V_1}^2}{2X_{12}} = 1.00$$

Excitation limited, $V_G = 1.4$:

$$max(-P_2) = \frac{{V_{th}}^2}{2X_{total}} = 0.72$$

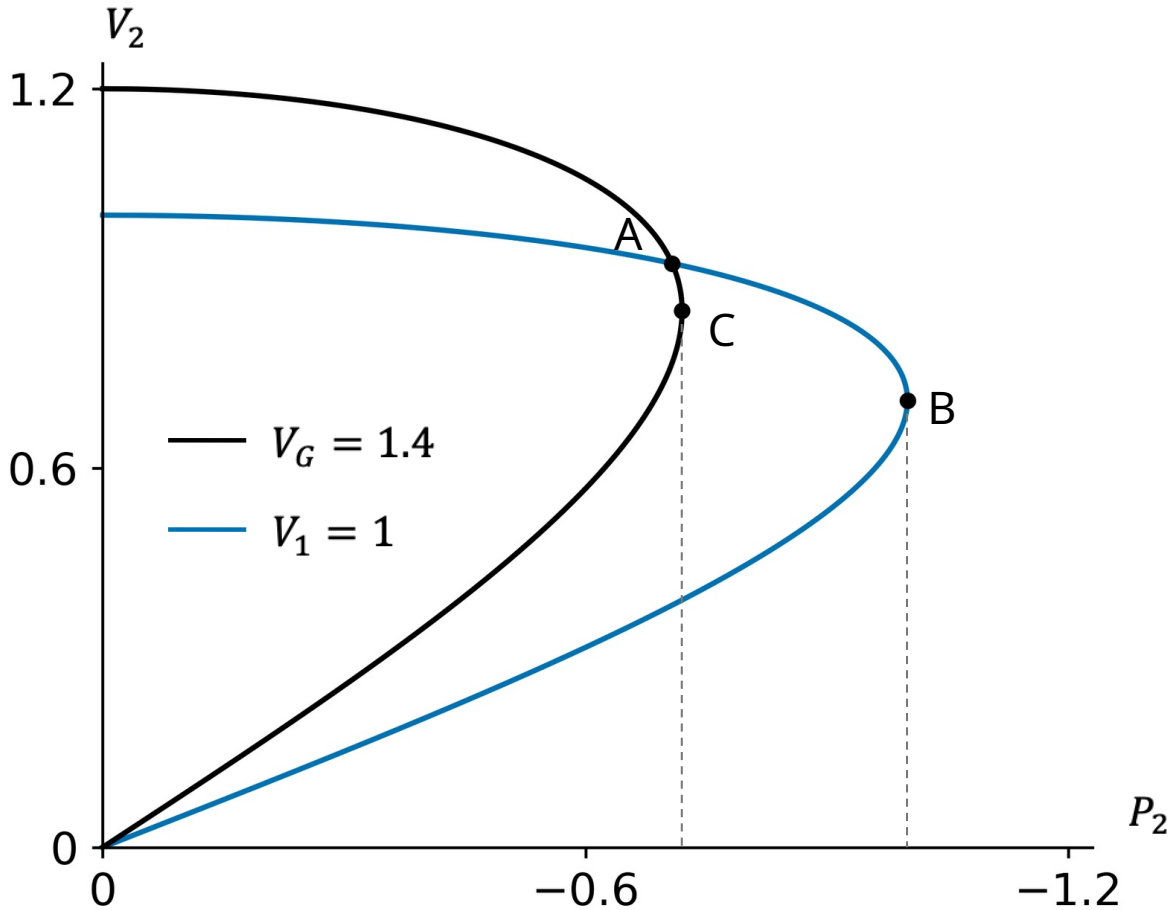
Voltage divider (applies at no load):

$$V_{th} = V_0 + \frac{X_{01}}{X_{01} + X_G}(V_G - V_0) = 1.2$$

Sources shorted: $X_{th} = \frac{X_{01}X_G}{X_{01}+X_G} = 0.5.$

Series line: $X_{total} = X_{th} + X_{12} = 1$

Exercise solution



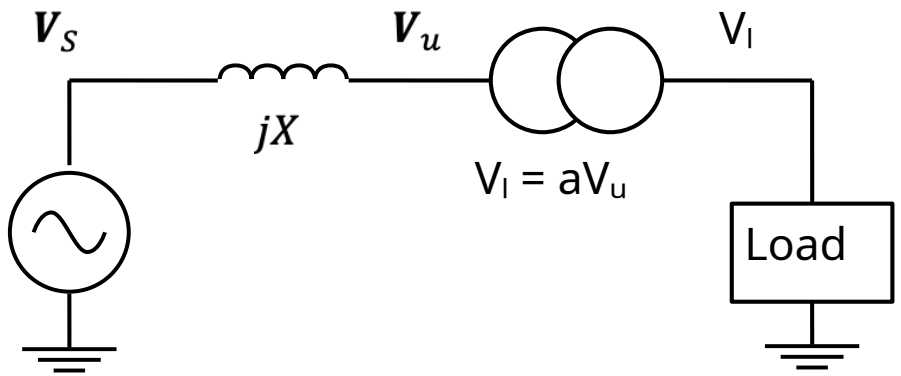
While the AVR holds $V_1 = 1$, the load voltage V_2 falls as load increases along the blue line.

At A, excitation reaches $V_G = 1.4$. Further loading now lowers V_1 below 1 and voltage follows the black line.

The maximum load is 0.72 at C, not 1.00 at B (the max without the excitation limit).

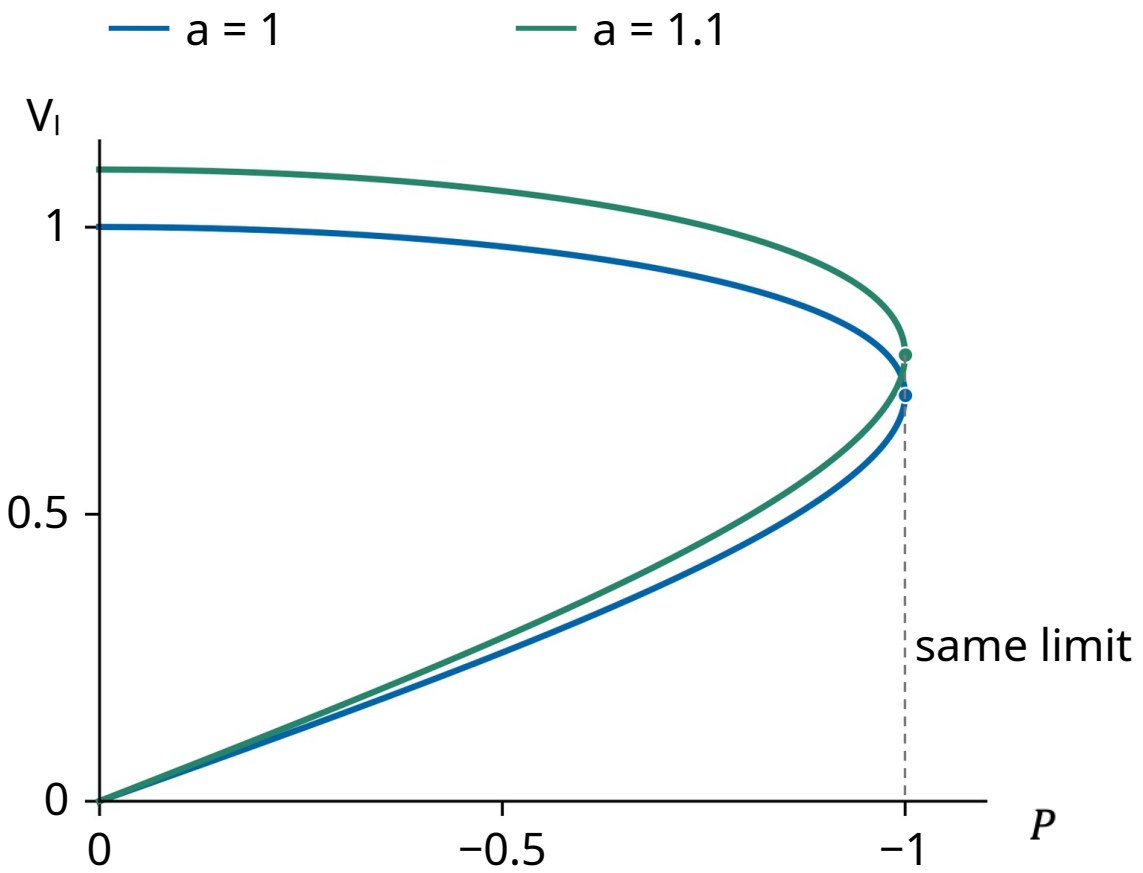
Thus $P_2 = -0.8$ has no steady solution after the excitation limit.

An ideal **receiving-end** tap changes voltage without changing the line's power limit.



Tapping up raises load voltage.

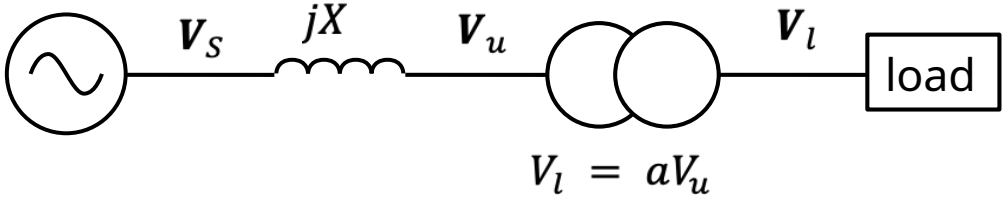
Upstream voltage and current stay the same.



$V_s = 1, \quad X = 0.5, \quad Q = 0$

Unusual distribution network response to tap-changes

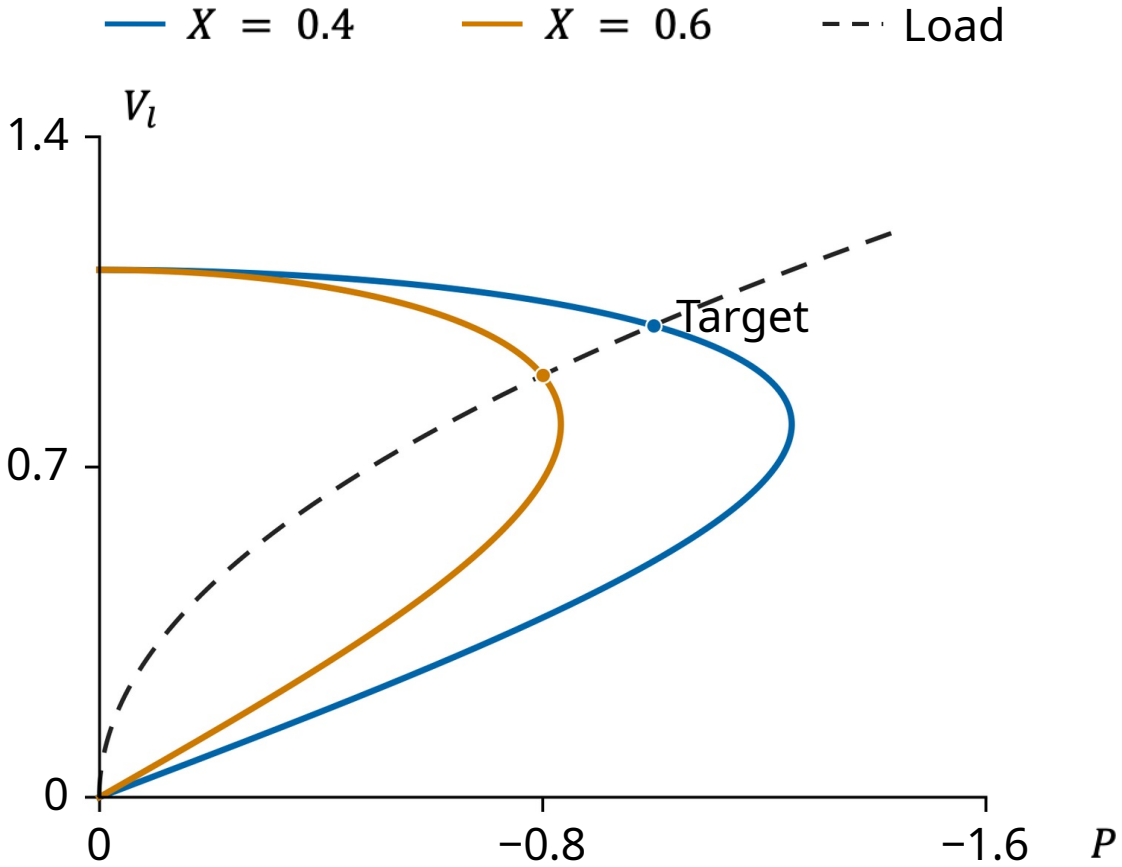
After an outage, restoring load voltage can require more power than the line can transfer (*assuming the network is load increases with voltage, e.g.* $P = -V_l^2$)



An outage increases X from 0.4 to 0.6, decreasing network voltage.

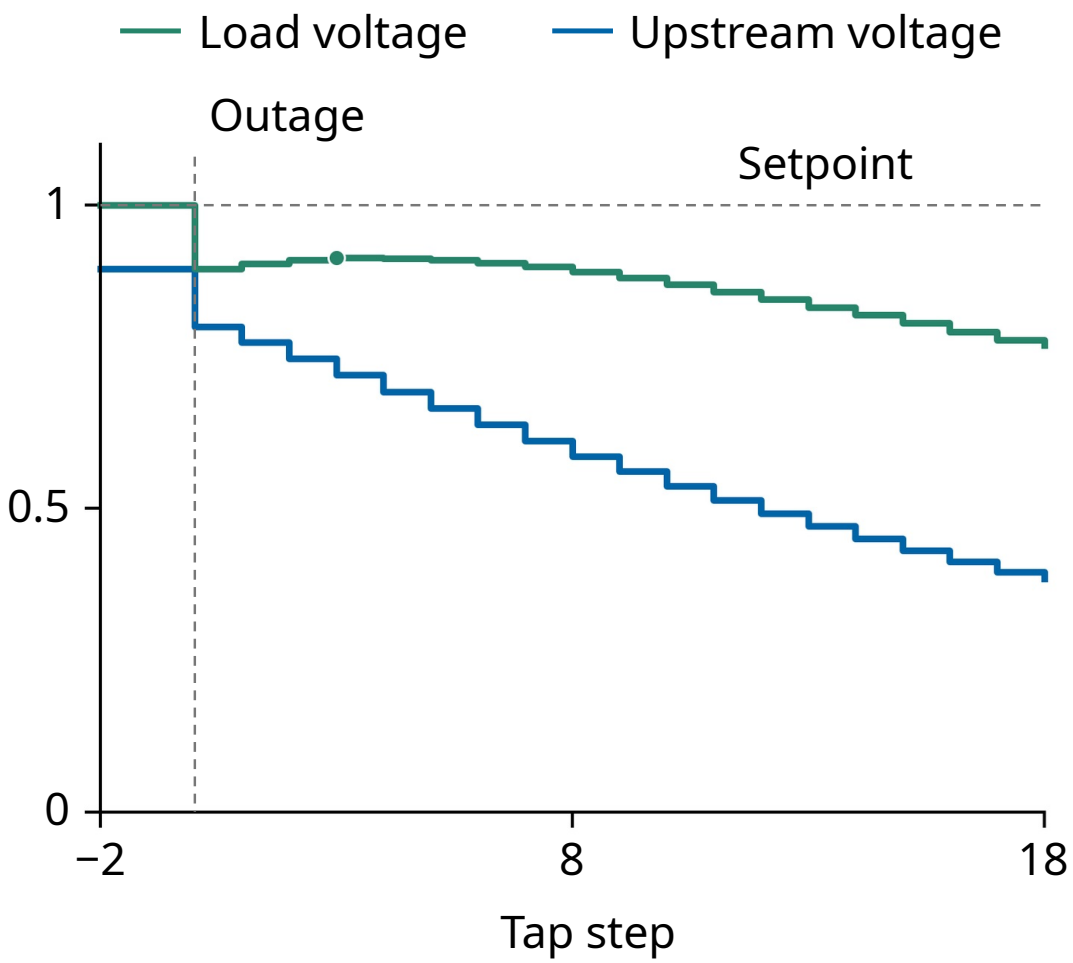
Tapping up raises voltage, initially restores some of the loads power demand.

Current rises and upstream voltage falls further, prompting more tap-ups.



Unusual distribution network response to tap-changes

Repeated tap-ups eventually lowers the load voltage, which is the opposite of the intention.



Extra slides

Analyzing voltage stability dynamics

(Why/when is the low voltage branch unstable?)

Dynamic Voltage Stability Analysis

Consider a load with $Q = 0$ and a fixed power target. Its operating point is $P^\circ < 0, V^\circ > 0$.

Its power consumption is described by the following transfer function:

$$H(s) = \frac{\Delta P(s)}{\Delta V_1(s)} = \frac{2P^\circ}{V^\circ}\frac{Ts}{1+Ts}$$

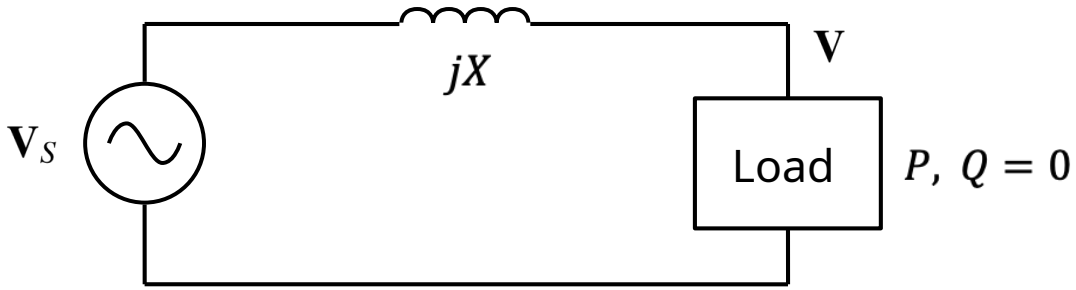
Immediately after a voltage change: $\Delta P \approx \frac{2P^\circ}{V^\circ}\Delta V$.

For a sustained imposed voltage change: $\Delta P \to 0$ (constant-power at steady state)

$T > 0$ is the transfer function time constant.

Model: David Hill https://doi.org/10.1109/59.221270

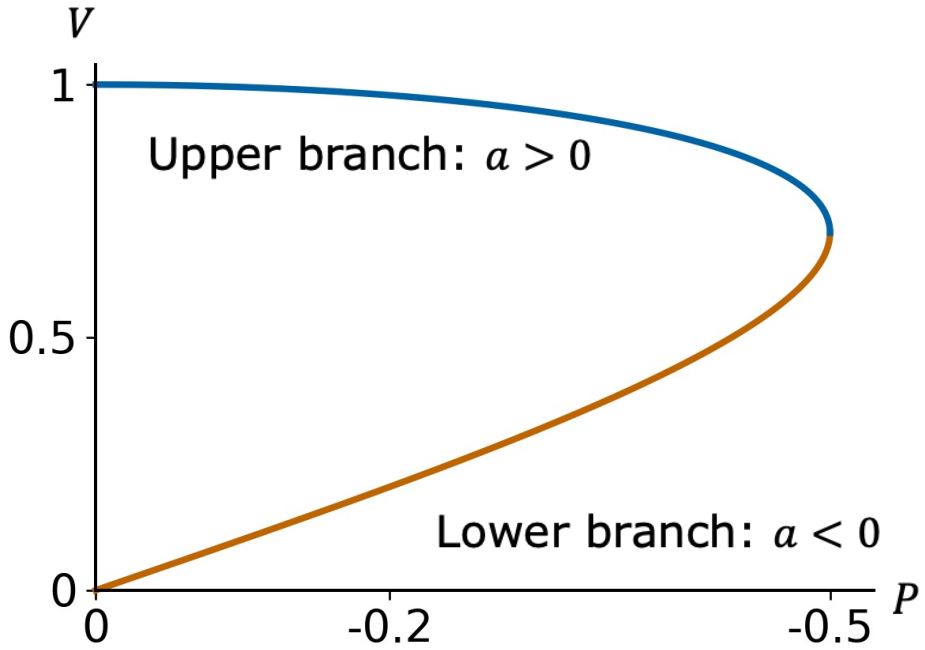
Dynamic Voltage Stability Analysis



The network is instantaneous relative to the load (thus, it does not have a transfer function).

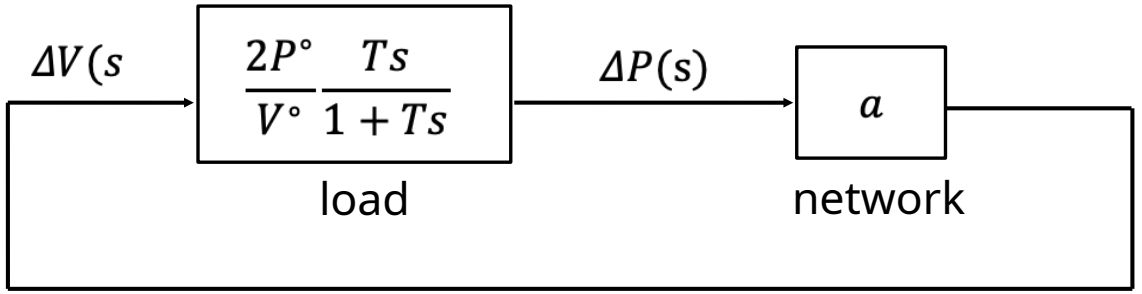
With $Q = 0$,

$$(XP)^2 + V^4 - V_S^2 V^2 = 0 \qquad a = \frac{dV}{dP}$$



(curve shown for $V_S = X = 1$)

Dynamic Voltage Stability Analysis

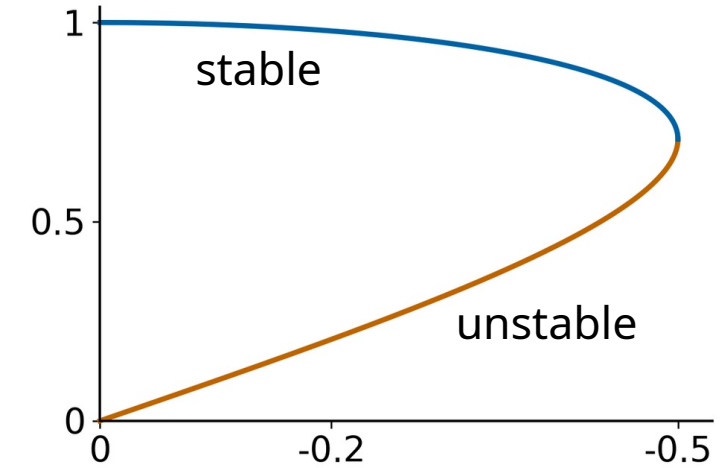


Closing the loop gives the pole $\mu = \frac{V_S^2 - 2(V^\circ)^2}{TV_S^2}$

(next slide)

Upper branch: $V^\circ > V_S/\sqrt{2}$ → $\mu < 0$, stable

Lower branch: $0 < V^\circ < V_S/\sqrt{2}$ → $\mu > 0$, unstable



Dynamic Voltage Stability Analysis

From the positive feedback formula,

$$\frac{\Delta V(s)}{\Delta P(s)} = \frac{a}{1 - aH(s)}$$

The poles satisfy $1 - aH(s) = 0$. Solving for s gives

$$1 - a\frac{2P^\circ}{V^\circ}\frac{Ts}{1+Ts} = 0 \quad \Rightarrow \quad \mu = -\frac{1}{T(1 - 2aP^\circ/V^\circ)}$$

For $Q = 0$ and fixed V_S, X, differentiate the network PV relation:

$$(XP)^2 + V^4 - V_S^2V^2 = 0 \quad \Rightarrow \quad a = \left.\frac{dV}{dP}\right|_\circ = -\frac{X^2P^\circ}{V^\circ[2(V^\circ)^2 - V_S^2]}$$

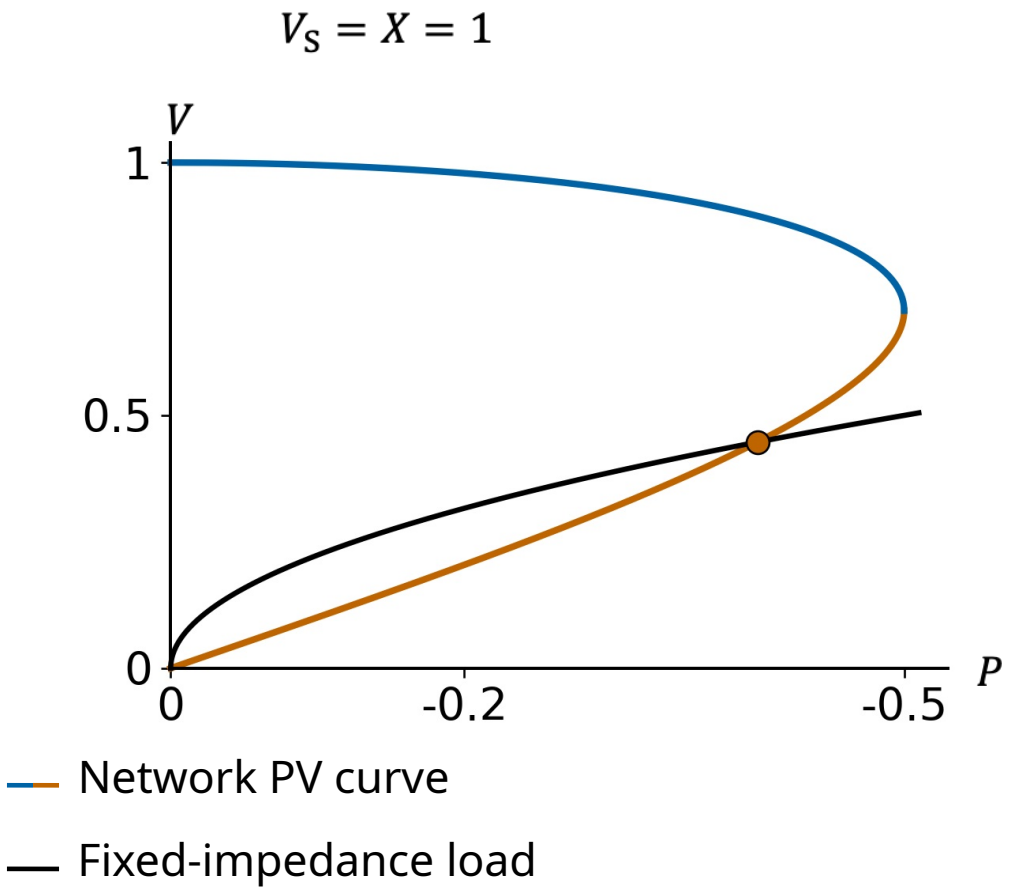
At equilibrium, $X^2(P^\circ)^2 = (V^\circ)^2[V_S^2 - (V^\circ)^2]$, so

$$1 - \frac{2aP^\circ}{V^\circ} = 1 + \frac{2[V_S^2 - (V^\circ)^2]}{2(V^\circ)^2 - V_S^2} = \frac{V_S^2}{2(V^\circ)^2 - V_S^2}$$

Substituting gives $\mu = \frac{V_S^2 - 2(V^\circ)^2}{TV_S^2}$

Dynamic Voltage Stability Analysis

The same low-voltage operating point is **stable when the load has fixed impedance**.



Use a fixed resistive load, $R = 0.5$, with $Q = 0$:

$$P = -\frac{V^2}{R}$$

Its positive-voltage equilibrium is the same lower point: $P^\circ = -0.4$, $V^\circ = \sqrt{0.2}$.

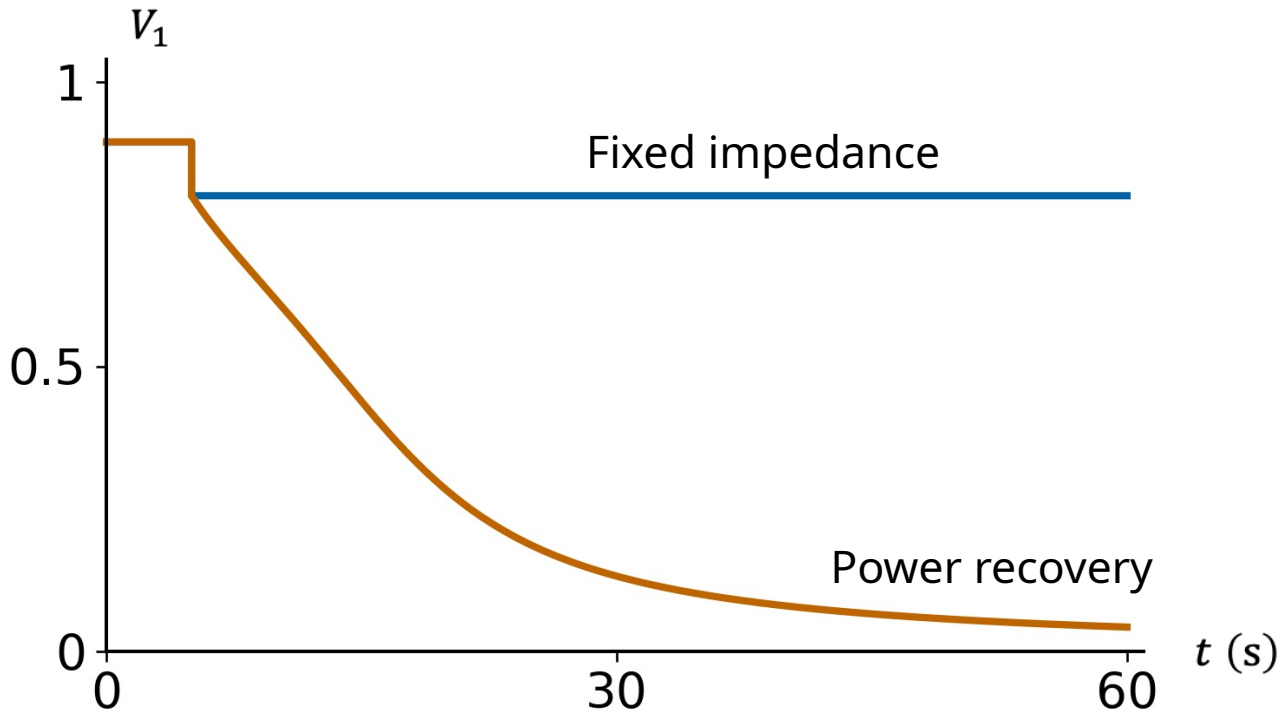
Consumption falls with voltage and does not recover.

For a fixed source and inductive line, $X = \omega L$, the remaining passive electrical transients decay:

$$\mathrm{Re}(\mu) = -\frac{R}{L} < 0$$

Dynamic Voltage Stability Analysis

Example: line outage leaves the weakened network unable to support the load's restored power.



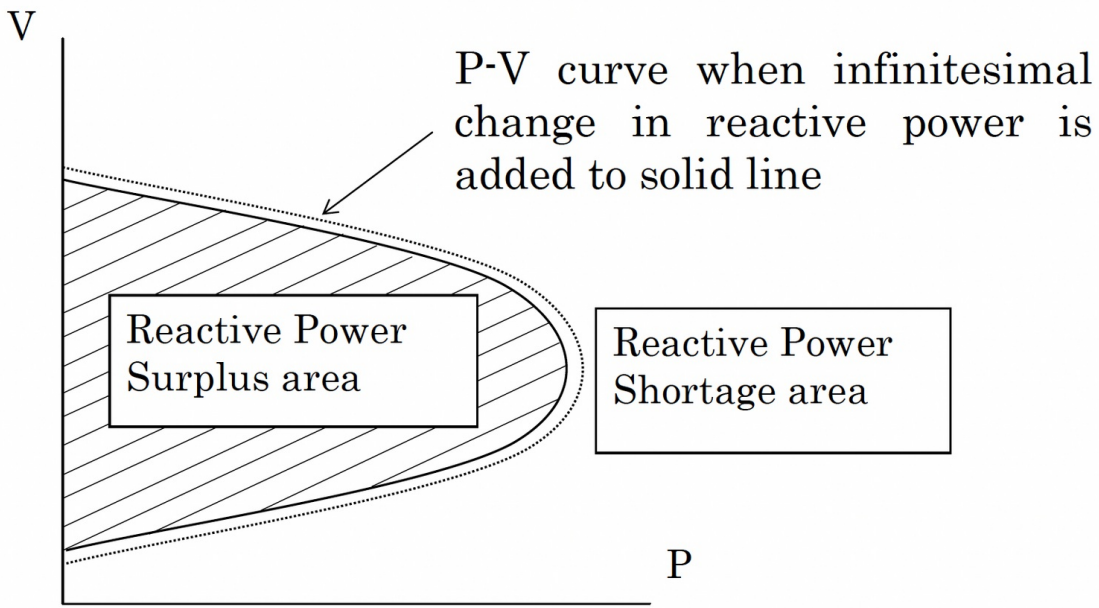
At $t = 5$ s, a line trips.

Voltage drops immediately.

The constant-power load causes voltage collapse, while the constant impedance load does not.

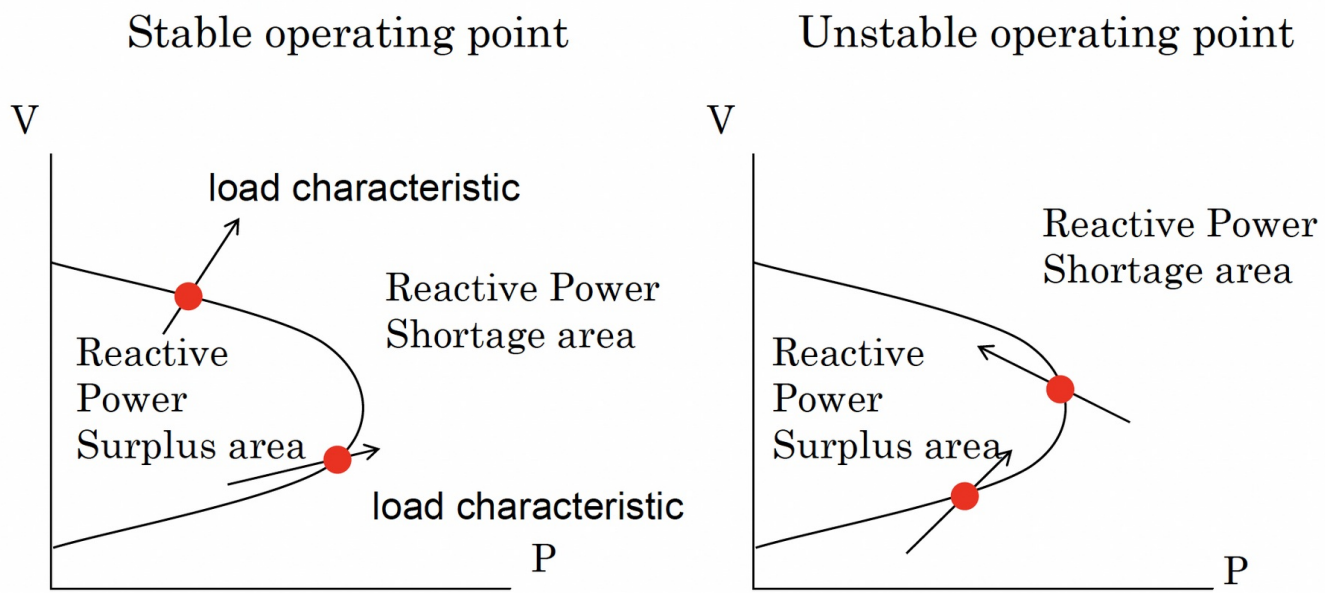
TEPCO has a different way of explaining the same phenomenon — voltage collapse is caused by the interaction of the network in a low-voltage operating point and load dynamics.

Static Voltage Stability (1)



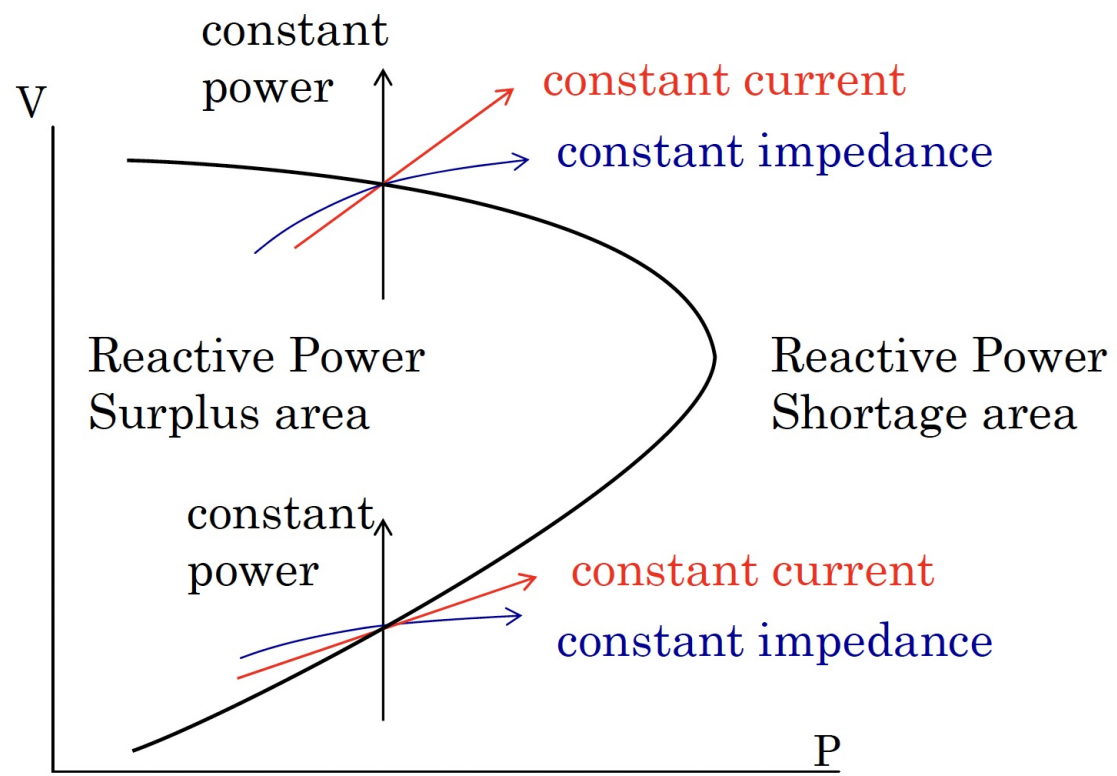
TEPCO (2010)

Static Voltage Stability (2)



TEPCO (2010)

Static Voltage Stability (3)



TEPCO (2010)