Voltage Control

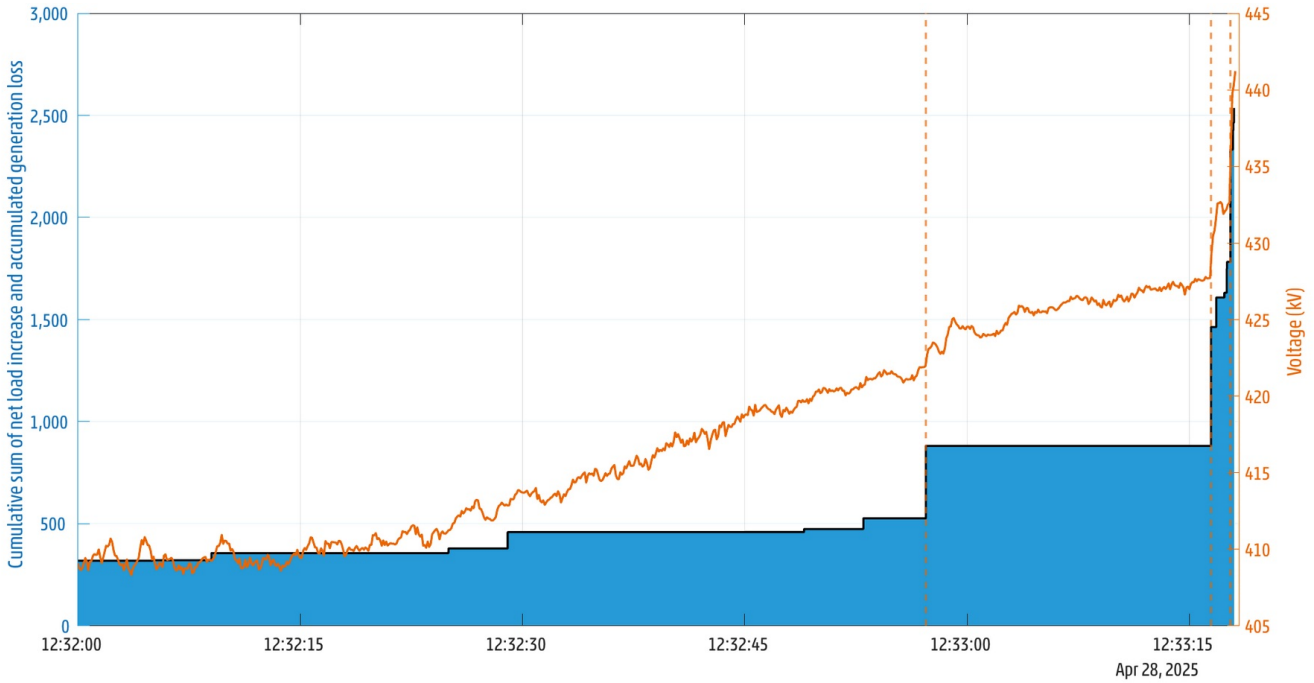
ELEN90060 Power System Analysis

Keith Moffat

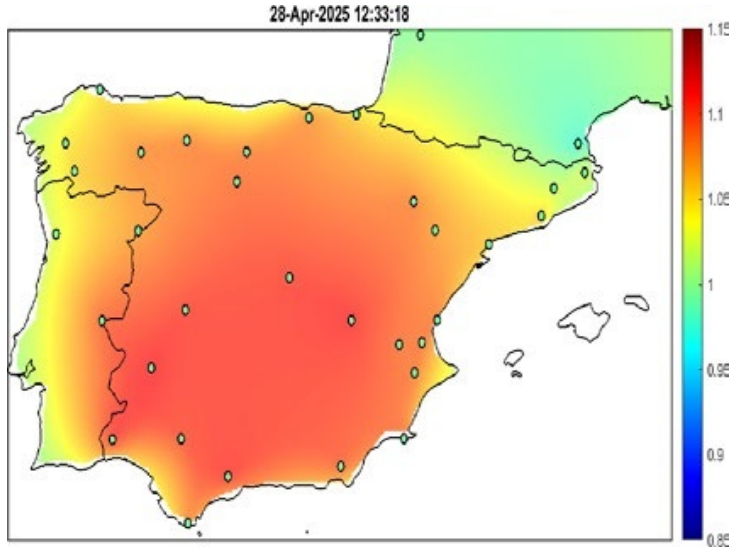


https://www.esa.int/ESA_Multimedia/Images/2025/05/Night-time_light_emissions_during_blackout_in_Spain_and_Portugal

What caused the 2025 Iberian blackout?



Voltage across Iberia at 12:33:18



Overvoltage tripped generation, which caused more overvoltage, which caused more trips.

https://www.entsoe.eu/publications/blackout/28-april-2025-iberian-blackout/

Iberian voltage as generators in the South trip offline

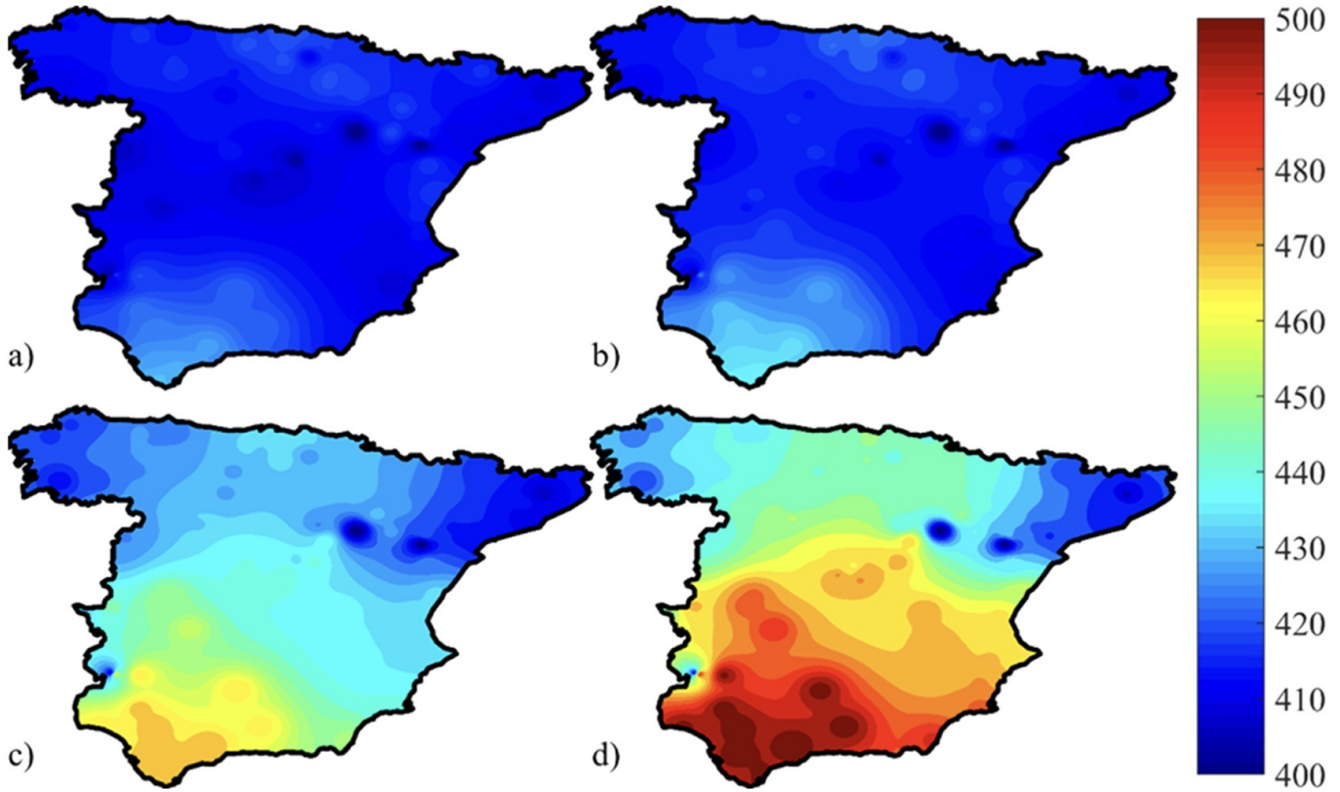


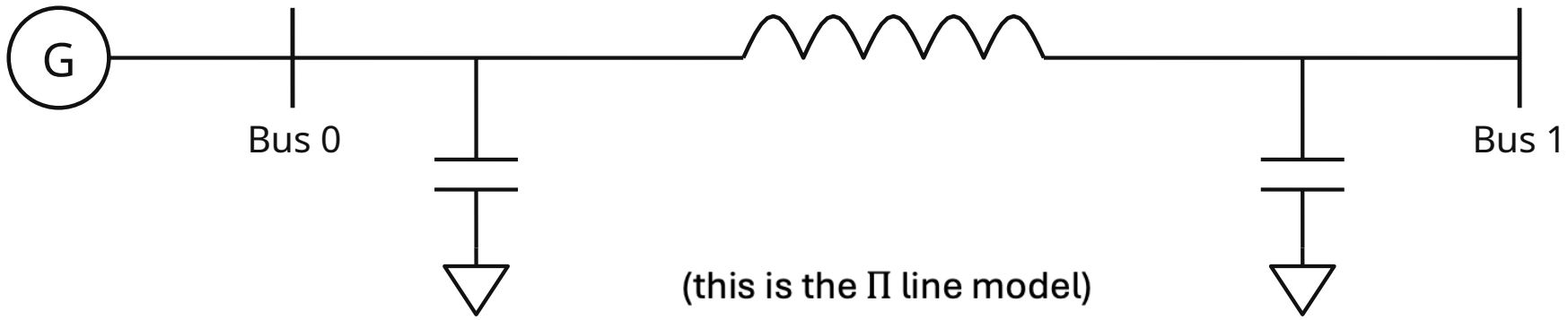
Fig. 11. Voltage heat maps at 400 kV grid level: a) initial situation, b) loss of 355 MW (event 1), c) loss of 1085 MW (event 1 +event 2), and d) loss of 1635 MW (event 1 +event 2 +event 3).

https://doi.org/10.1016/j.segan.2026.102125 , Rouco et al

Ferranti effect on a lightly loaded line

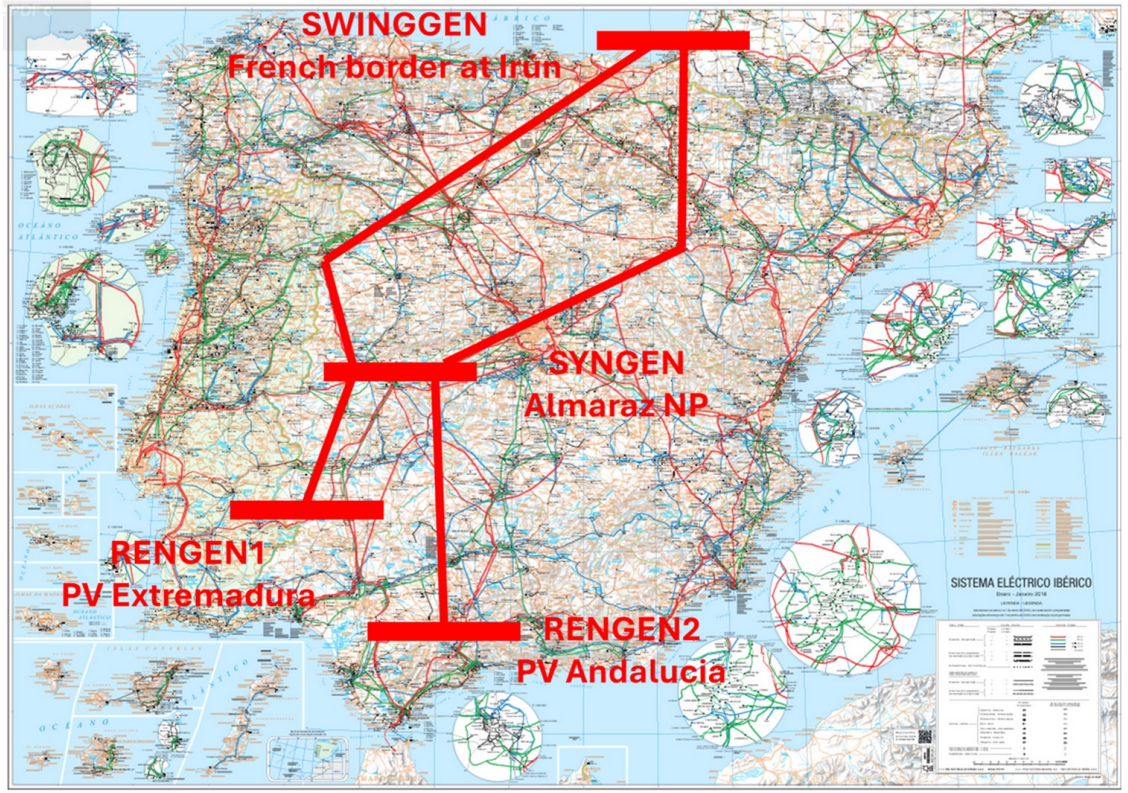
Charging current into the Bus 1 capacitance increases Bus 1 voltage

$$V_0 < V_1$$



The Ferranti effect is most pronounced when there is no real power injection at Bus 1.

The generators tripping offline in the South reduced the real power injection, increasing voltage.



SWINGGEN held at constant voltage

RENGEN1 falls from 600 to 0 MW

while RENGEN2 stays at 400 MW

SYNGEN tries to hold its bus at 400 kV but reaches its limit

Illustrative model. Source: Rouco et al., Fig. 5, https://doi.org/10.1016/j.segan.2026.102125

VAR support capability of SYNGEN

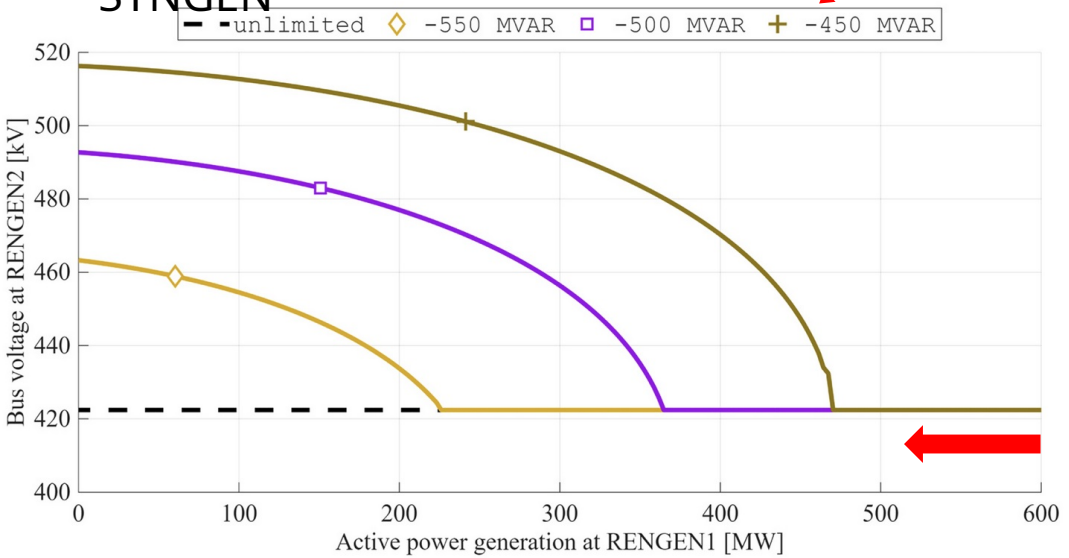


Fig. 8. Evolution (from right to left) of the voltage of the RENGEN2 node with the loss of renewable generation at RENGEN1.

Why didn’t inverters help by extracting Q to lower voltage?

The Spanish renewable plant grid code in April 2025 specified a close-to-unity power factor.

Is there a better way to control inverters?

Spoiler: yes, Volt-VAR control. Australia has mandated it since 2021.

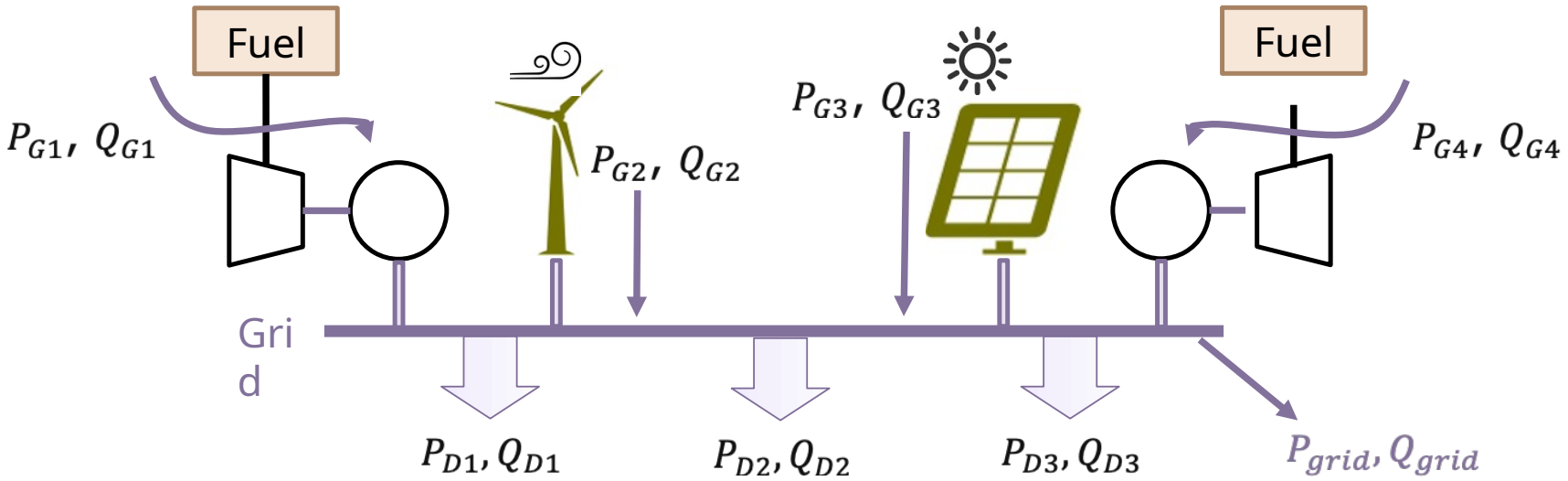
Voltage (Magnitude) Regulation

Why regulate transmission voltage?

- Keep equipment within voltage limits. (This prevents overvoltage collapse!)
- Preserve power-transfer and voltage-stability margins
- Limit current and resistive losses for a given power transfer

Why regulate distribution voltage?

- Keep customer voltage within an acceptable range
- Preserve power-transfer and voltage-stability margins
- When needed, voltage can alter grid-connected load and generation



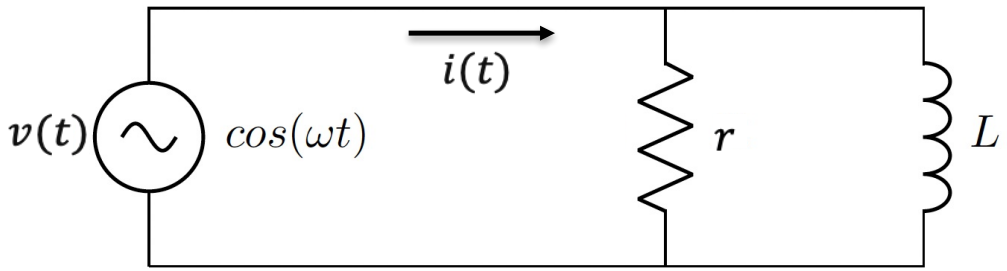
Steady-state power balance:

$$\sum_k P_{Gk} = \sum_i P_{Di} + P_{grid} \iff f$$

$$\sum_k Q_{Gk} = \sum_i Q_{Di} + Q_{grid} \iff V$$

What is reactive power?

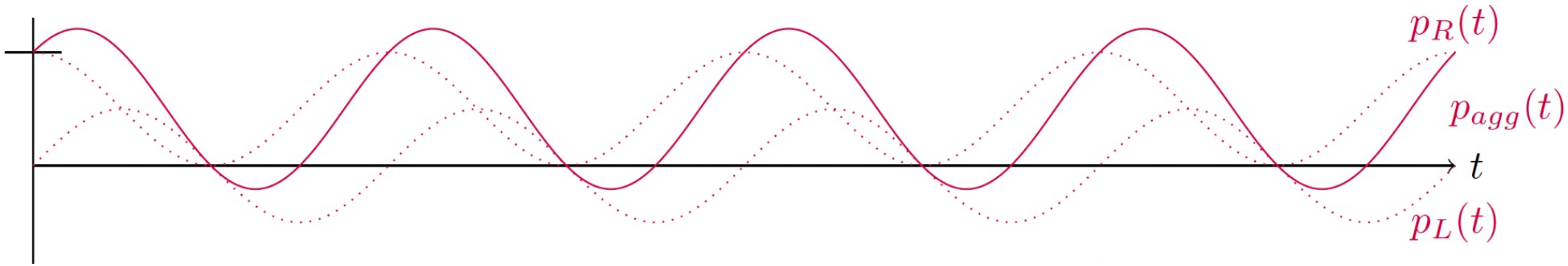
AC current waves can be split into a part in phase with voltage and a part 90° out of phase.



For this simple circuit,

Q is the amplitude of $p_L(t)$

P is the amplitude of $p_R(t)$



The instantaneous power drawn from the voltage source is

$$p(t) = v(t)i(t) = P[1 + \cos(2\omega t)] + Q\sin(2\omega t)$$

What is reactive power?

Reactive power Q (VARs) is a concept used by power system engineers because it is useful.

Like real power P (Watts), **reactive power is conserved**:

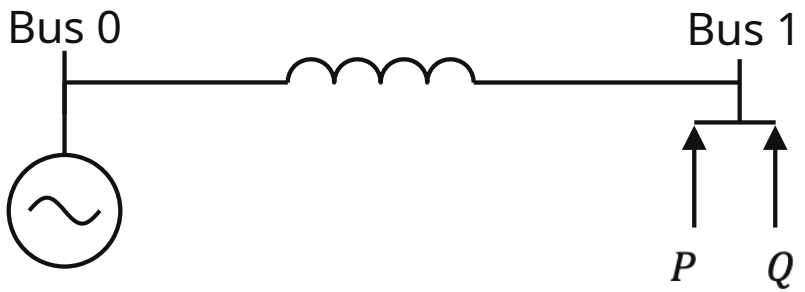
$$\sum_{i \in \text{all injections/extractions}} Q_i = 0$$

Why do power system engineers care about reactive power?

Q affects line currents and voltages.

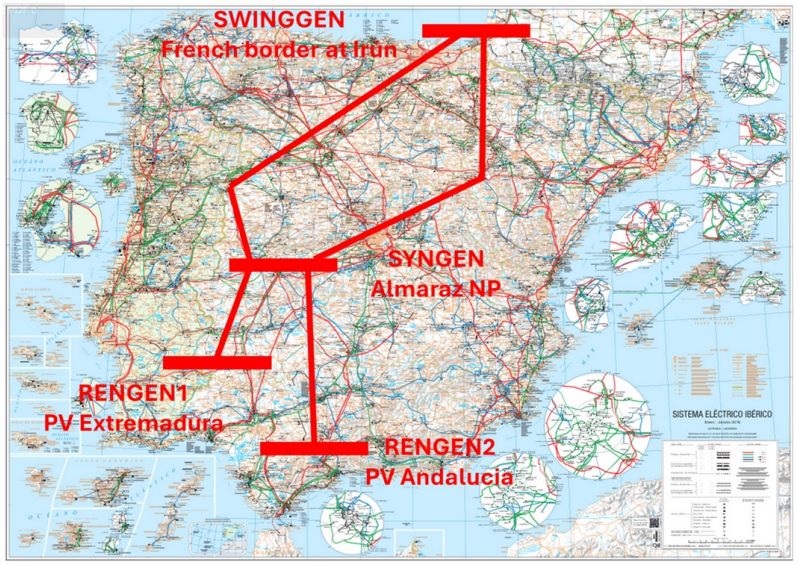
Thus, where Q is supplied changes bus voltage magnitudes and line currents.

What is reactive power?



With the source voltage fixed, increasing reactive injection at Bus 1 increases its voltage.

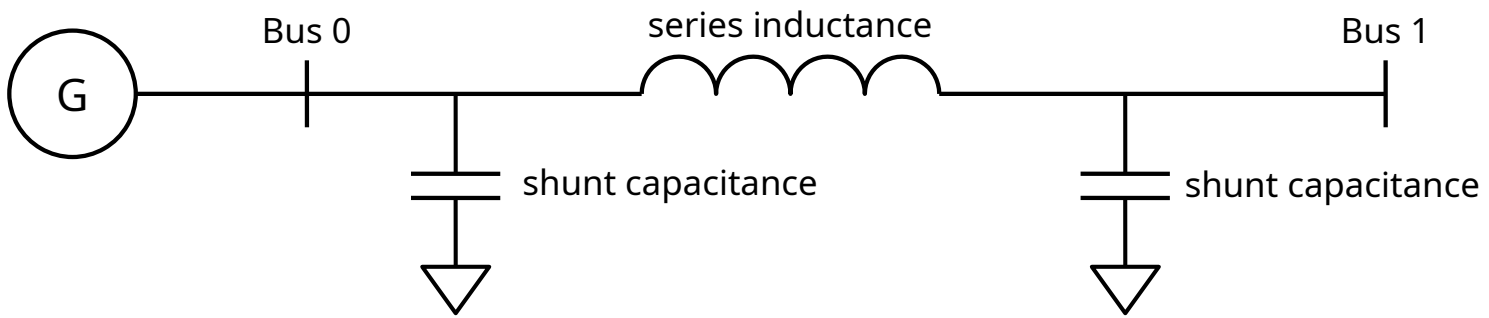
This simple intuition extends to more complex networks.



Uncontrolled reactive-power injections

- Loads
  - Inductive loads, such as motors, absorb reactive power
  - Capacitive loads (less common) inject reactive power

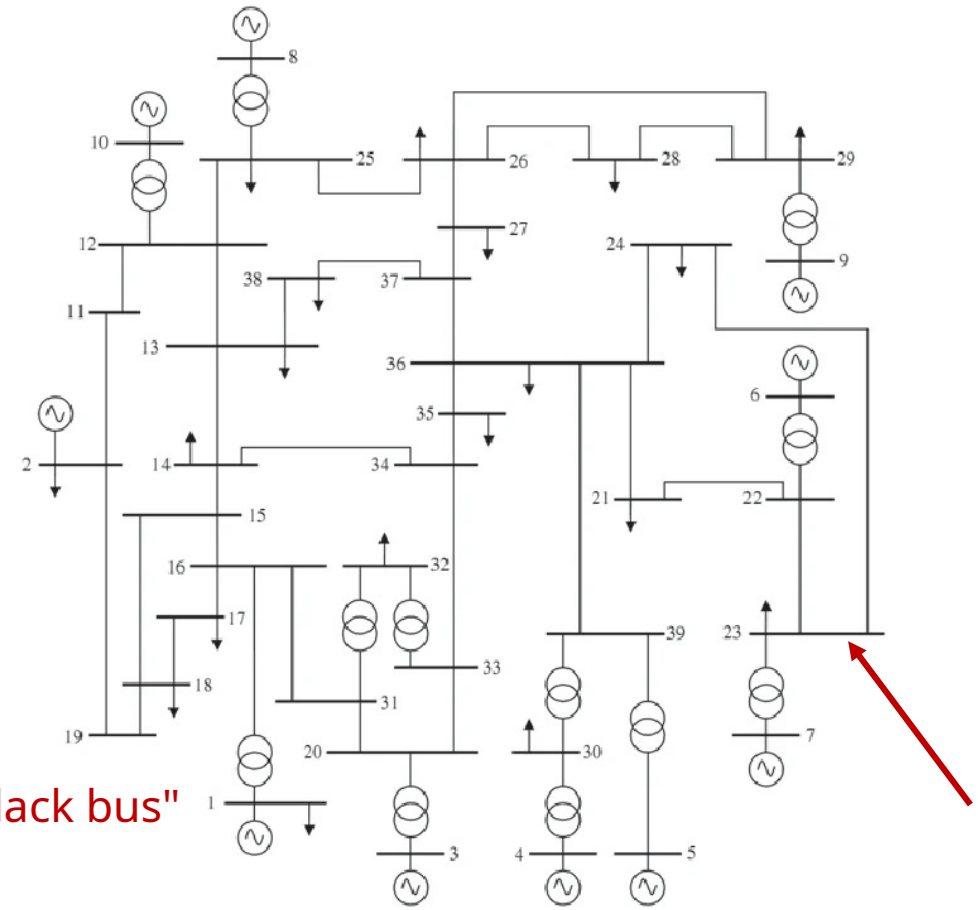
- Power lines
  - series inductance absorbs reactive power; shunt capacitance injects it



Reactive-power control devices

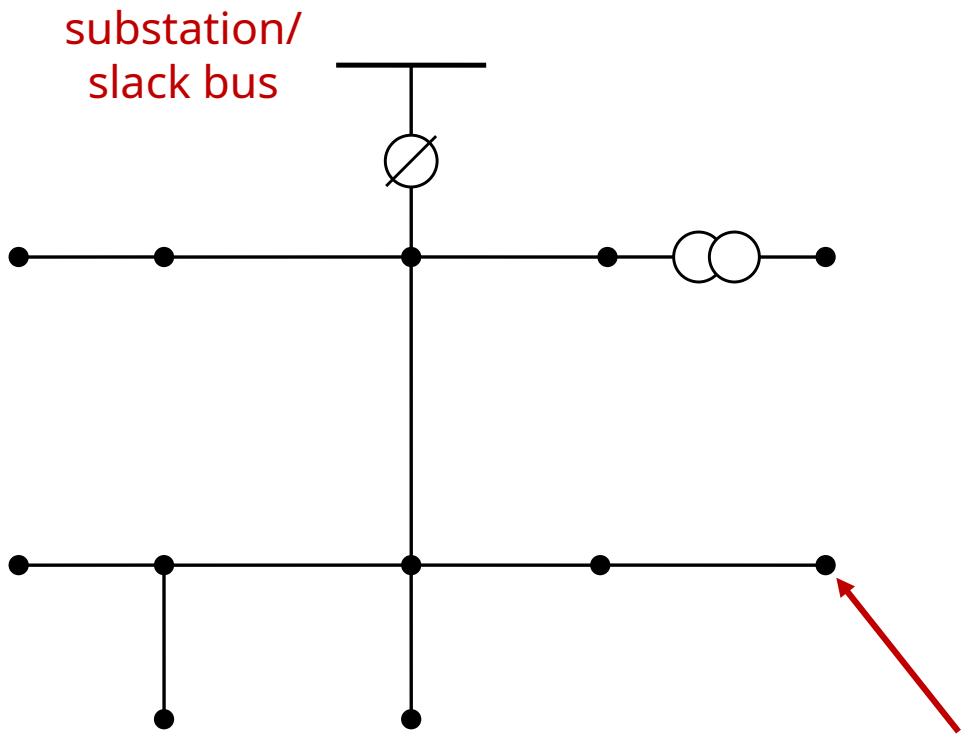
- Synchronous generators are the main source of reactive power in a network
  - Over-excited machine → reactive power generation
  - Under-excited machine → reactive power absorption
- Capacitors inject reactive power
- Inductors absorb reactive power
- Static Var Compensators (SVCs) and Static Compensators (STATCOMs) are power electronics devices that inject/extract reactive power to control voltages
- Any inverter can also inject/extract reactive power. The max amount depends on the hardware, control, and operating condition.

Transmission Networks



"slack bus"

Distribution Networks

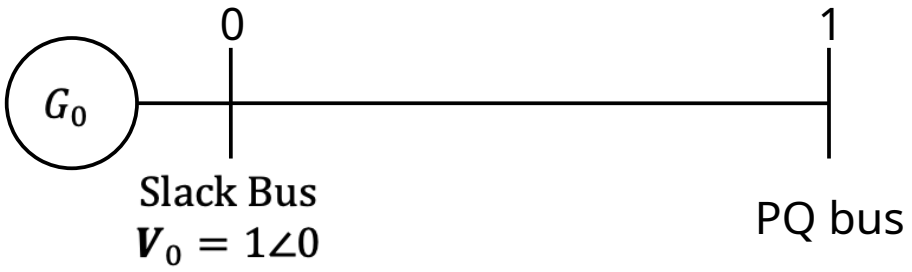


If we inject real or reactive power at a given location, how do the voltages on the network change?

The slack bus

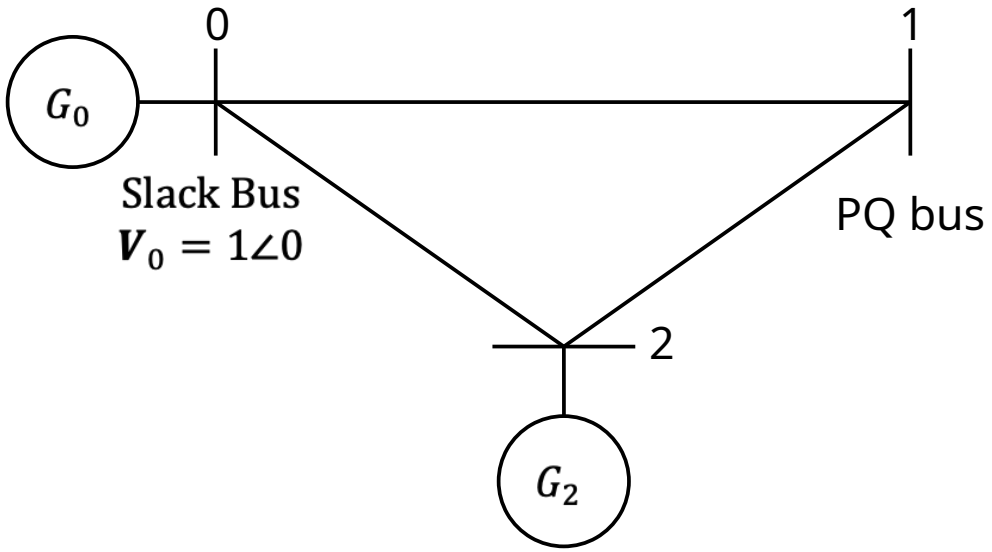
Slack power adjusts its power such that the net real and reactive power injections for the entire network are zero

Two Bus Network



$$\Delta I_0 = -\Delta I_1$$

Three Bus Network



$$\Delta I_0 = -(\Delta I_1 + \Delta I_2)$$

Voltage Magnitude Sensitivity

From the Power Flow module, recall the power-flow linearization:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}\approx\begin{bmatrix}J_1 & J_2\\ J_3 & J_4\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix}$$

ΔP and $\Delta\theta$ contain buses 1,...,n; bus 0 is the fixed slack bus.

ΔQ and ΔV contain only PQ buses.

At PV buses, voltage magnitude is fixed and Q adjusts.

At flat nominal voltage ($\boldsymbol{V}=1\angle 0$), with no shunt admittances:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}\approx\begin{bmatrix}-B_{PP} & G_{PQ}\\ -G_{QP} & -B_{QQ}\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix}$$

G and B are the conductance and susceptance matrices.
Matrix subscripts: P : non-slack buses; Q : PQ buses.

Voltage Magnitude Sensitivity

At flat nominal voltage ($\boldsymbol{V} = 1\angle 0$), with no shunt admittance,

$$\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix} \approx \begin{bmatrix}-B_{PP} & G_{PQ} \\ -G_{QP} & -B_{QQ}\end{bmatrix}^{-1} \begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix}$$

$$=: \begin{bmatrix}X_{PP} & -R_{PQ} \\ R_{QP} & X_{QQ}\end{bmatrix} \begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix}$$

$$\Delta V \approx R\Delta P + X\Delta Q$$

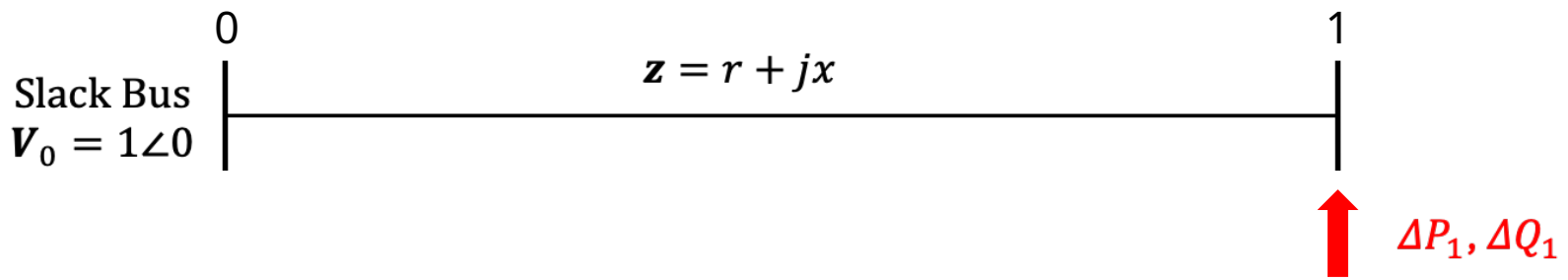
For simplicity, subscripts are omitted in $\Delta V \approx R\Delta P + X\Delta Q$ throughout this module.
ΔP contains all non-slack buses. ΔV and ΔQ contain only PQ buses.
$R = R_{OP}$ and $X = X_{OO}$ are the corresponding voltage-sensitivity matrices.

The matrices R and X can be calculated from the Jacobian of the power flow manifold or estimated from data.

In this class, we will assume R and X are given.

Example 1

For a two-bus network with no shunts, only bus 1 remains in the reduced voltage equations



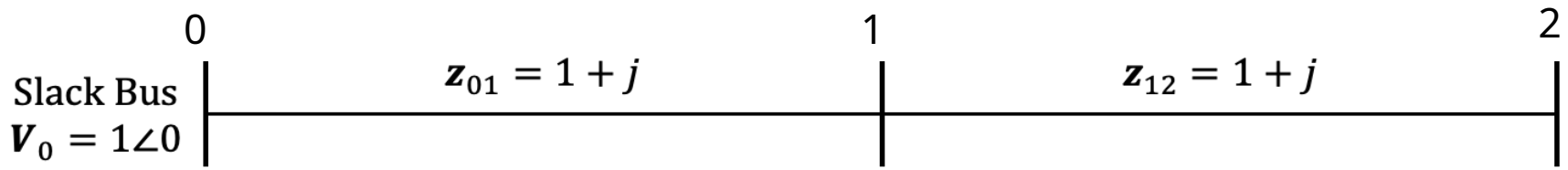
True, nonlinear relationship of power to voltage

$$\Delta V_1 = \frac{(r\Delta P_1 + x\Delta Q_1) + j(x\Delta P_1 - r\Delta Q_1)}{\overline{V}_1}$$

Linear approximation of power to voltage magnitude

$$\Delta V \approx R\Delta P + X\Delta Q \quad \Rightarrow \quad \Delta V_1 \approx r\Delta P_1 + x\Delta Q_1$$

Example 2



$$\Delta V \approx R\Delta P + X\Delta Q$$

$$R = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

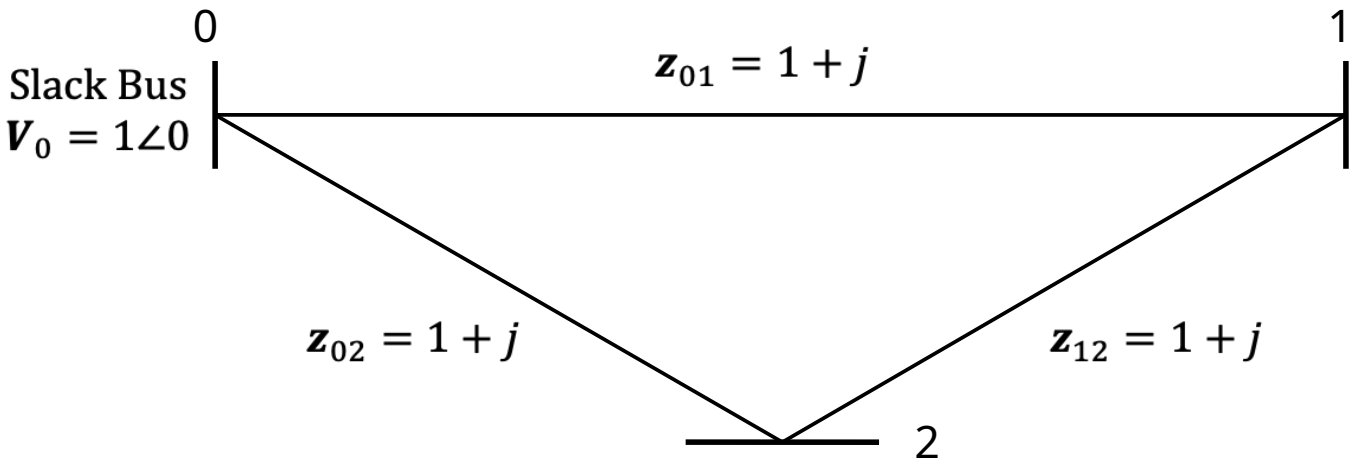
The i, k^{th} entry of R gives the sensitivity of bus i's voltage to a real power injection at bus k

The i, k^{th} entry of X gives the sensitivity of bus i's voltage to a reactive power injection at bus k

Example 1: ΔP_2 affects bus 1 and bus 2's voltage, but it affects bus 2's voltage twice as much.

Example 2: ΔQ_1 affects bus 1 and bus 2's voltage equally.

Example 3



$$\Delta V \approx R\Delta P + X\Delta Q$$

$$R = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad X = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

The i, k^{th} entry of R gives the sensitivity of bus i's voltage to a real power injection at bus k

The i, k^{th} entry of X gives the sensitivity of bus i's voltage to a reactive power injection at bus k

Example 1: ΔP_1 affects bus 1 and bus 2's voltage, but it affects bus 1's voltage twice as much.

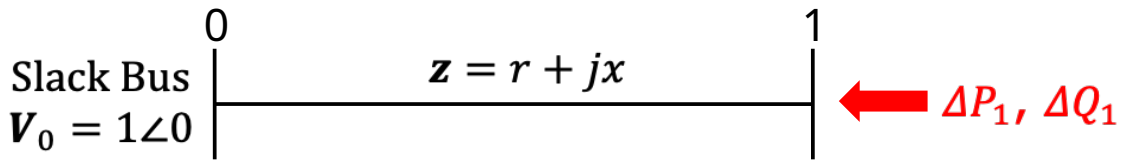
Example 2: ΔQ_2 affects bus 1 and bus 2's voltage, but it affects bus 2's voltage twice as much.

How accurate is $\Delta V_1 \approx r\Delta P_1 + x\Delta Q_1$?

2 bus example, inductive load at Bus 1

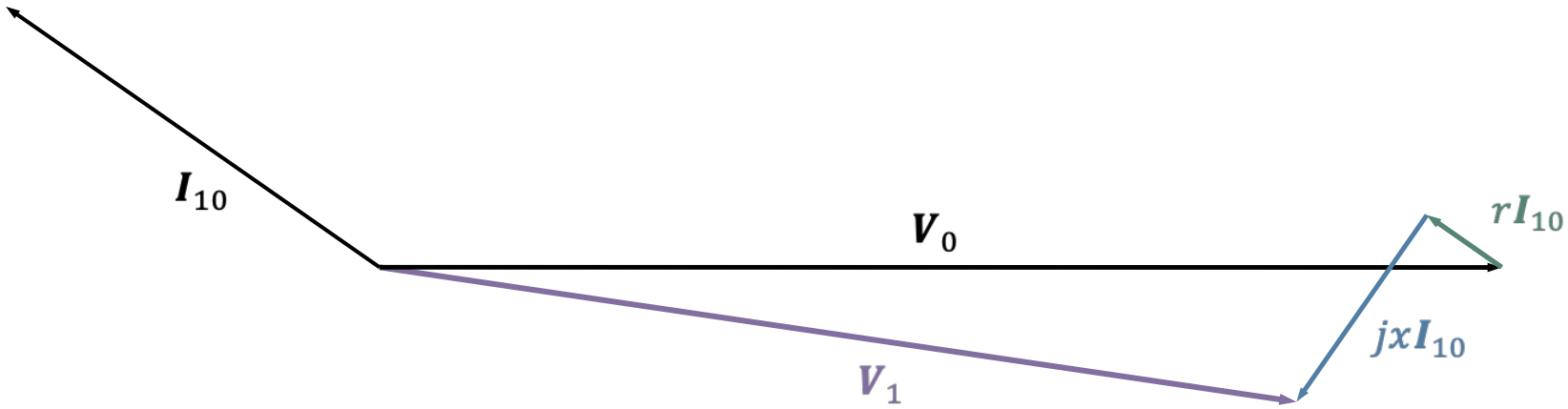
How does a change in the load affect voltage magnitude?

$r = 0.2, \qquad x = 0.5$



Given the load current phasor $\boldsymbol{I}_{10}$, it is easy to determine $\boldsymbol{V}_1 = \boldsymbol{V}_0 + (R + jX)\boldsymbol{I}_{10}$

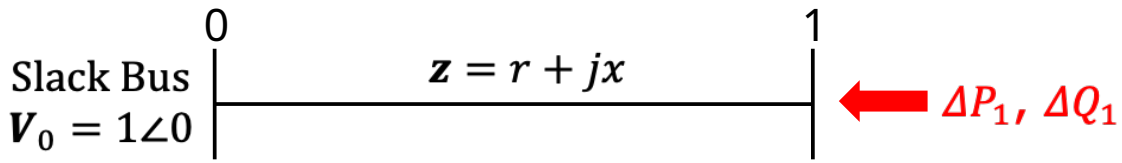
$$\Delta V_1 = V_1 - V_0 = -0.17$$



2 bus example, inductive load at Bus 1

How does a change in the load affect voltage magnitude?

$r = 0.2, \qquad x = 0.5$



Rarely do we have the current phasor $\boldsymbol{I}_{10}$ relative to the "slack bus" $\boldsymbol{V_0}$

Thus, we use $\Delta V_1 \approx rP_1 + xQ_1$

$$P_1 = -0.3\ , \qquad Q_1 = -0.15$$

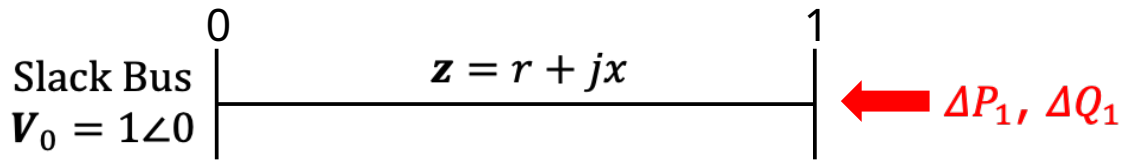
Estimated voltage change: $\Delta V_1 \approx -0.14$

(the exact answer on the previous slide was -0.17)

2 bus example, capacitive load at Bus 1

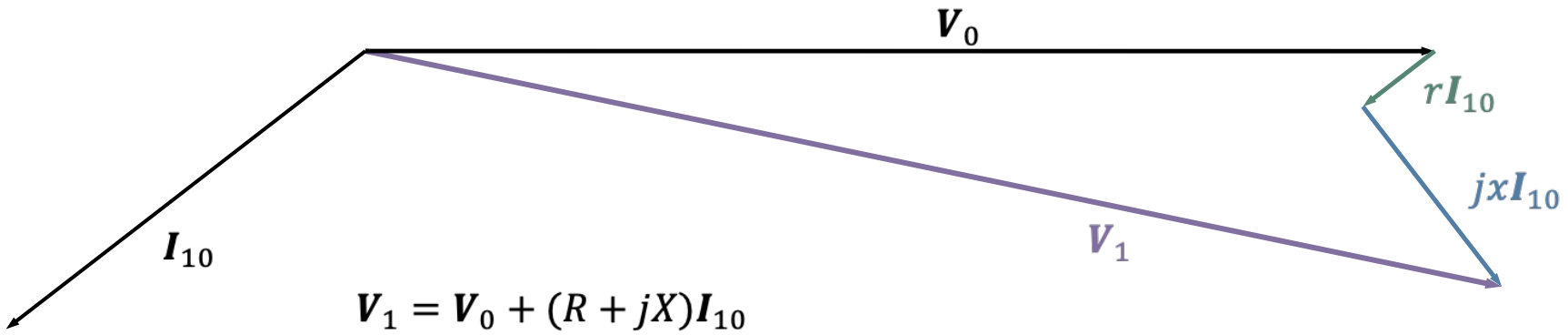
How does a change in the load affect voltage magnitude?

$r = 0.2, \qquad x = 0.5$



Given the load current phasor $\boldsymbol{I}_{10}$, it is easy to determine $\boldsymbol{V}_1 = \boldsymbol{V}_0 + (R + jX)\boldsymbol{I}_{10}$

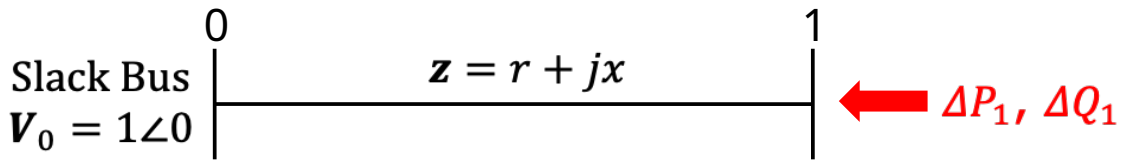
$$\Delta V_1 = V_1 - V_0 = 0.08$$



2 bus example, capacitive load at Bus 1

How does a change in the load affect voltage magnitude?

$r = 0.2, \qquad x = 0.5$



Rarely do we have the current phasor $\boldsymbol{I}_{10}$ relative to the "slack bus" $\boldsymbol{V_0}$

Thus, we use $\Delta V_1 \approx rP_1 + xQ_1$

$$P_1 = -0.3\ , \qquad Q_1 = 0.3$$

Estimated voltage change: $\Delta V_1 \approx 0.09$

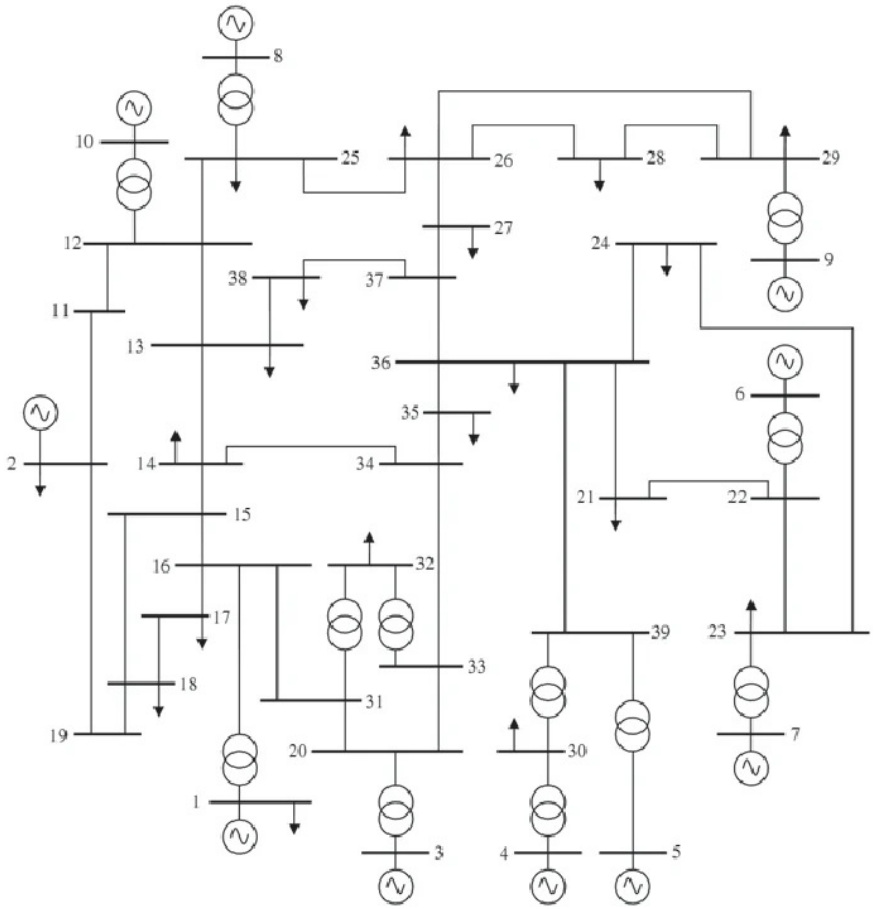
(the exact answer on the previous slide was 0.08)

The 2 bus intuition extends from single line networks to multi-bus, multi-line networks.

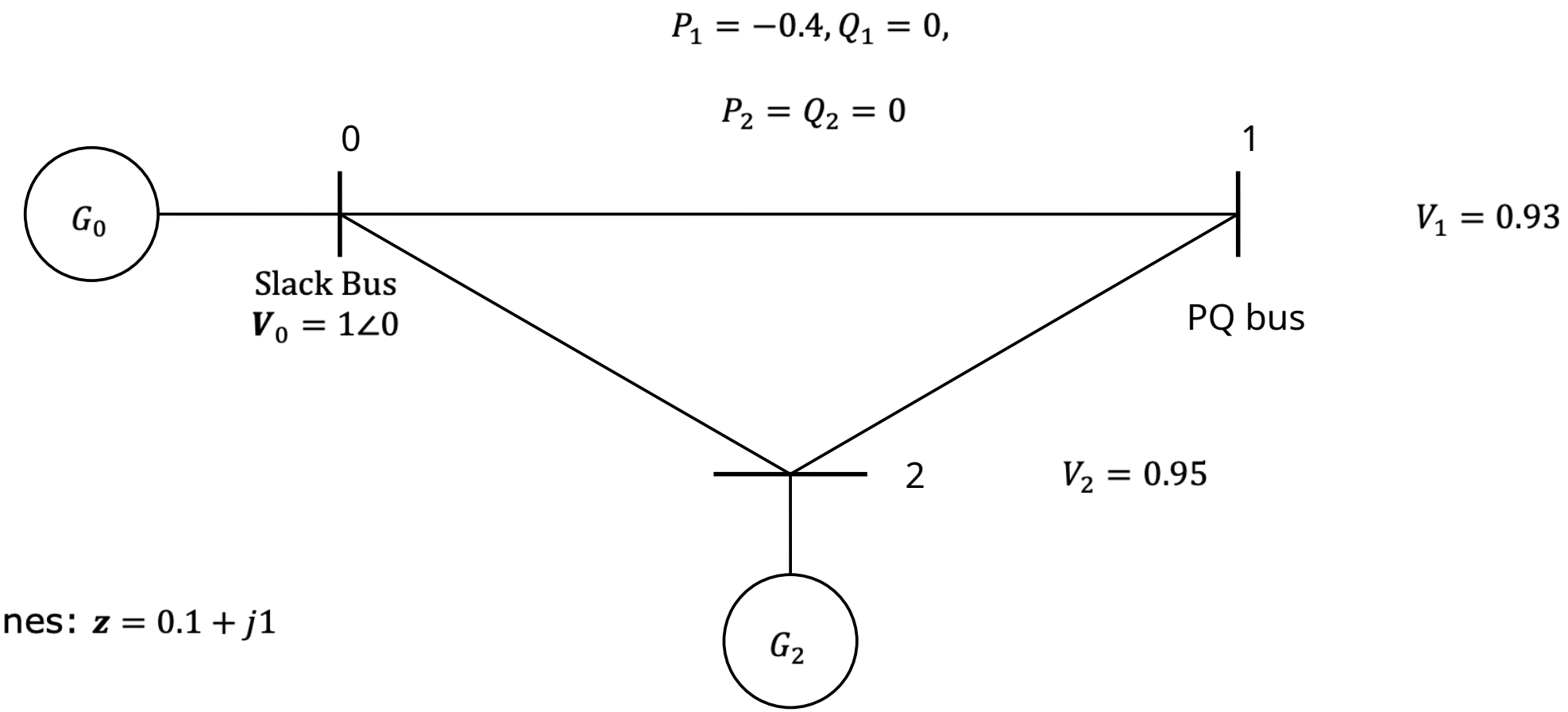
$\Delta V \approx R\Delta P + X\Delta Q$ is the equation we use to assess voltage sensitivity.

Implicit in this equation is the assertion that the slack bus maintains power balance.

Next: Transmission network voltage regulation

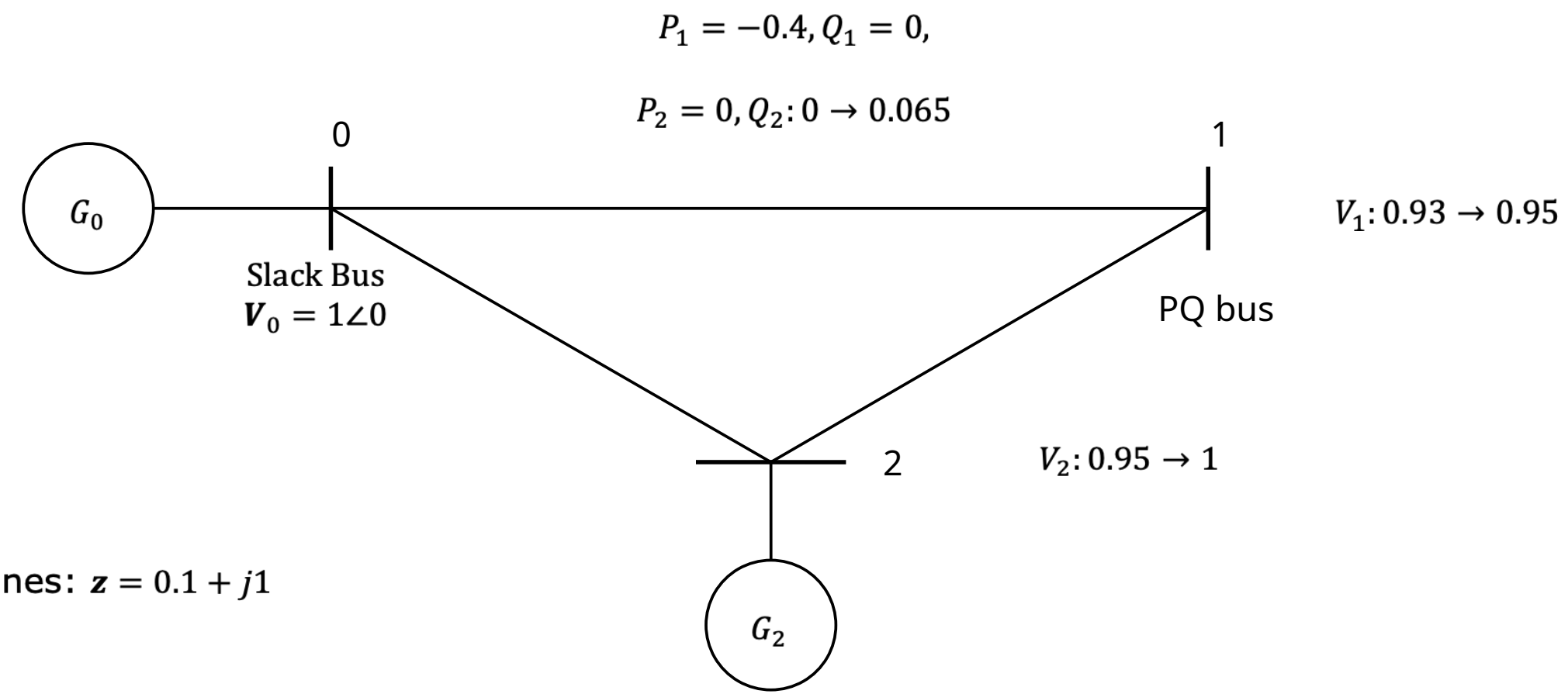


Real-power load lowers voltage: $\Delta V \approx R\Delta P + X\Delta Q$



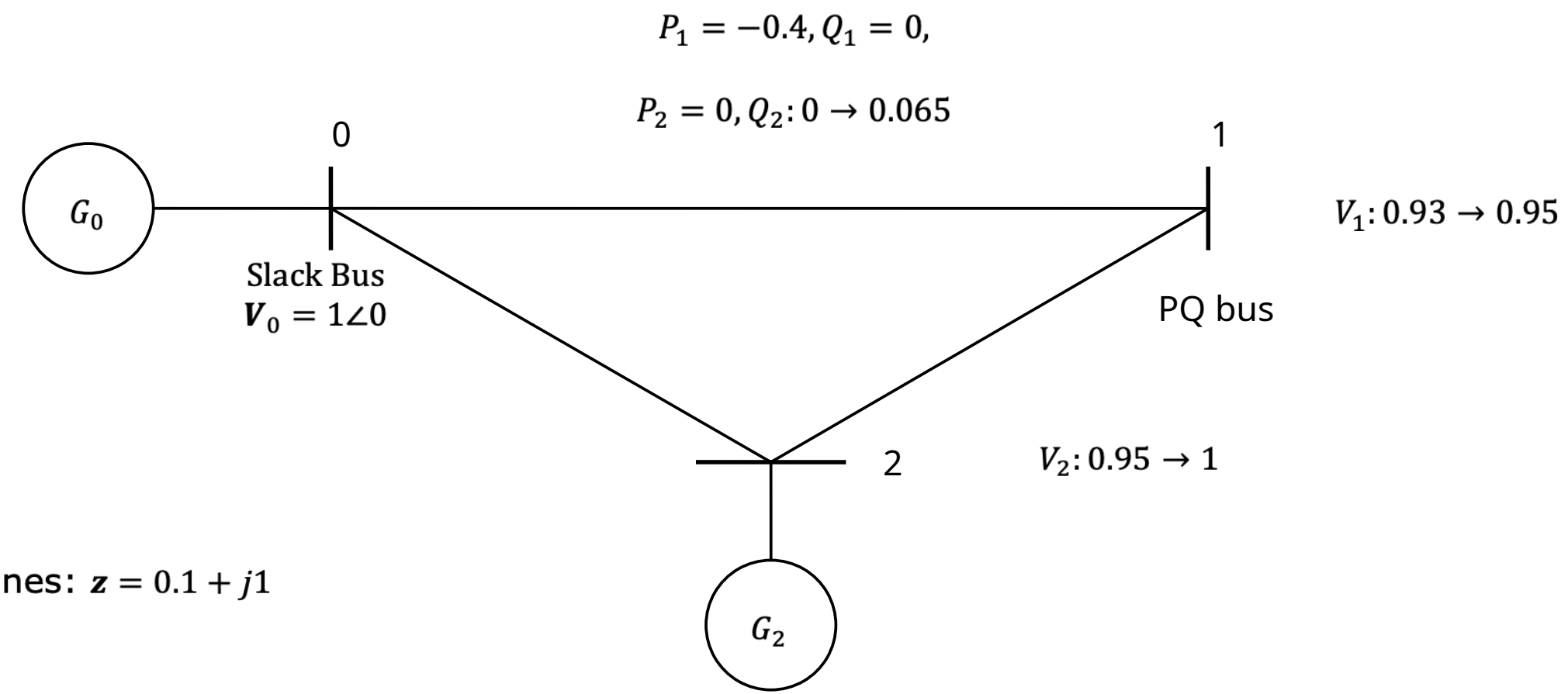
All lines: $\boldsymbol{z} = 0.1 + j1$

Reactive power injections raise voltage: $\Delta V \approx R\Delta P + X\Delta Q$

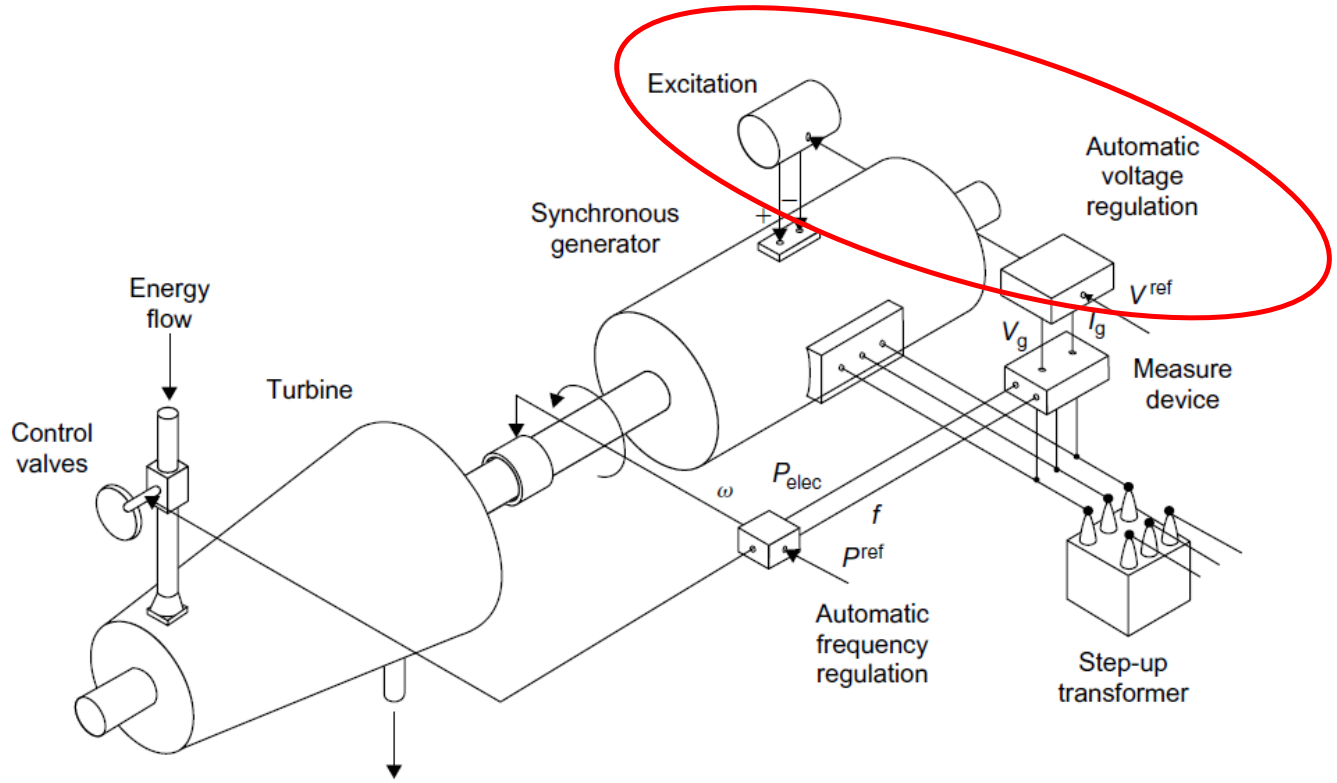


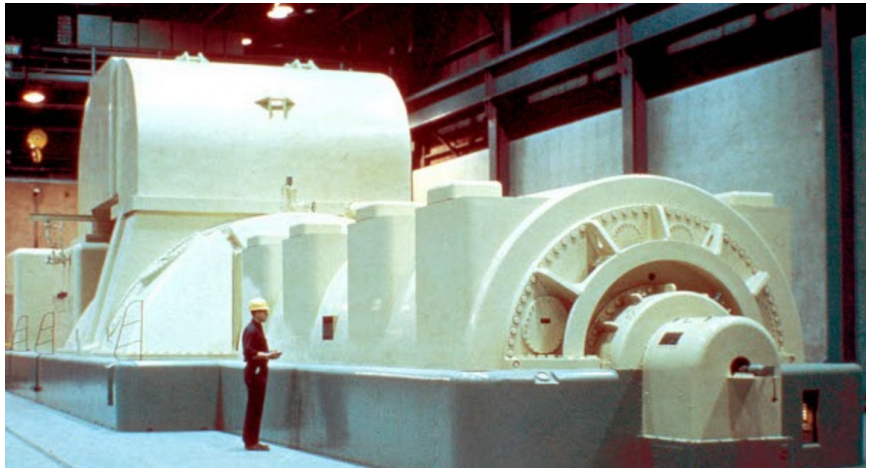
All lines: $\boldsymbol{z} = 0.1 + j1$

This same result could have been achieved if Bus 2 was a constant voltage bus with $V_2 = 1$



How do rotating machines maintain output voltage magnitude?

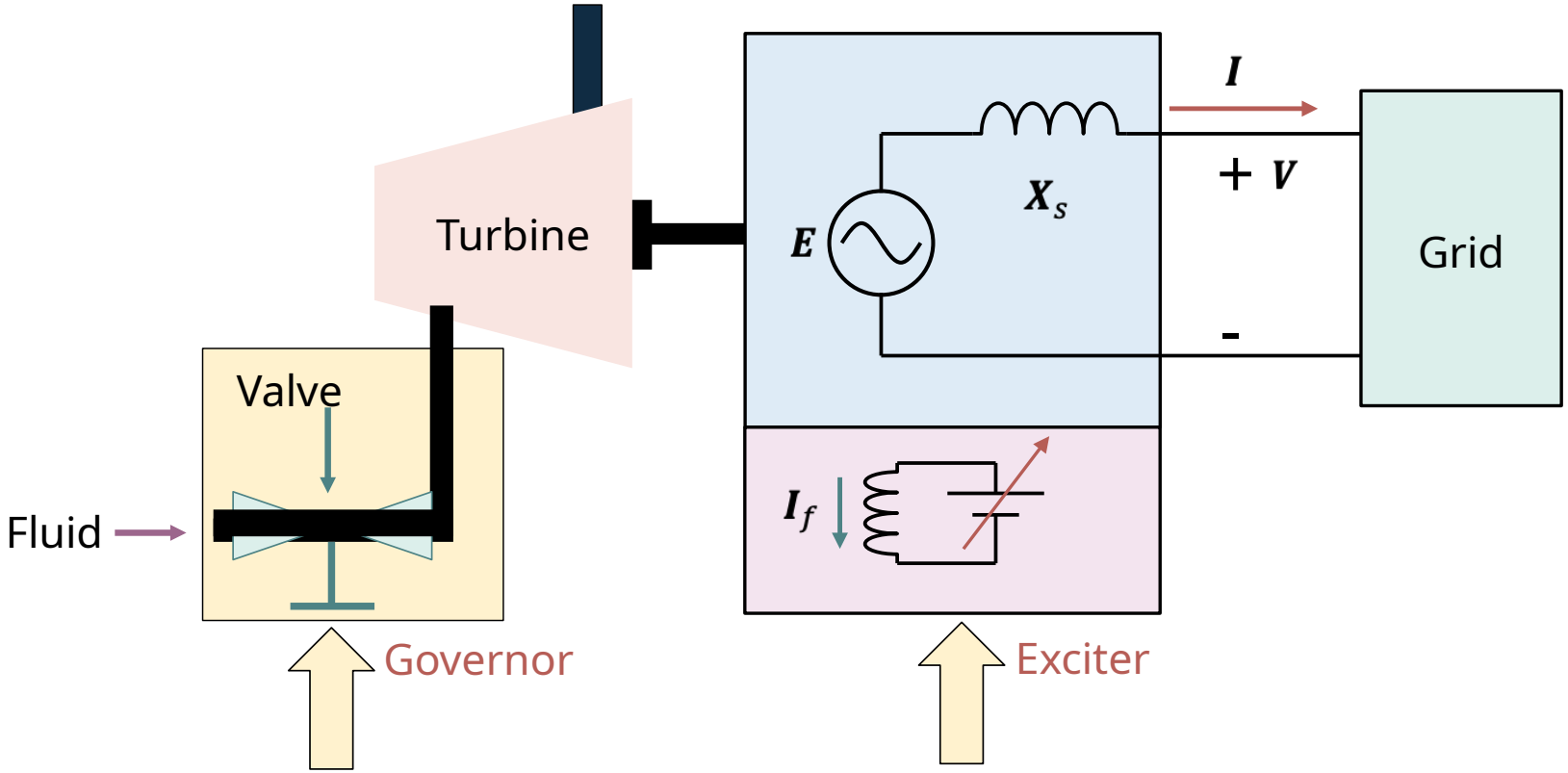




Source: Gomez-Exposito et al.

The AVR is a feedback loop that determines the Q necessary to maintain the output voltage.

Controls of a synchronous generator



Control of mechanical power

→ Shaft speed

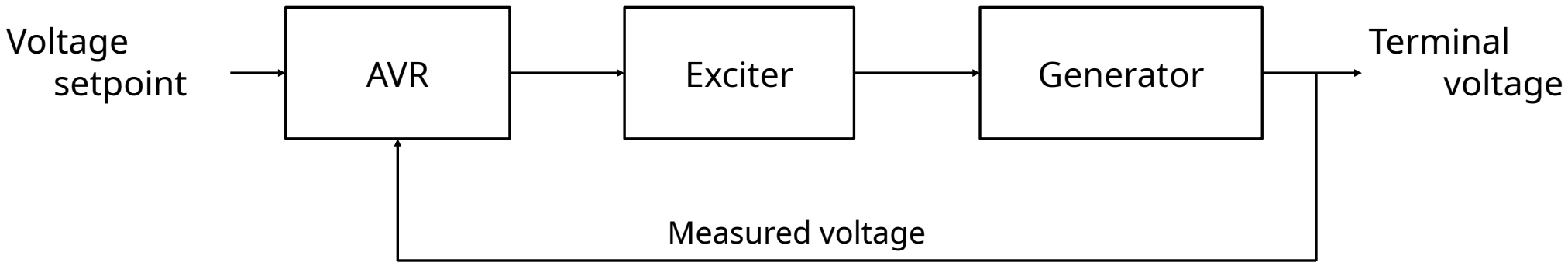
→ Electrical frequency

Control of field current

→ Magnitude of internal e.m.f.

→ Magnitude of terminal voltage

Automatic Voltage Regulation (AVR)

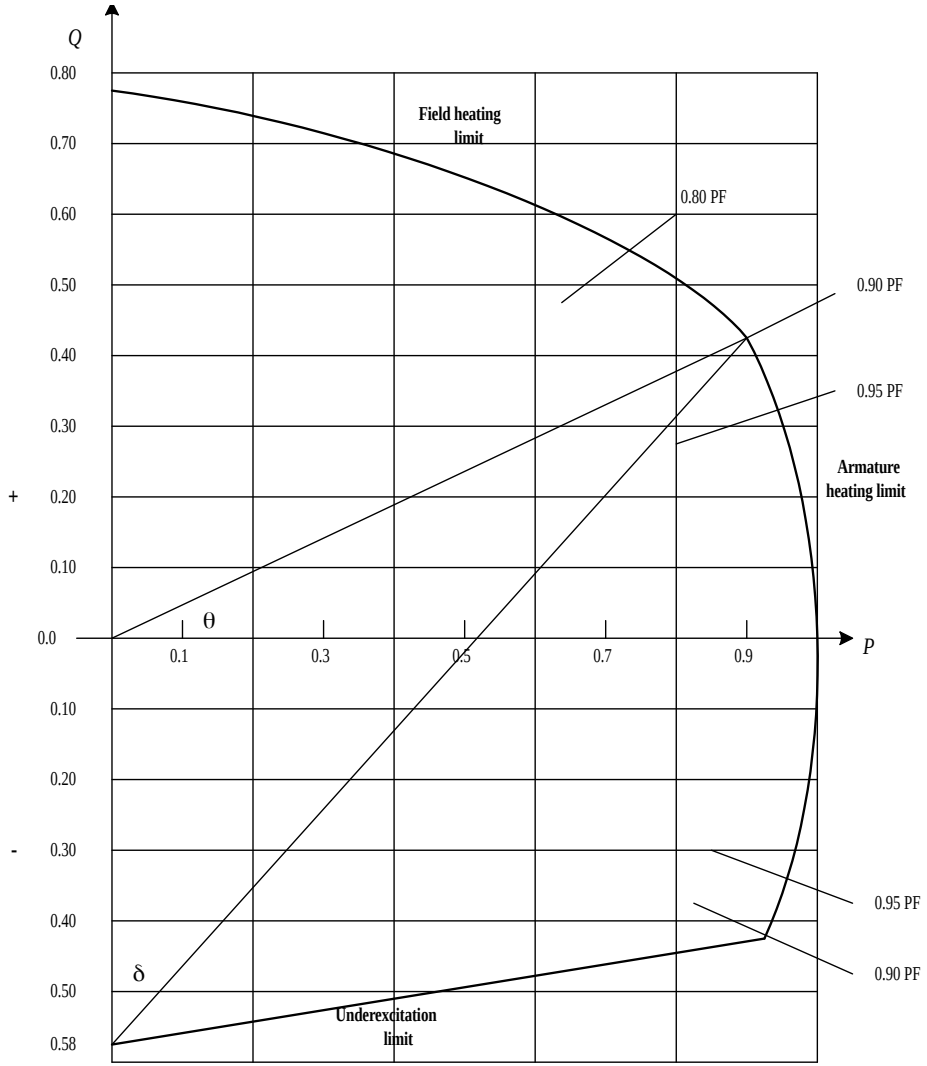


If voltage falls, the AVR raises field current. At fixed real power, the generator supplies more reactive power Q which flows into the network and raises the voltage back to the setpoint.

This creates a PV bus, which has a set voltage at steady state.

If the generator hits its reactive power limit, it becomes a PQ bus with Q set by the limit.

A generator can hold voltage only within its reactive capability.



Exercise: Machine control

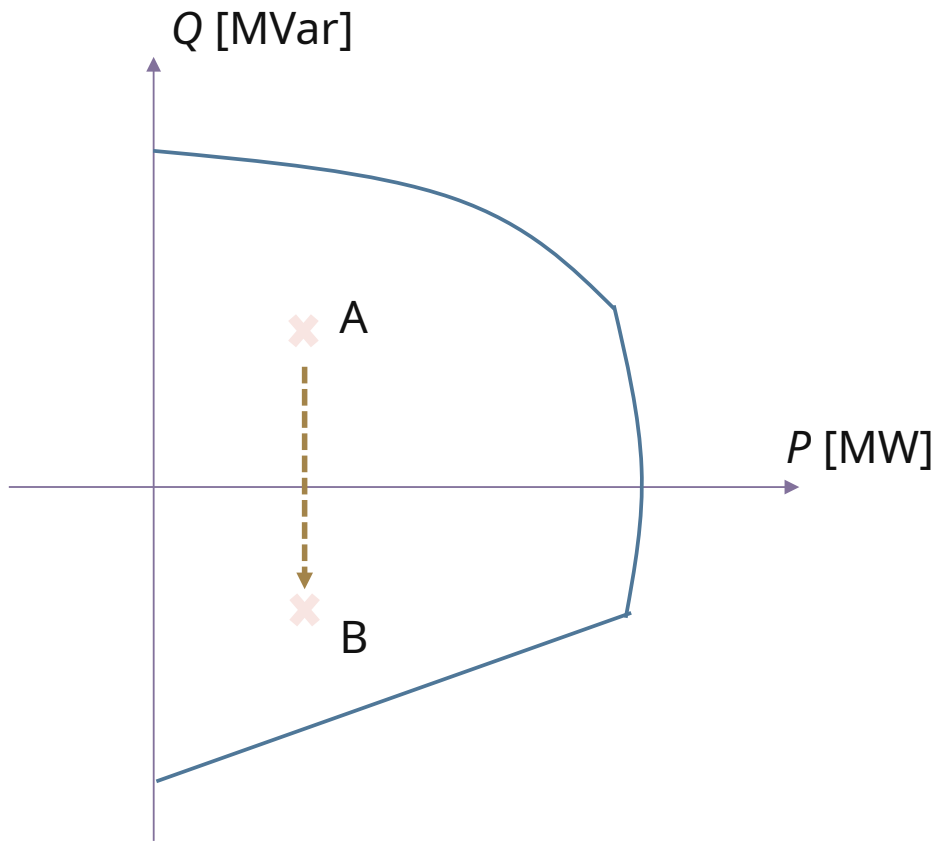
At fixed real power, the operating point moves from A to B. **Which action causes this**?

a. Increase excitation current

b. Reduce excitation current

c. Increase turbine torque

d. Reduce turbine torque



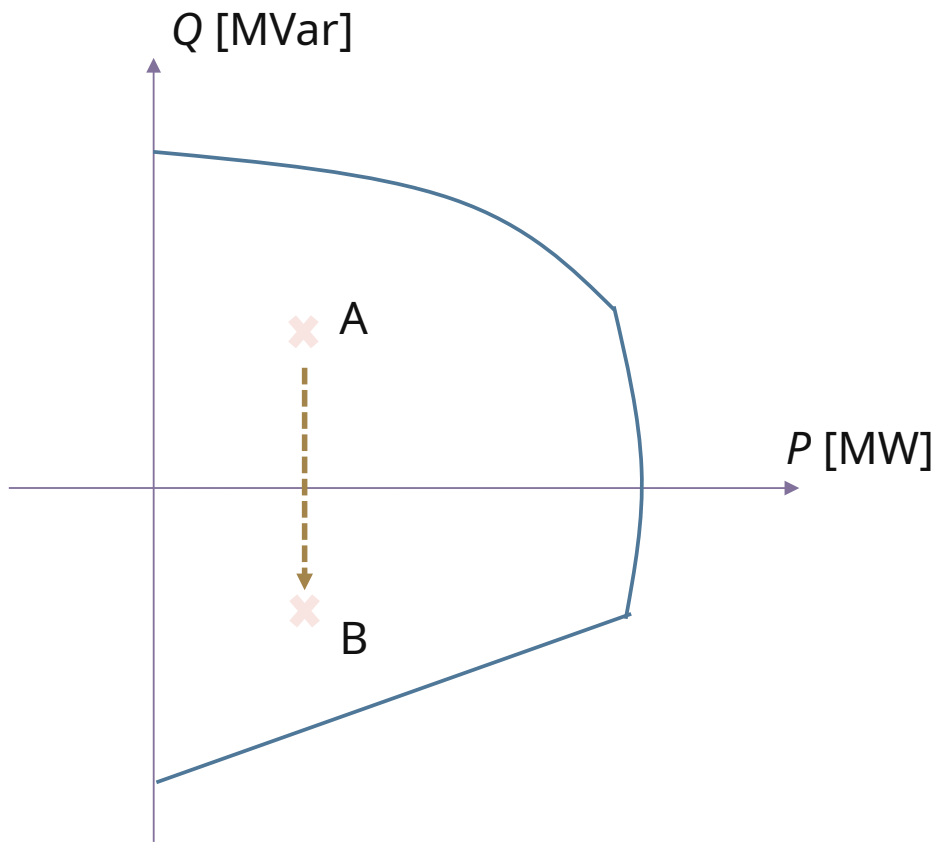
Exercise: solution

b. Reduce excitation current

(it becomes negative)

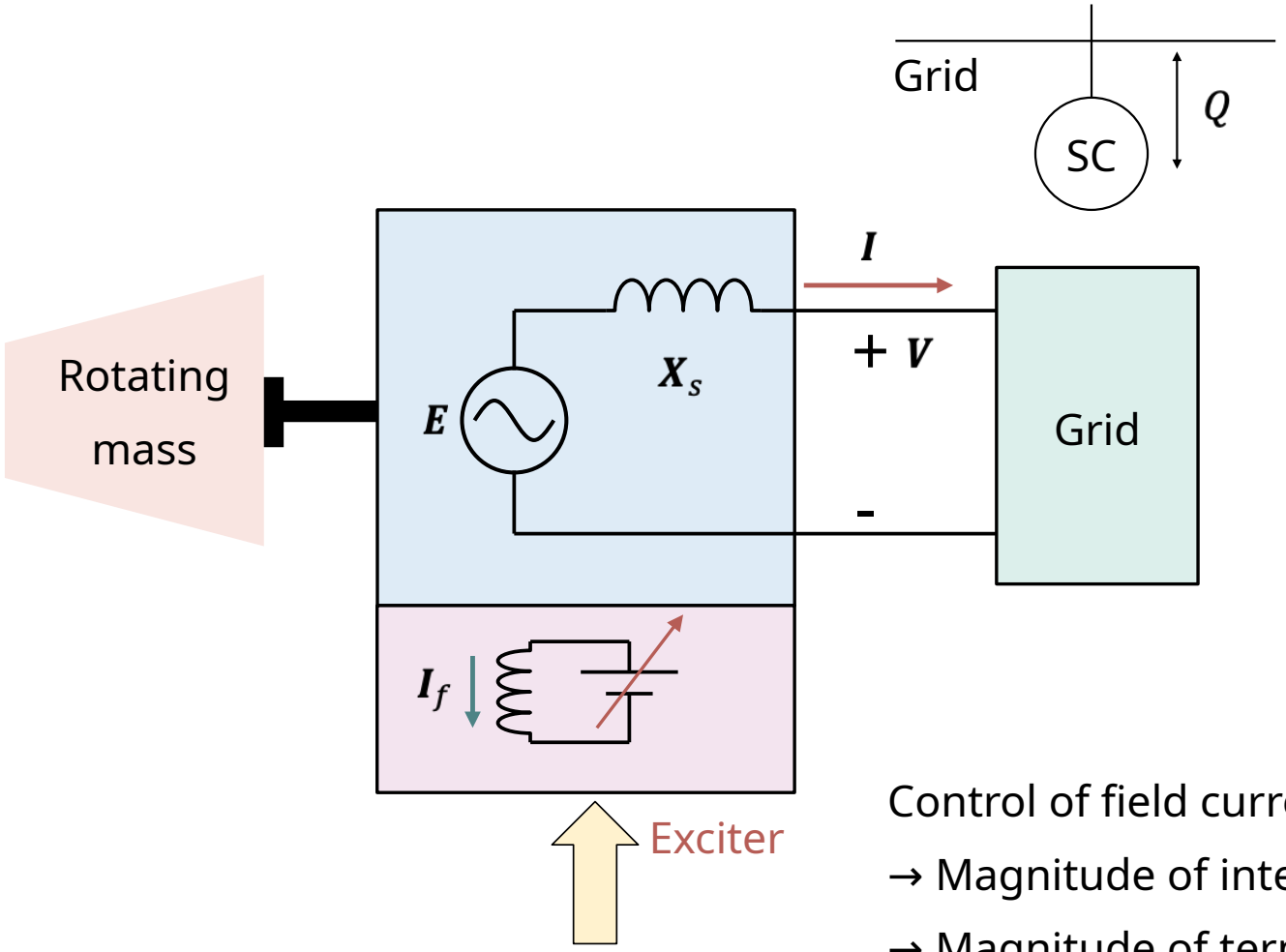
$$P = \text{constant}, \qquad Q \downarrow$$

The turbine torque is unchanged because the real power is unchanged.



Voltage Support Devices

SynCon (Synchronous Condenser): Rotating machine without a prime mover



at steady state, neglecting losses: $P = 0$



Synchronous condenser installation at Templestowe substation, Melbourne, Victoria, Australia. Built by ASEA in 1966, the unit is hydrogen-cooled and capable of three-phase power at 125 MVA.

Control of field current

→ Magnitude of internal e.m.f.

→ Magnitude of terminal voltage

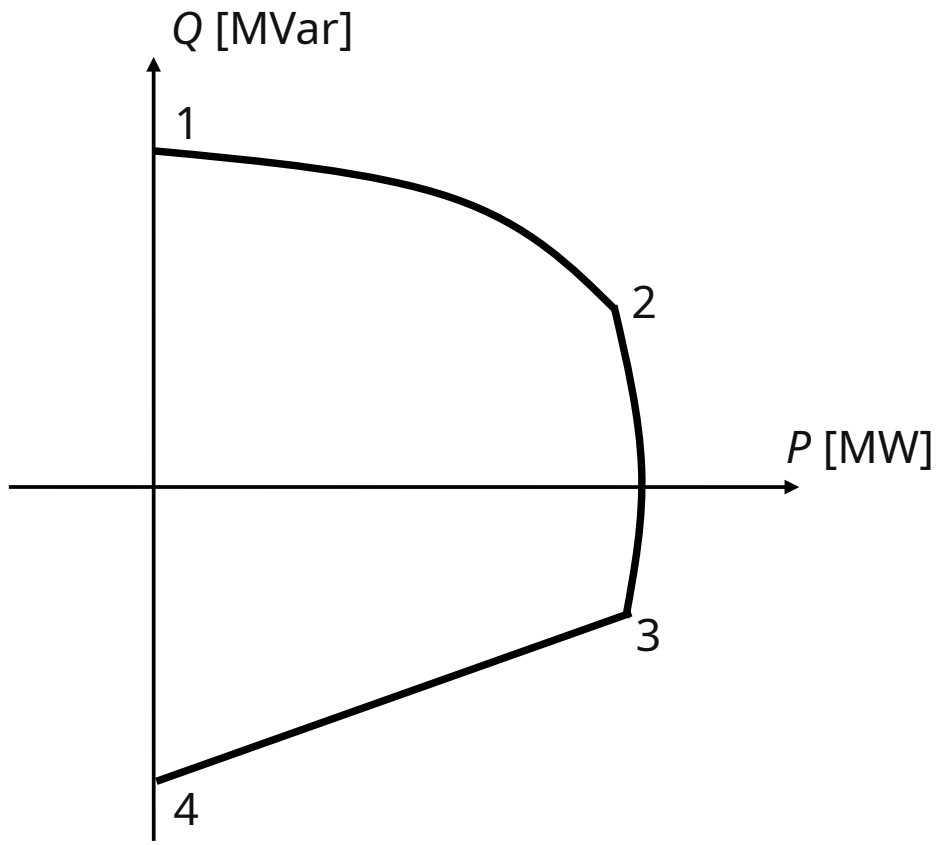
“Overexcited": injects reactive power. "Underexcited": absorbs reactive power.

Class exercise: synchronous condenser

The curve bounds the feasible $P-Q$ region of a synchronous generator *at steady state*.

Remove the prime mover and operate it as a synchronous condenser.

Which portion of the chart remains feasible?

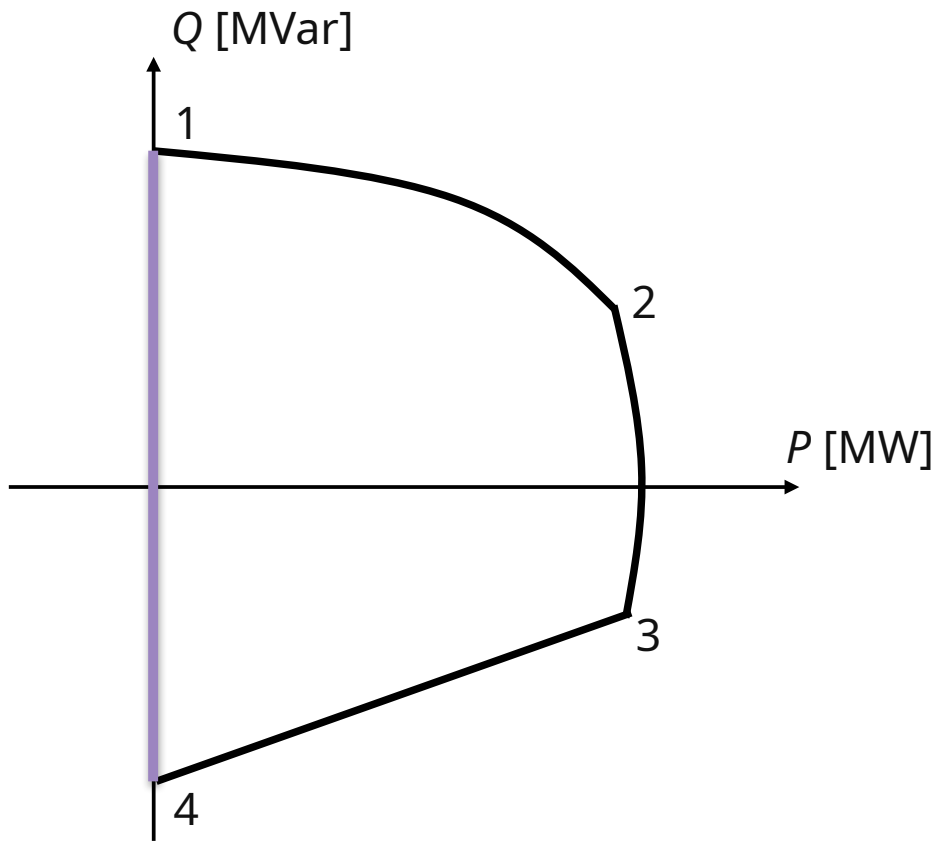


Synchronous condenser: solution

At steady state, $P = 0$

The feasible operating points lie on segment 4–1 of the Q-axis.

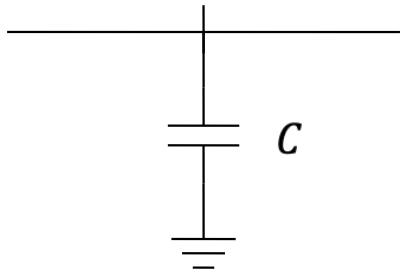
The machine can inject or absorb reactive power within its limits.



Note: this does not mean the SynCon cannot output real power during a transient, which it does when it provides an inertial response to the grid

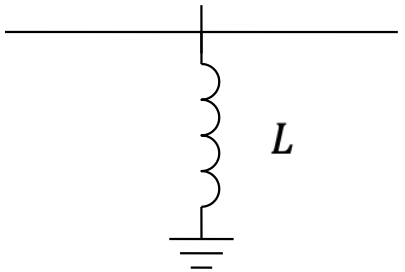
Voltage Support Devices

Fixed shunt devices (capacitors, inductors) inject/extract reactive power:



capacitor: inject Q

$Q_C = \omega C V^2$

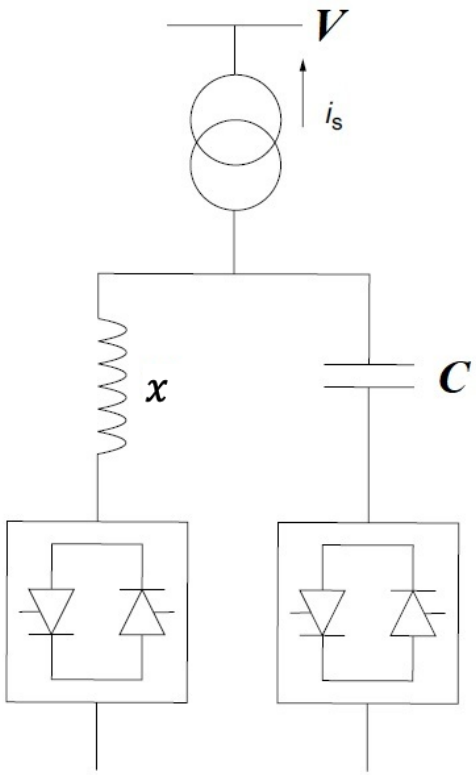


Inductor/reactor: absorb Q

$Q_{reactor} = -V^2/(\omega L)$

Voltage Support Devices

SVC (Static VAR Compensator): Bank of capacitors and inductors that can be switched on

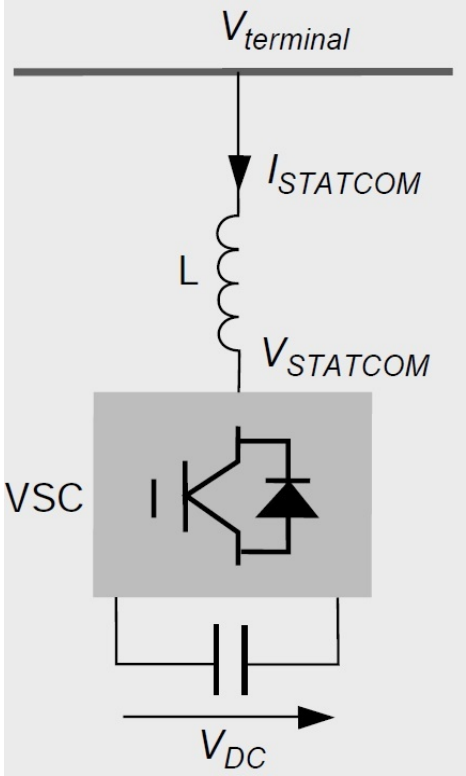


Has a finite amount of capacitance. It reaches its capacitive limit when all available capacitor banks are connected and reactor absorption is minimized.

At its capacitive limit, support falls with the square of voltage: $Q = B_{\mathrm{SVC}} V^2$

Voltage Support Devices

STATCOM (Static Synchronous Compensator): an inverter dedicated to reactive-power support



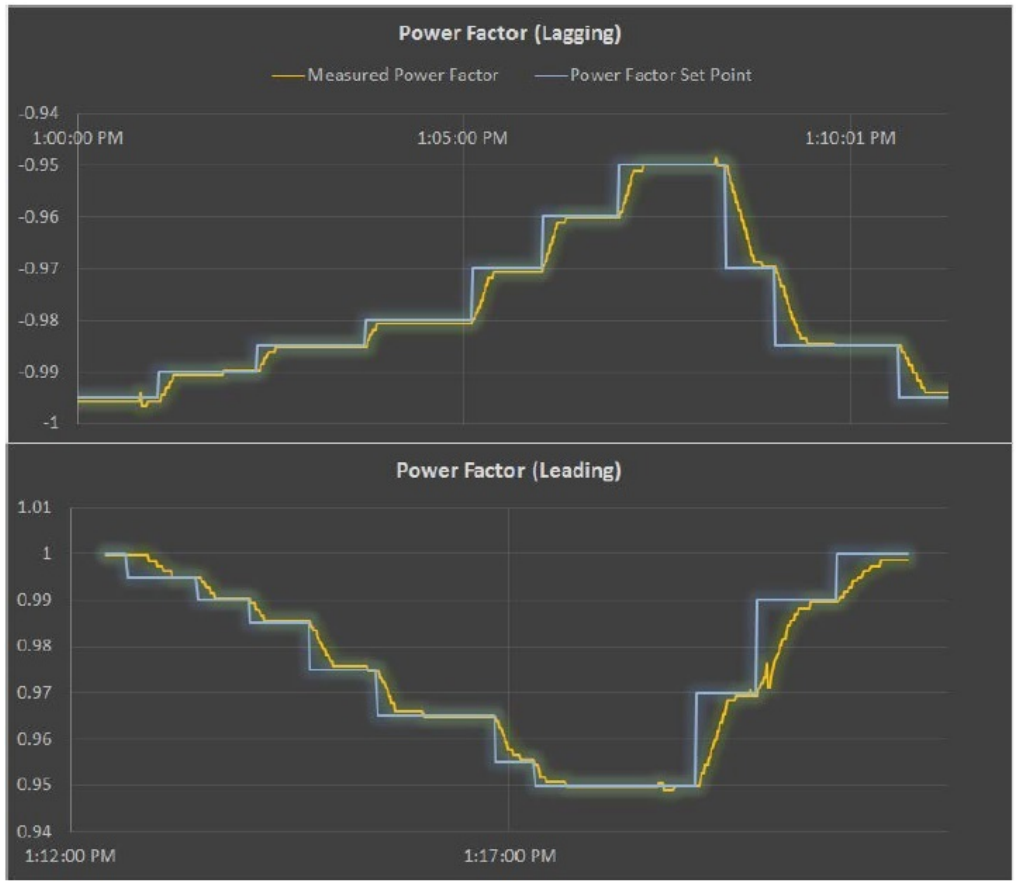
Programmable reactive power injection.

At its current limit, available reactive power ∝ voltage: $Q_{\max} \approx VI_{\max}$

What about inverters?

Real-life example 1: Inverters providing reactive power support in California

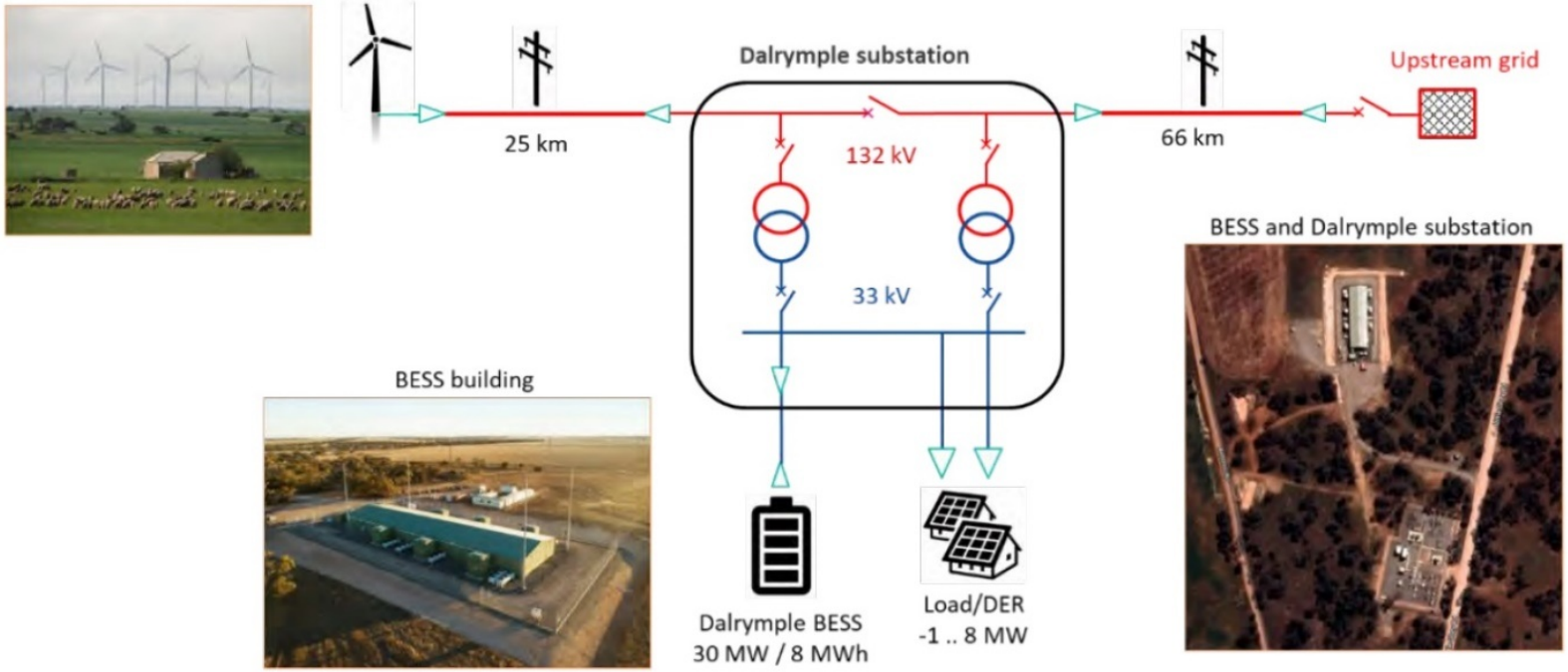




National Renewable Energy Laboratory (NREL), "Demonstration of essential reliability services by a 300-MW solar photovoltaic power plant", Technical Report, March 2017.

Real-life example 2: Inverters demonstrating islanding capability in Australia

30 MW / 8 MWh battery on South Australia's Yorke Peninsula.

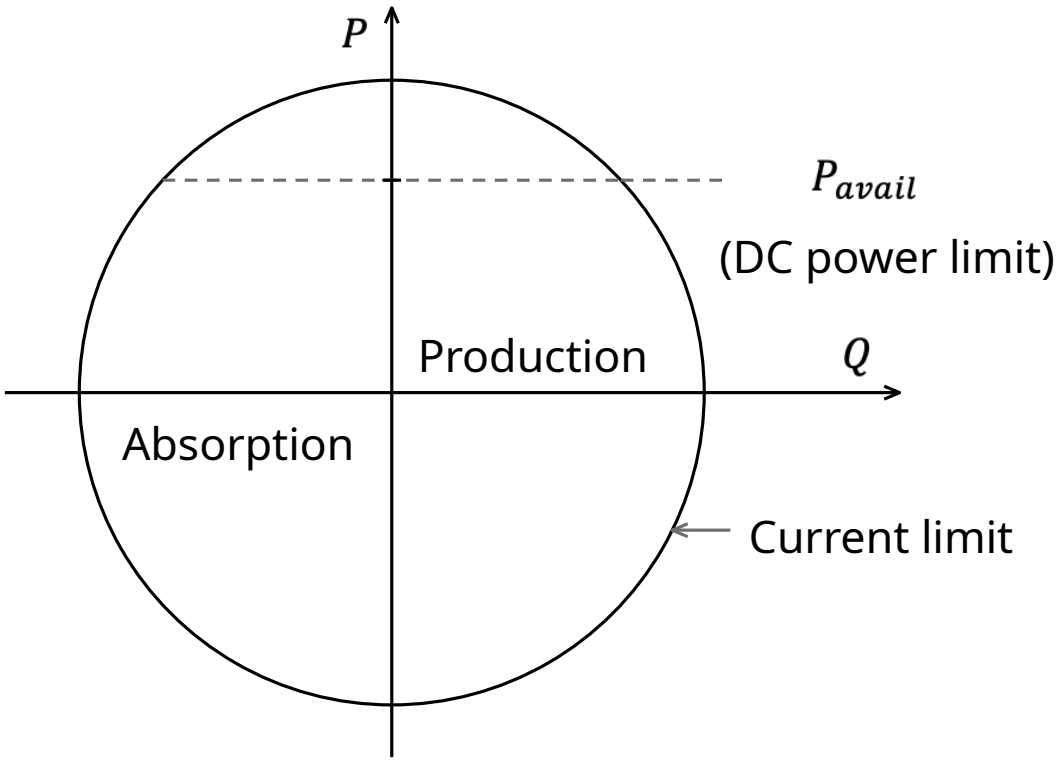


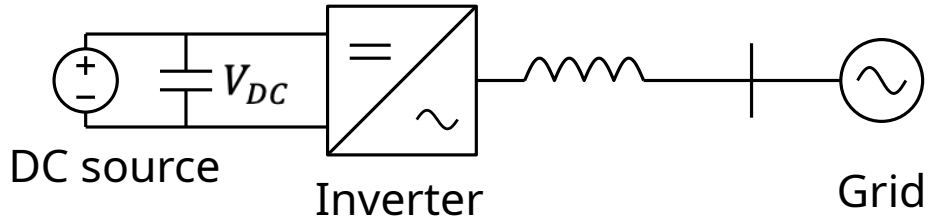
ElectraNet," ESCRI-SA battery storage project operational report#3, August 2020.

Island capability demonstrated during 28.12.2023 outages.

Inverter Capability

At a given terminal voltage, inverter current limits bound real and reactive output: $P^2 + Q^2 \leq (VI_{\max})^2$





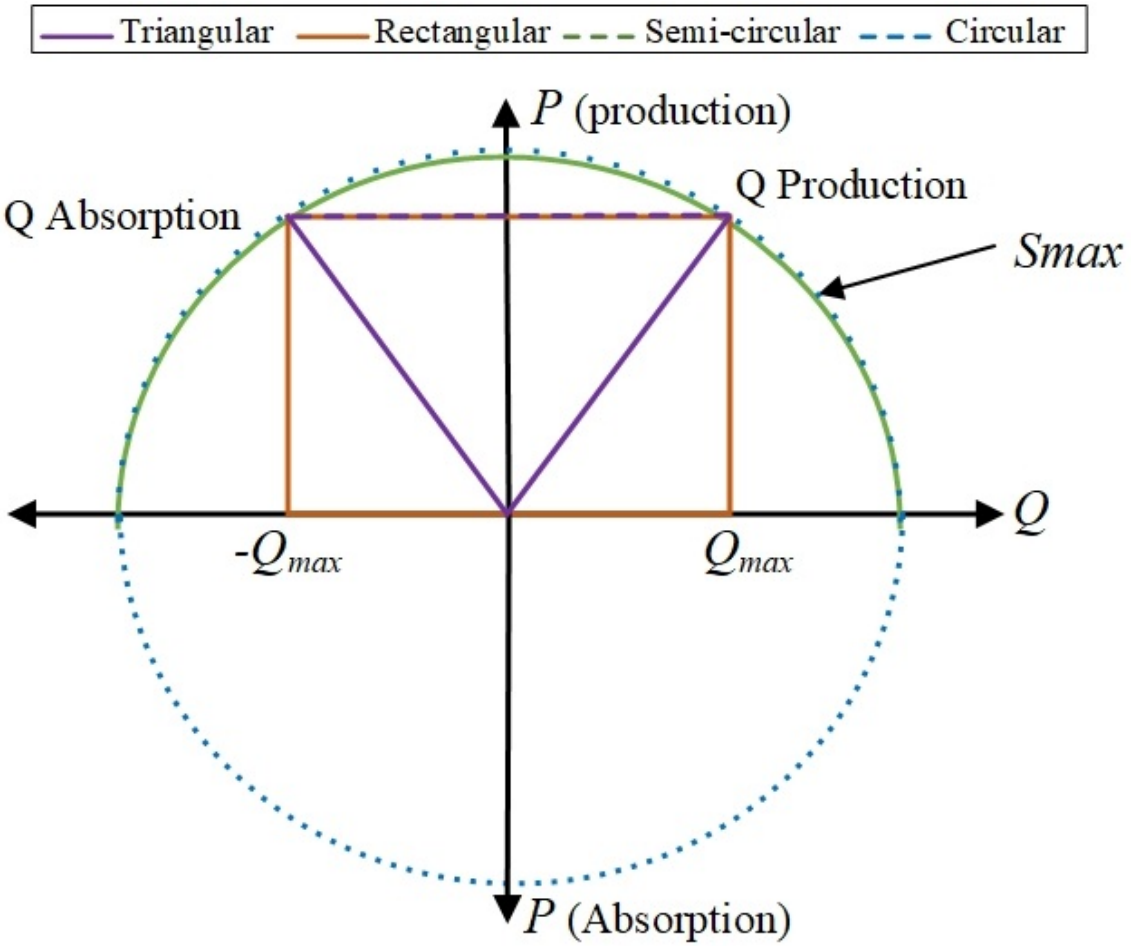
DC-link voltage and modulation also limit the inverter's AC voltage/reactive power injection.

Available DC power limits P, not Q

Negative P requires a DC-side sink (e.g., battery)
Negative Q does not

Inverter Capability

Other, hardware/manufacturer-specific constraints may apply.



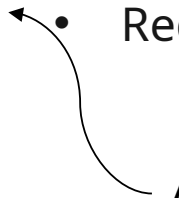
When should inverters trip?

Stay connected

- Support voltage after a contingency
- Brief sag or swell
  - Provide fault currents to trip protection
  - Reduce ROCOF after a contingency

Trip

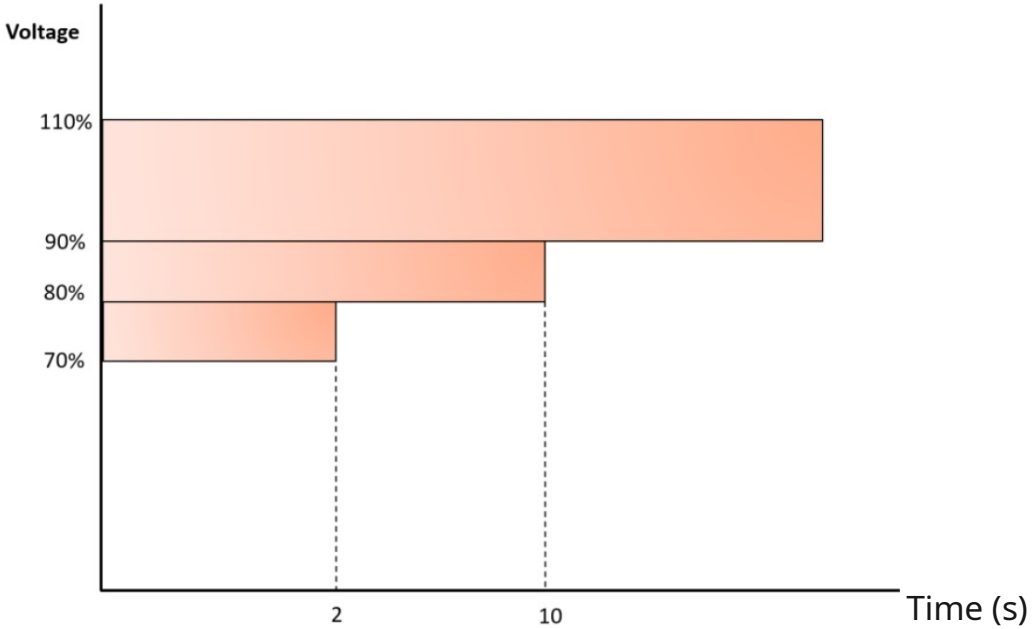
- Isolated feeder: avoid supplying a disconnected line (safety for grid workers)
- Sustained extreme voltage: protect equipment



As increasingly depend on inverters, this “grid-forming” behavior is increasingly important

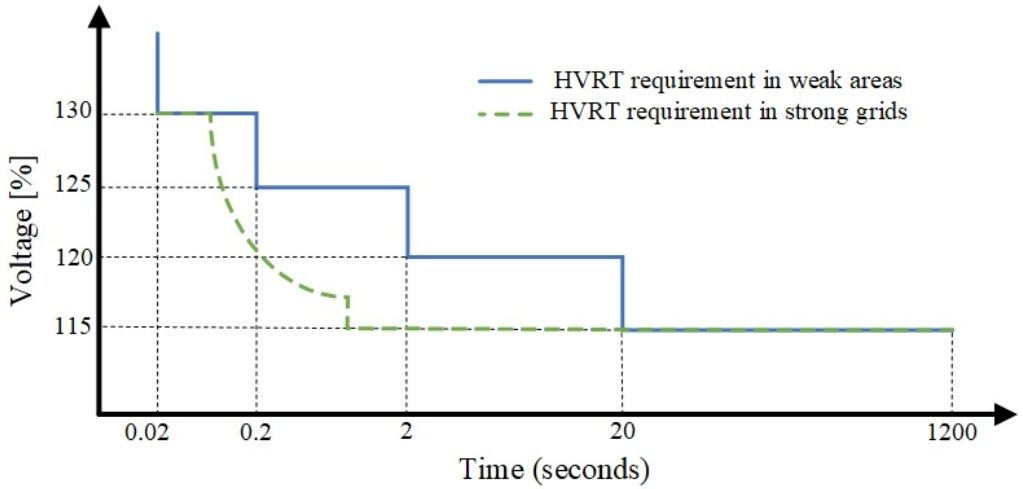
Minimum ride-through times

Low voltage



example: at 0.85 pu, at least 10 s

High voltage



example: at 1.22 pu, at least 2 s (blue curve)

These are minimum stay-connected times, not mandatory trip times.

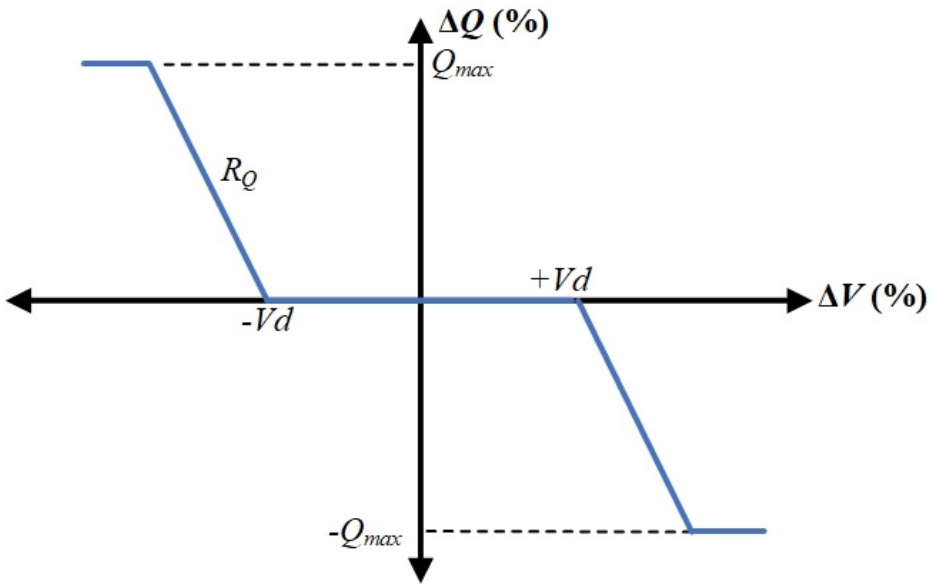
Inverters are the future of grid (voltage) control.

There are many inverter manufacturers, thus it is not viable for grid operators to directly control all inverters.

The behavior of inverters can be controlled by setting grid codes ahead of time.

Volt-VAR Control

How much reactive power should an inverter inject or absorb?



The inverter measures its terminal voltage and follows a preset voltage–reactive power curve.

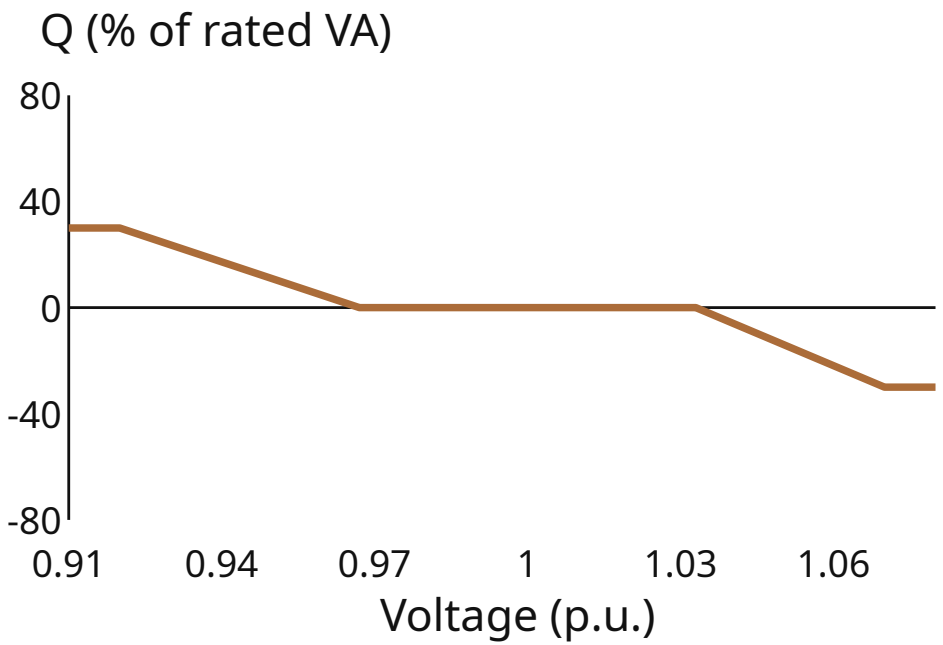
$$\Delta V \approx R\Delta P + X\Delta Q$$

At low voltage it injects reactive power, raising local voltage.

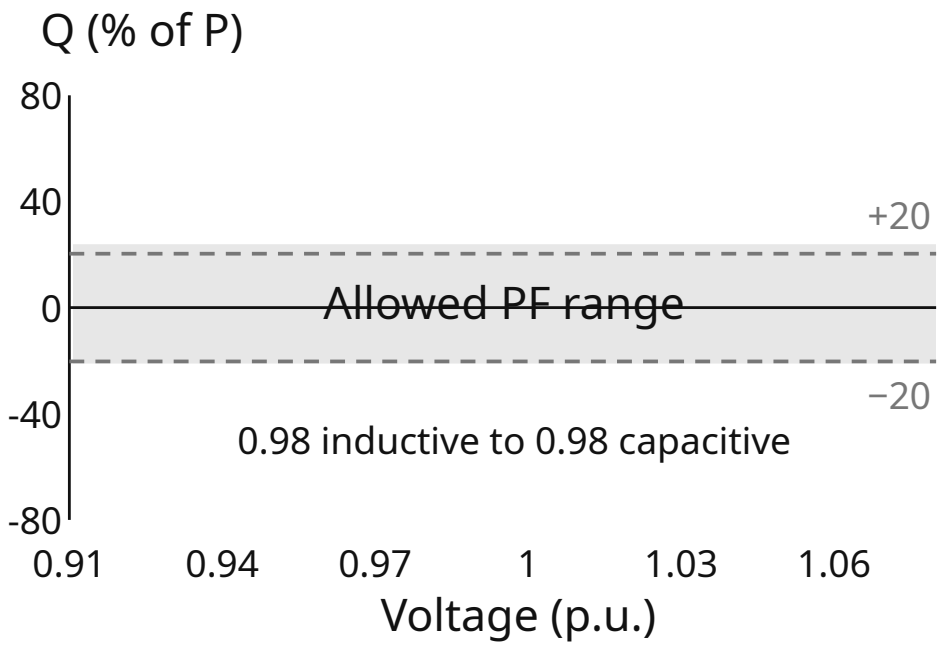
At high voltage it absorbs reactive power, lowering local voltage.

Volt–VAR vs. Set Power Factor

Volt–VAR control



Spanish power-factor rule, April 2025



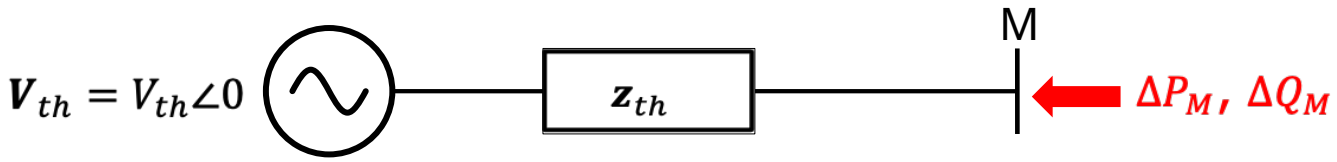
No automatic Q–V response required

Why is Volt-VAR control a good idea?

Inverters automatically respond to high/low voltages, keeping voltage in bounds

Thevenin-Equivalent Circuit Modelling

The voltage sensitivity at any bus can be approximated by a Thevenin-Equivalent impedance:



z_{th}: fictitious impedance between the grid location of interest and a fictitious voltage source.

$V_{th}$is usually chosen to be V_{nom} at M.

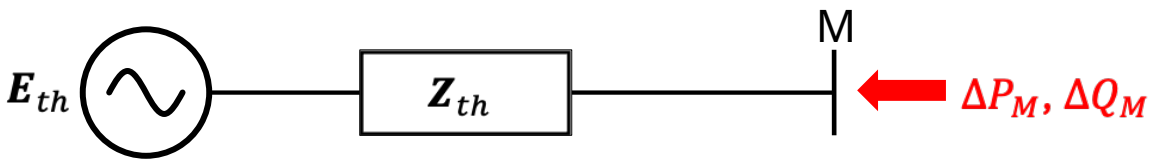
Accurate to the extent that the grid behaves like voltage sources behind a network of impedances.

While this approximation is not strictly accurate, it is critical for power systems intuition.

Voltage magnitude sensitivity from the Thevenin-equivalent circuit:

$$\Delta V_M \approx r_{th}\Delta P_M + x_{th}\Delta Q_M$$

Who maintains reactive power balance?



For the Thevenin circuit, changes in Q are balanced by the Thevenin voltage source.

This approximates the true system, which maintains power balance in a distributed manner.

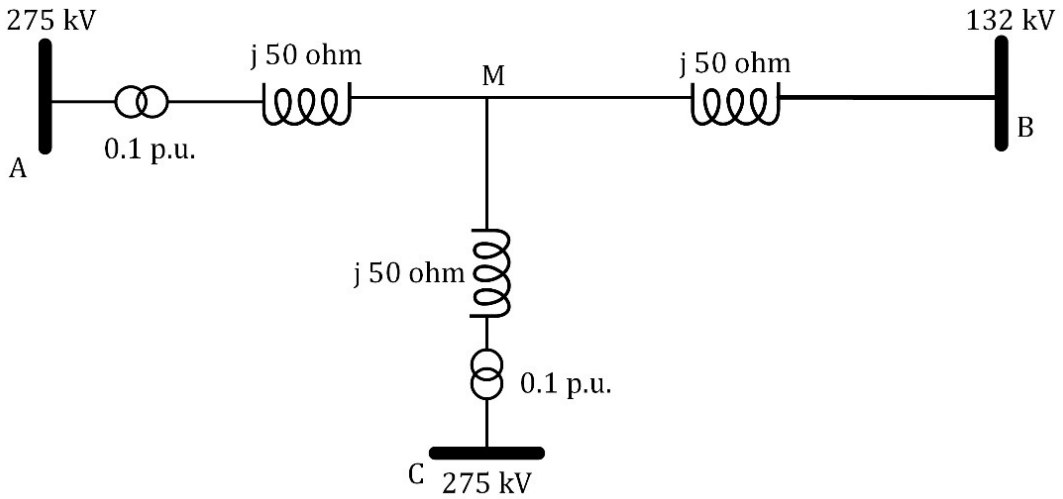
Exercise: reactive compensation at a point using Thevenin-equivalent impedance

A and C are $275\ kV$ sources; B is a $132\ kV$ source. Their voltages remain fixed.

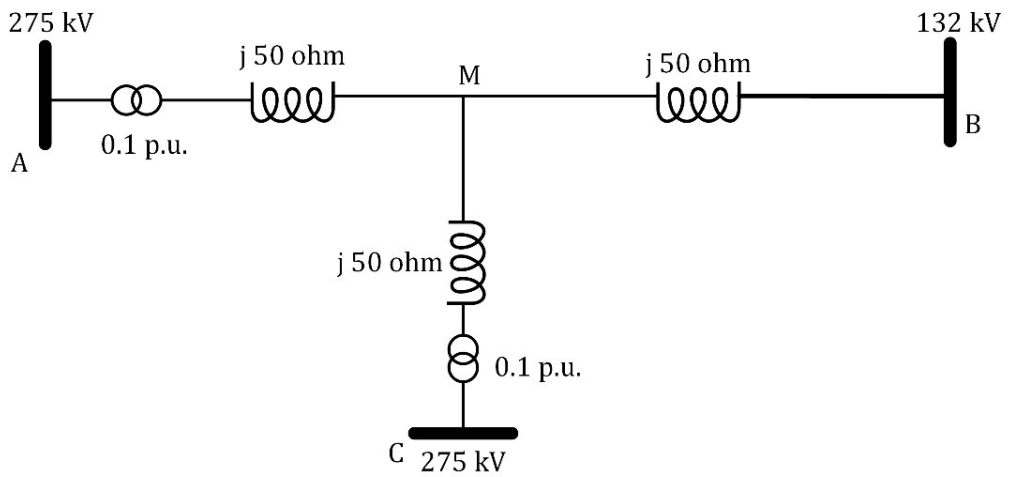
Use a $500\ MVA$ base, $132\ kV$ at M, and nominal $275/132\ kV$ transformer ratios.

The 0.1 pu transformer reactances are on this base. Neglect resistance and shunts.

The voltage at M falls by $5\ kV$. **Estimate the reactive injection needed to restore it.**



Exercise first step: find z_{th}

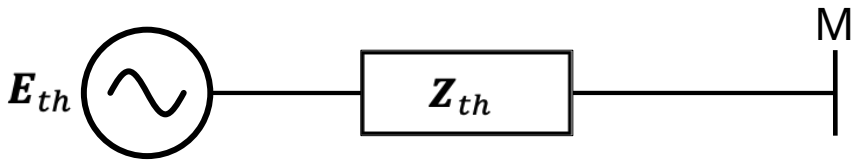


The three lines are on 132 kV

$$Z_{\text{base},M} = \frac{(132\,\text{kV})^2}{500\,\text{MVA}} \approx 34.85\,\Omega$$

$$x_{\text{line}} = \frac{50\,\Omega}{34.85\,\Omega} \approx 1.43\,\text{pu}$$

$$x_A = x_C \approx 1.53\,\text{pu}, \qquad x_B \approx 1$$

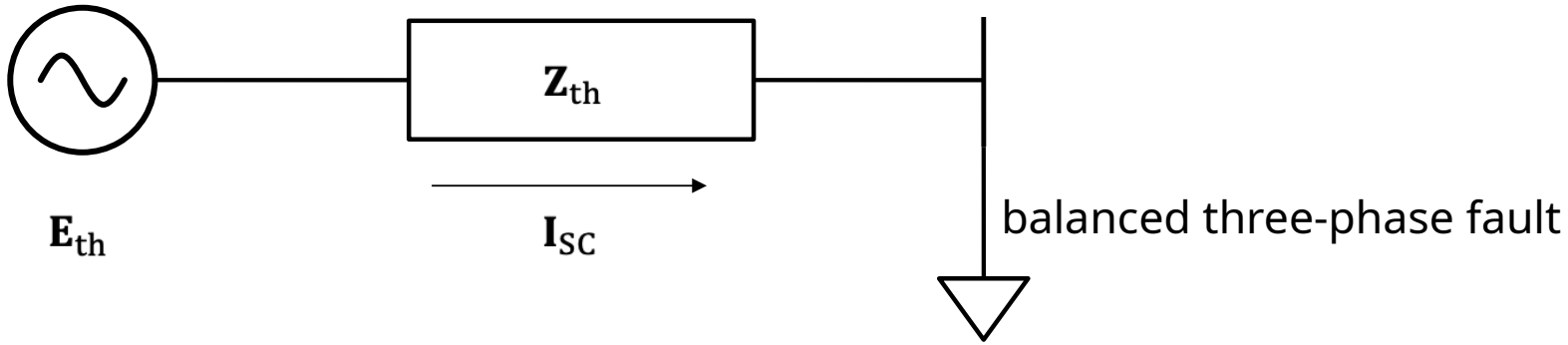


$$Z_{th} = j\left(\frac{1}{x_A} + \frac{1}{x_B} + \frac{1}{x_C}\right)^{-1} \approx \text{j}0.50\,\text{pu}$$

Now estimate the reactive injection needed to restore the voltage at M by 5kV

Short Circuit Current and Thevenin Impedance

Often, grid operators know the short circuit current $\mathbf{I}_{\mathrm{SC}}$ from running fault simulations.



$$\mathbf{Z}_{\mathrm{th}} = R_{\mathrm{th}} + jX_{\mathrm{th}}, \qquad \mathbf{I}_{\mathrm{SC}} = \frac{\mathbf{E}_{\mathrm{th}}}{\mathbf{Z}_{\mathrm{th}}}$$

$$Z_{\mathrm{th}} = \frac{E_{\mathrm{th}}}{I_{\mathrm{SC}}} = \sqrt{R_{\mathrm{th}}^2 + X_{\mathrm{th}}^2}$$

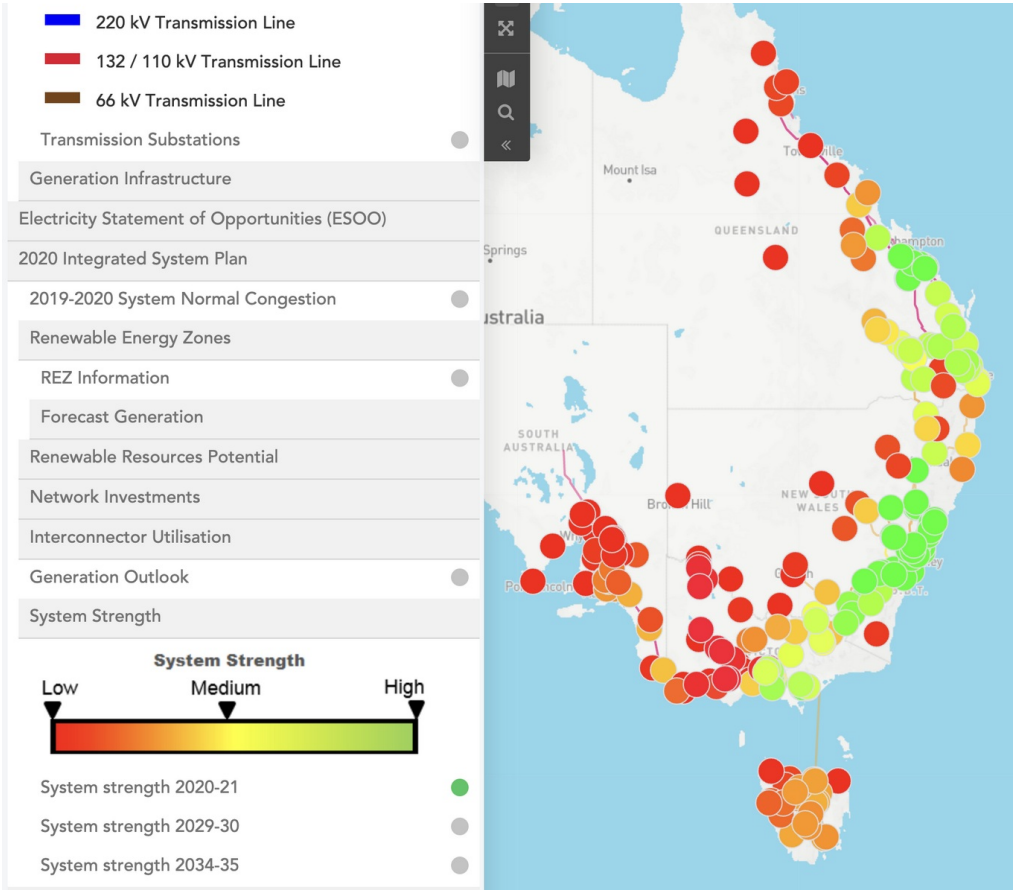
Assuming $V_{th} = V_{nom}$, I_{sc} tells us the magnitude of z_{th}.

“Grid Strength”

$$I_{\text{SC}} = \frac{V_{\text{th}}}{z_{\text{th}}}$$

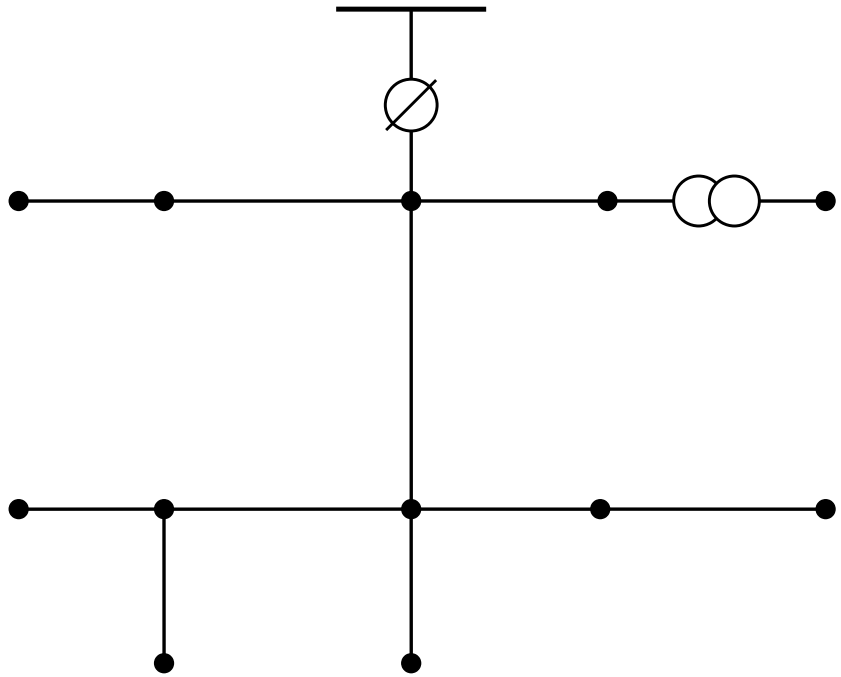
AEMO used to use short circuit current.

We will cover what they do now later in the course.



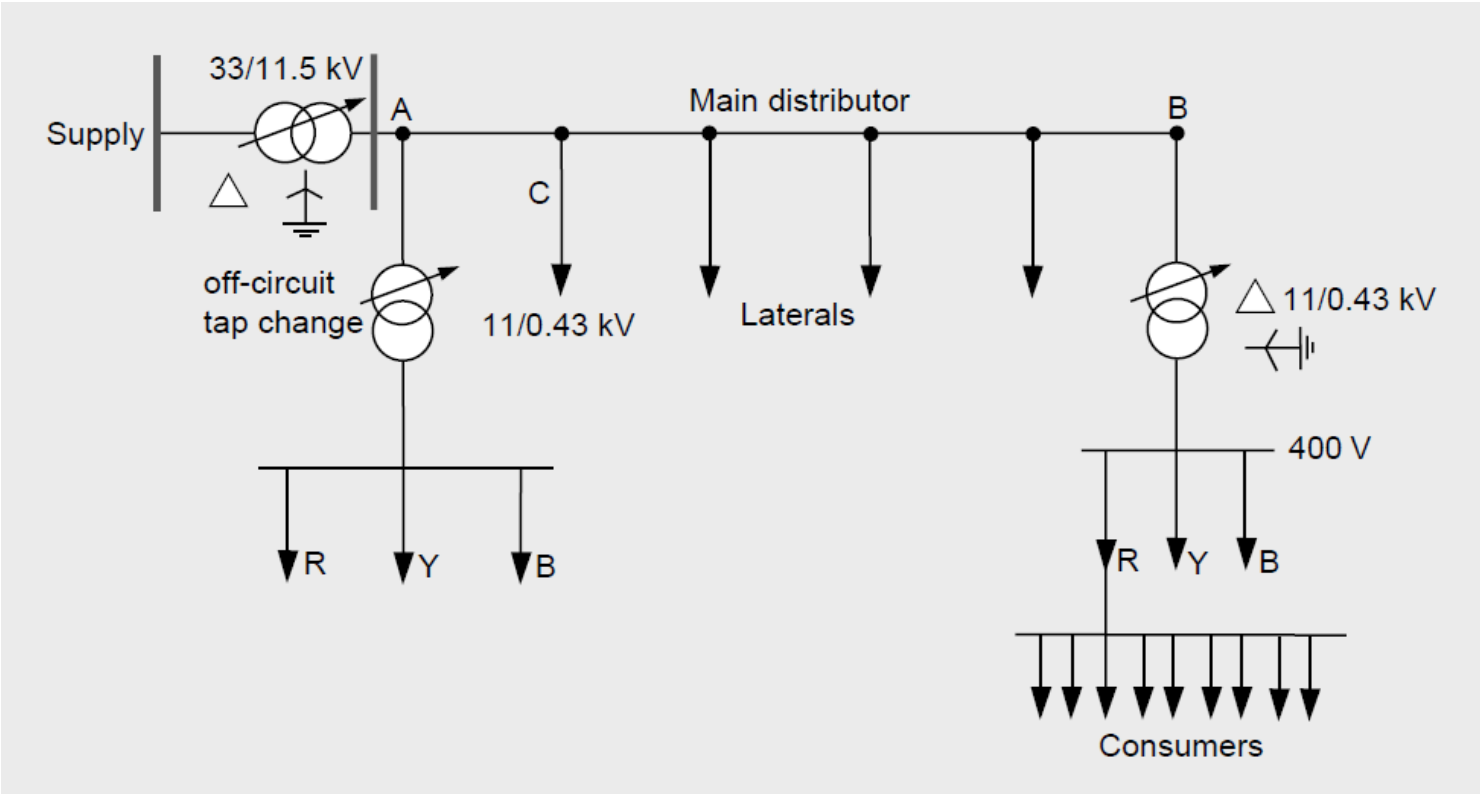
https://www.aemo.com.au/aemo/apps/visualisations/map.html

Next: Distribution network voltage regulation



On a distribution feeder, the substation sets upstream voltage.

Loads and distributed generation change injections along the feeder.

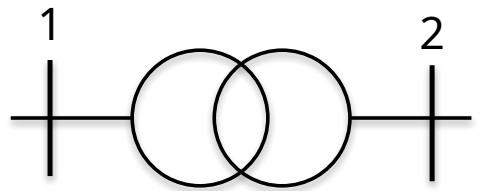


Distribution voltage limits in Victoria, January 2026

STANDARD NOMINAL VOLTAGE VARIATIONS					
	Voltage Level in kV	**Voltage Range for Time Periods**			**Impulse voltage**
		Steady State	**Less than 1 minute**	**Less than 10 seconds**	
1	<1	*AS* 61000.3.100*	+ 13% - 10%	Phase to Earth +50%, -100% Phase to Phase +20%, -100%	6 kV peak
2**		+ 13% - 10%			
3	1 – 6.6	± 6% (± 10% Rural Areas)	± 10%	Phase to Earth +80%, -100% Phase to Phase +20%, -100%	60 kV peak
4	11				95 kV peak
5	22				150 kV peak
6	66	± 10%	± 15%	Phase to Earth +50%, -100% Phase to Phase +20%, -100%	325 kV peak

https://www.esc.vic.gov.au/sites/default/files/documents/Electricity%20Distribution%20Code%20of%20Practice%20-%20Version%203_0.pdf

Tap-Changing Transformers



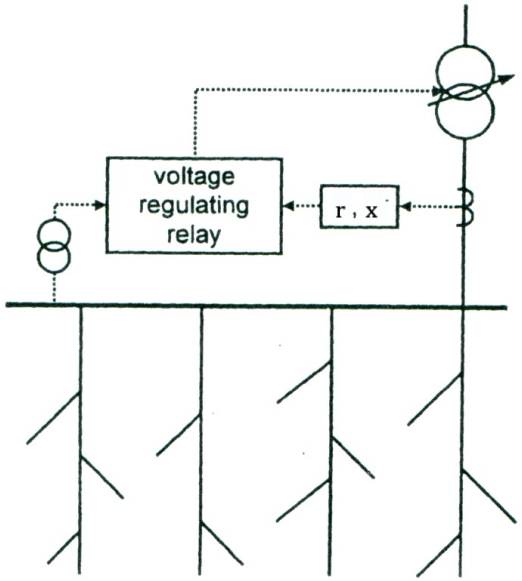
$$tV_1 = V_2$$

Off-load tap changer

- Aka as no-load tap changer
- Fixed transformation ratio by changing number of windings
- Manual tapping only when there is no load (hence the name)
- Common in Australian distribution transformers (e.g., 22kV/0.4kV)

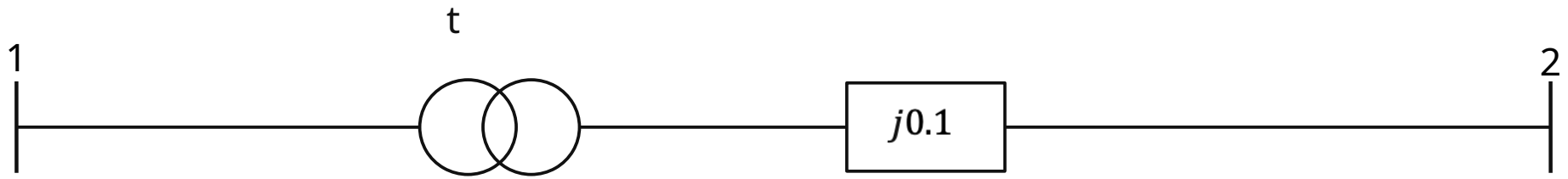
On-load tap changer (OLTC)

- In Australia found in primary substations (e.g., 66kV/22kV) and upstream
- Taps are automatically controlled under load following a setting
- It can control the voltage of a remote bus (see schematic)
- In Australia, it typically controls the voltage of the secondary bus



Exercise: OLTC tap settings

The tap t is downstream voltage gain: $V_2 \approx tV_1 + xQ_2$.

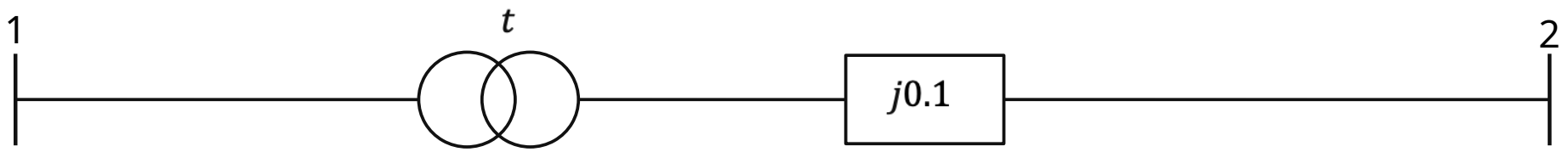


Condition	P_2	Q_2	V_1
Heavy load	-0.4	-0.3	0.97
Light load	-0.16	-0.12	1.03

1. **Find t for $V_2 = 1$ in each condition.**
2. **At light load, find V_2 if the heavy-load tap is retained.**

Exercise Solution:

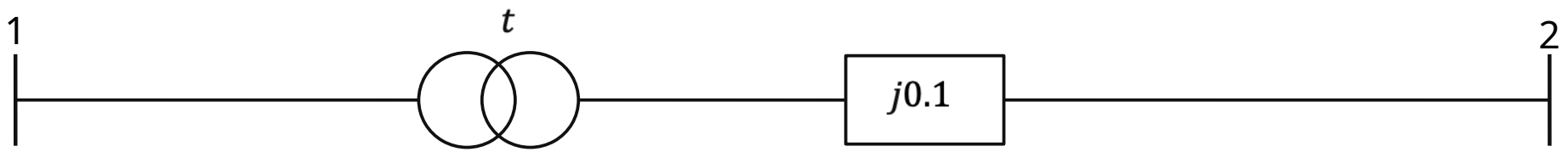
Heavy load



$$XQ_2 = 0.1(-0.3) = -0.03$$

$$1 \approx 0.97t - 0.03 \quad \Rightarrow \quad t \approx 1.06$$

Light load

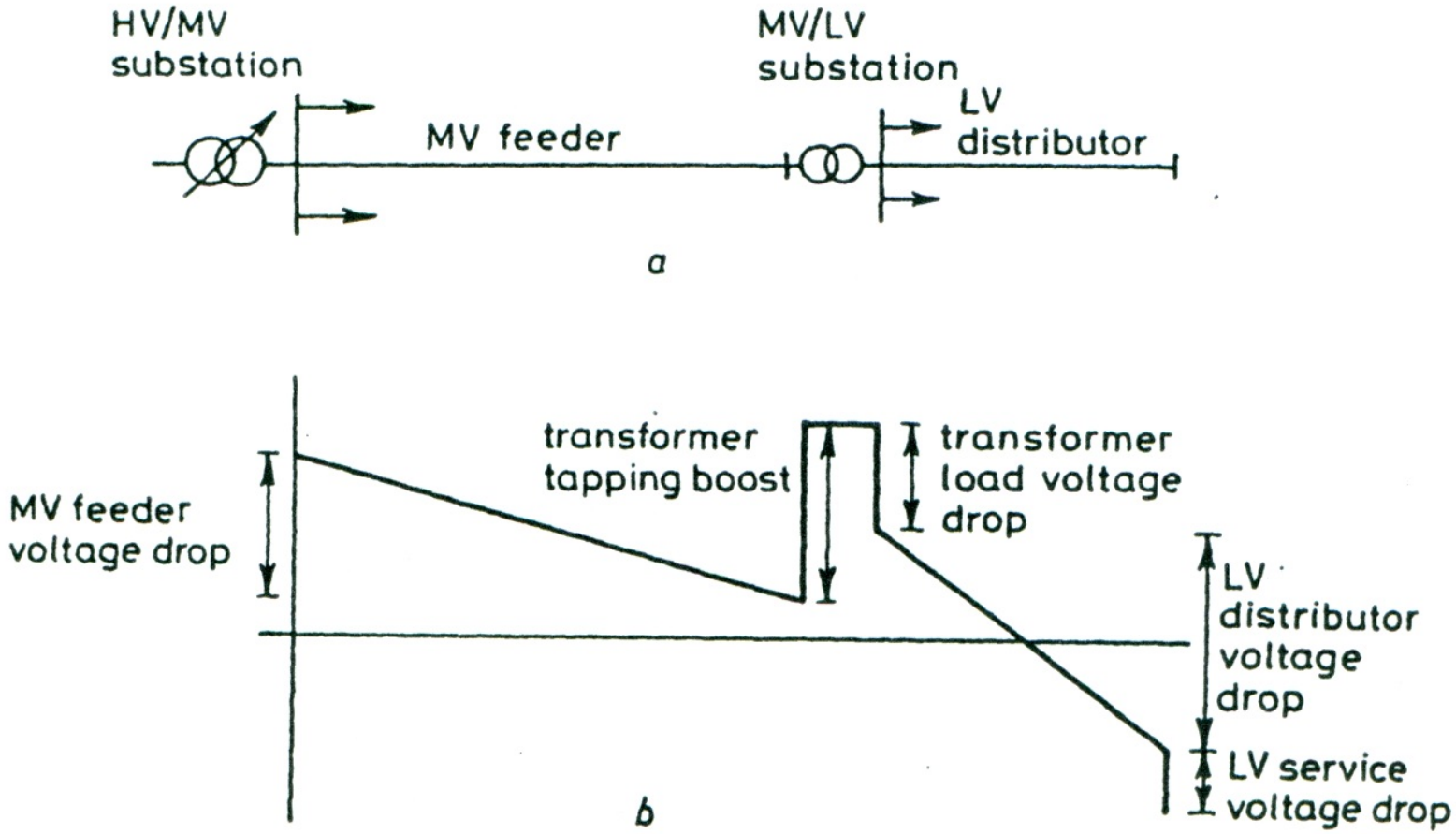


$$XQ_2 = 0.1(-0.12) = -0.012$$

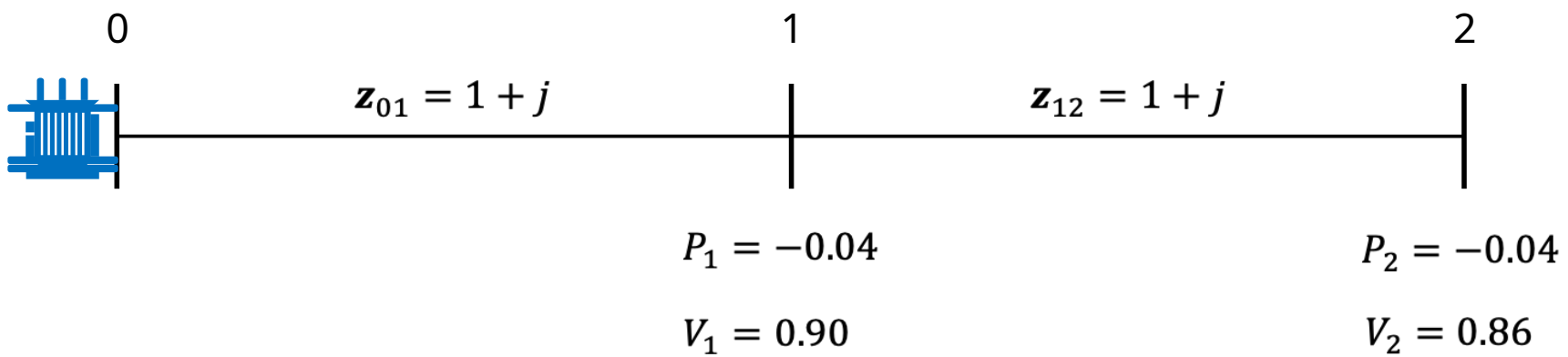
$$1 \approx 1.03t - 0.012 \quad \Rightarrow \quad t \approx 0.98$$

$$t = 1.06 \quad \Rightarrow \quad V_2 \approx 1.08$$

Tap changes shift the feeder voltage profile.

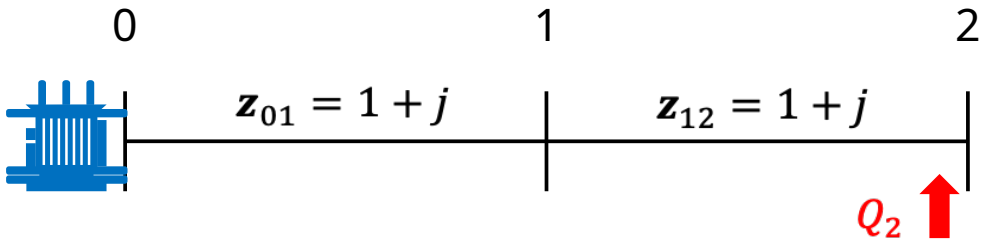


As with transmission, real-power loads lower feeder voltage.



On distribution networks there are two popular methods to control voltage.

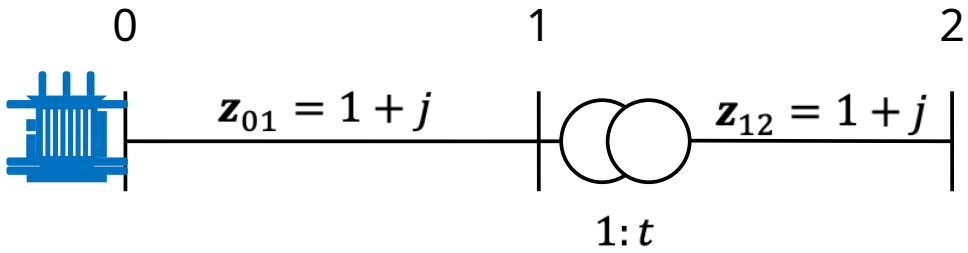
Reactive injection: $Q_2 = 0.078$



$P_1 = -0.04$ $P_2 = -0.04$

$V_1 = 0.97$ $V_2 = 1$

Transformer tap change: $t = 1.15$



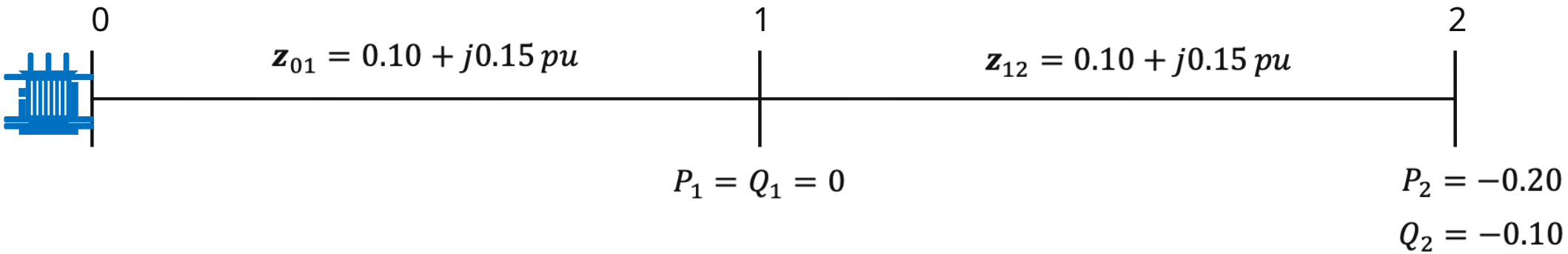
$P_1 = -0.04$ $P_2 = -0.04$

$V_1 = 0.90$ $V_2 = 1$

Exercise: feeder voltage control

Use the linear voltage equations at a flat 1 pu operating point:

$$\Delta V \approx R\Delta P + X\Delta Q \qquad R = \begin{bmatrix} r_1 & r_1 \\ r_1 & r_1 + r_2 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 & x_1 \\ x_1 & x_1 + x_2 \end{bmatrix}$$



$V_0 = 1\,pu$, $100\,MVA$ base, allowable voltage $0.95–1.05\,pu$

Does raising V_0 to $1.01\,pu$ restore bus 2 within the limit?

Does installing a $10\,MVAR$ capacitor restore bus 2 within the limit?

Solution:

$$\Delta V \approx R\Delta P + X\Delta Q \qquad R=\begin{bmatrix} r_1 & r_1 \\ r_1 & r_1+r_2 \end{bmatrix} \qquad X=\begin{bmatrix} x_1 & x_1 \\ x_1 & x_1+x_2 \end{bmatrix}$$

$$\begin{aligned} \Delta V_2 &\approx (r_1 + r_2)P_2 + (x_1 + x_2)Q_2 \\ &\approx 0.20(-0.20) + 0.30(-0.10) = -0.07\,\text{pu} \end{aligned}$$

$0.93\,\text{pu} < 0.95\,\text{pu}$ → Bus 2 needs voltage support

Compare the two controls:

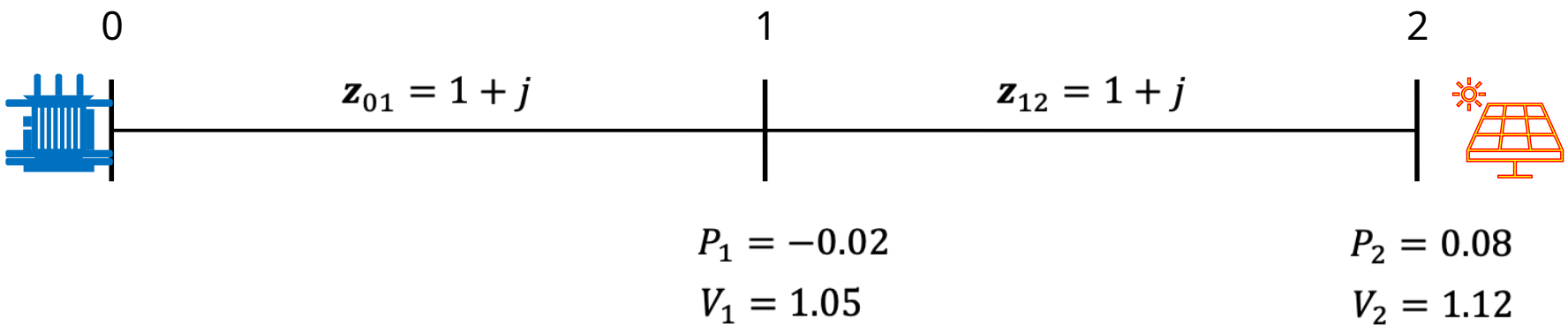
With injections fixed, a change in V_0 shifts V_2 by approximately the same ammount:

$$V_2 \approx 0.93 + 0.01 = 0.94\,\text{pu}$$ ❌

Install the 10 MVAr capacitor at bus 2 (linearized $\Delta Q_2 \approx +0.10\,\text{pu}$):

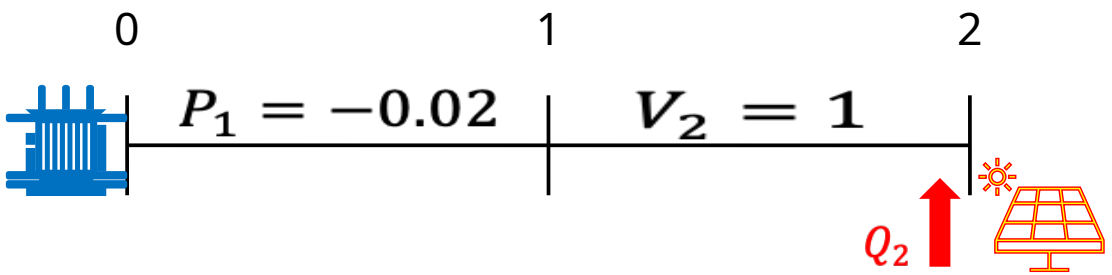
$$V_2 \approx 0.93 + (0.15 + 0.15)(0.10) = 0.96\,\text{pu}$$ ✅

Distribution networks can also have over-voltage, e.g. due to rooftop solar power



Like under-voltage, both voltage control methods can also control over-voltage

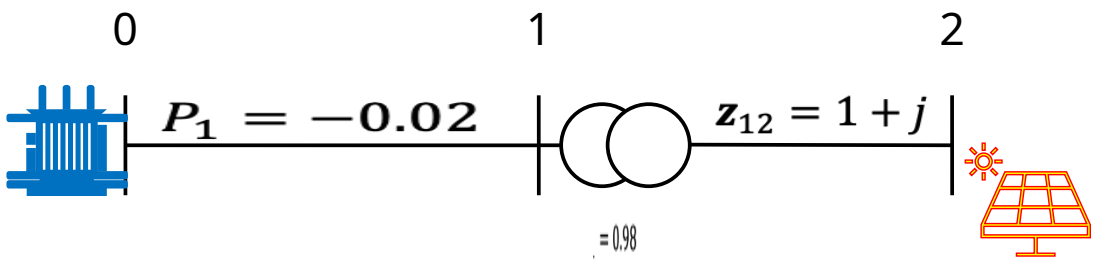
Reactive absorption: $Q_2 = -0.053$



$P_1 = -0.02$
$V_1 = 0.98$

$P_2 = 0.08$
$V_2 = 1$

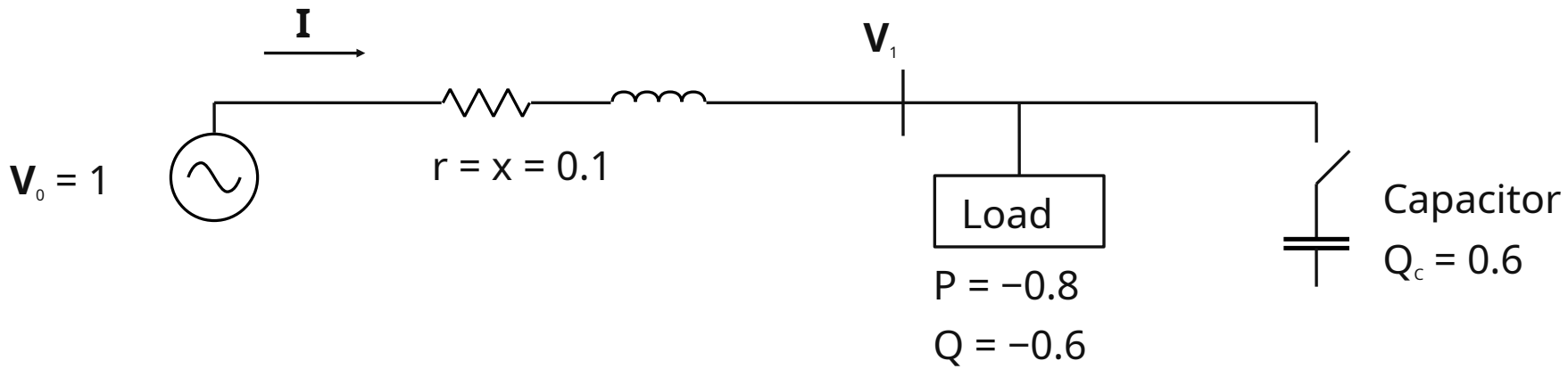
Transformer tap change: $t = 0.88$



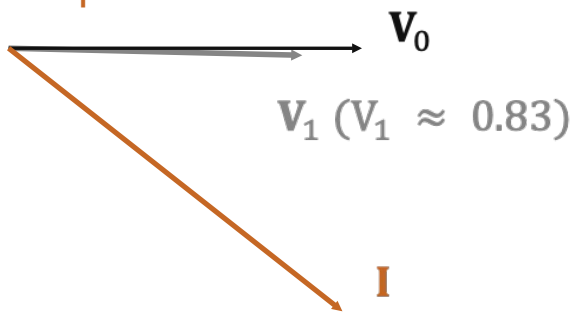
$P_1 = -0.02$
$V_1 = 1.04$

$P_2 = 0.08$
$V_2 = 1$

Reactive power injection can also reduce system losses

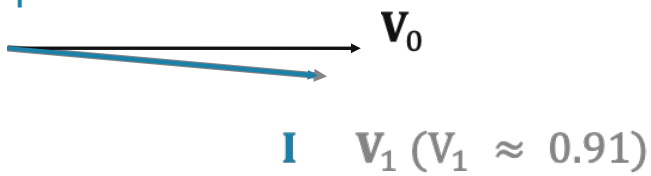


No capacitor



$I \approx 1.2, \quad \text{PF at bus 1} = 0.8$

Capacitor on



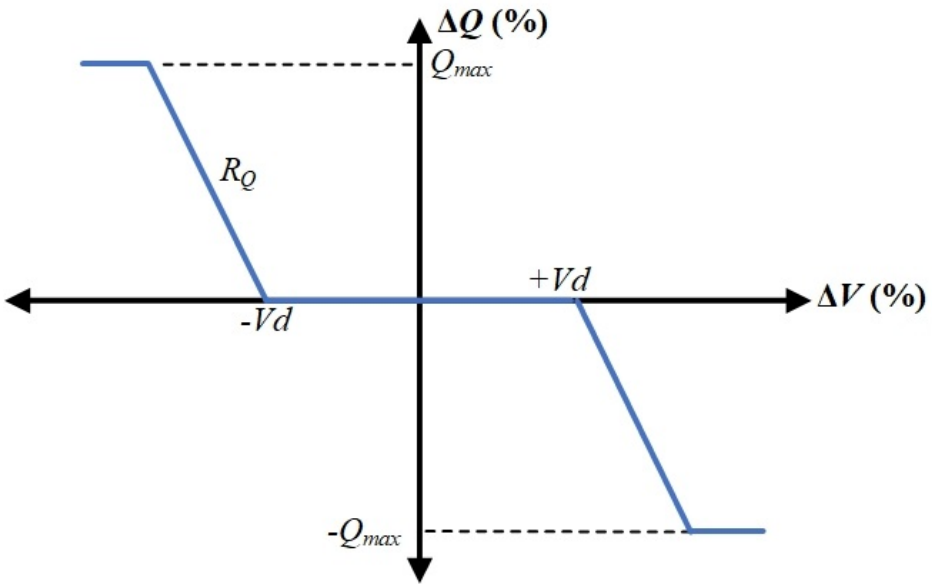
$I \approx 0.88, \quad \text{PF at bus 1} = 1$

Supplying Q locally raises the load voltage and reduces feeder current.

$P_{loss} = I^2R \approx 0.14$

$P_{loss} = I^2R \approx 0.08$

Volt–VAR control reduces voltage violations



Reactive injection raises local voltage, absorption lowers it.

$$\Delta V \approx R\Delta P + X\Delta Q$$

With high solar export, reactive absorption offsets part of the voltage rise.

Under heavy load, reactive injection offsets part of the voltage drop.

Australia

2015: Australia A included optional Volt–VAR, off by default

2019: Victorian distributors required Volt–VAR for most new inverter connections

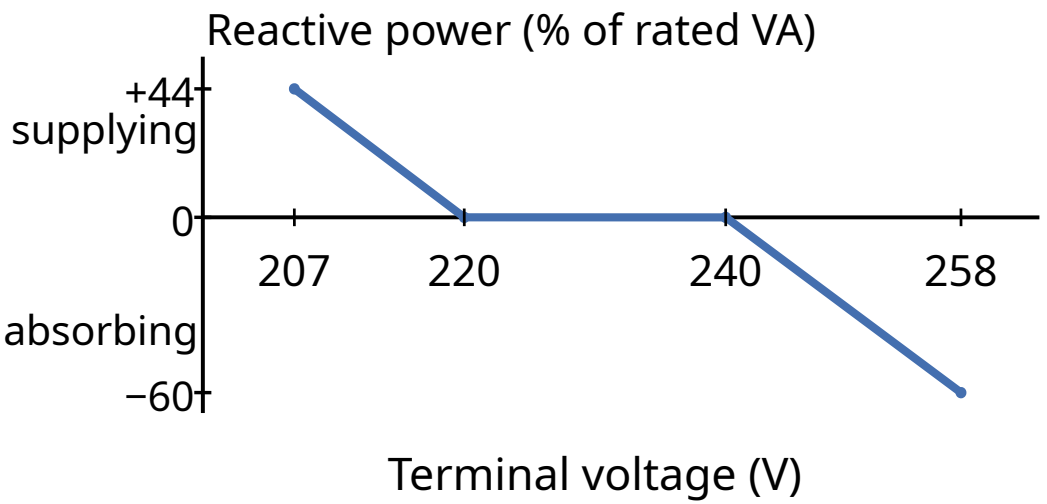
2021: Australia A required for new small-scale inverters; Volt–VAR on by default

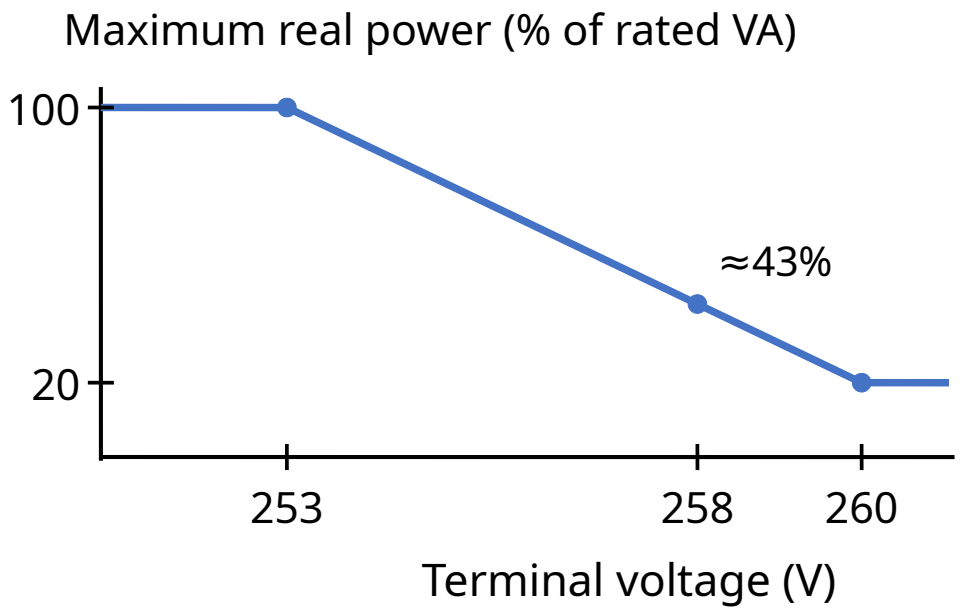
California

2017: Rule 21 required Volt–VAR capable smart inverters for new connections

2018: Volt–VAR active by default in those tariffs, with reactive-power priority

Volt–VAR and Volt–Watt settings in Australia ("Australia A")





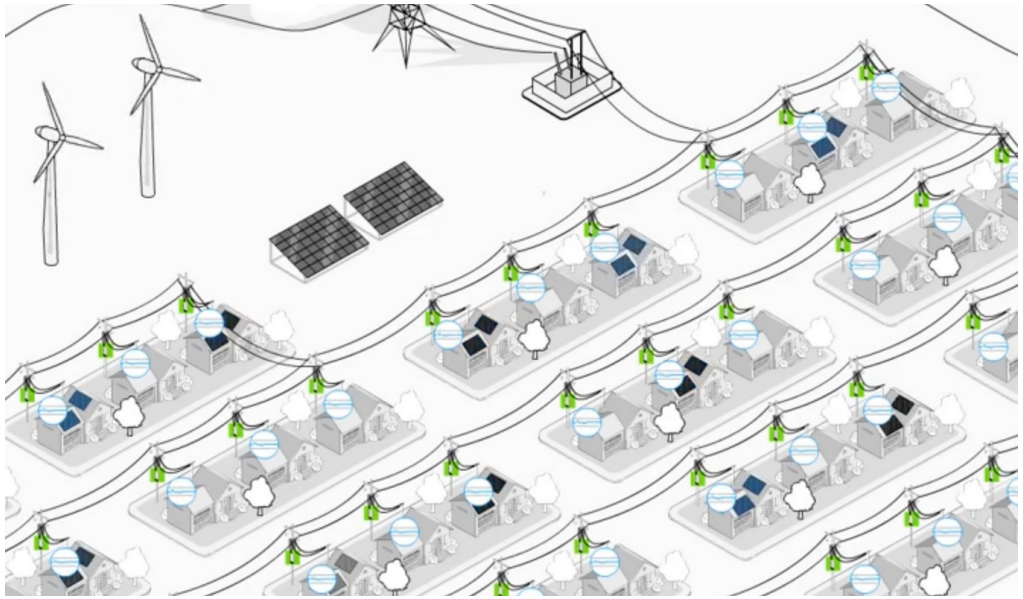
$$\Delta V \approx R\Delta P + X\Delta Q$$

Exercise:

You're an investor. You are meeting with Raventec, a hot grid optimization startup to determine if you will invest in their company.

The Raventec rep states that their devices mount on utility poles and optimize distribution grid flows, providing the following benefits:

- Energy savings
- Peak-demand reductions
- Increased solar hosting capacity
- Fewer transformer tap changes



Do you invest?

2013: Varentec's ENGO devices inject/extract reactive power on distribution networks





https://www.youtube.com/watch?v=16vC9IqtDVI

Varentec sold devices for reactive-power support.



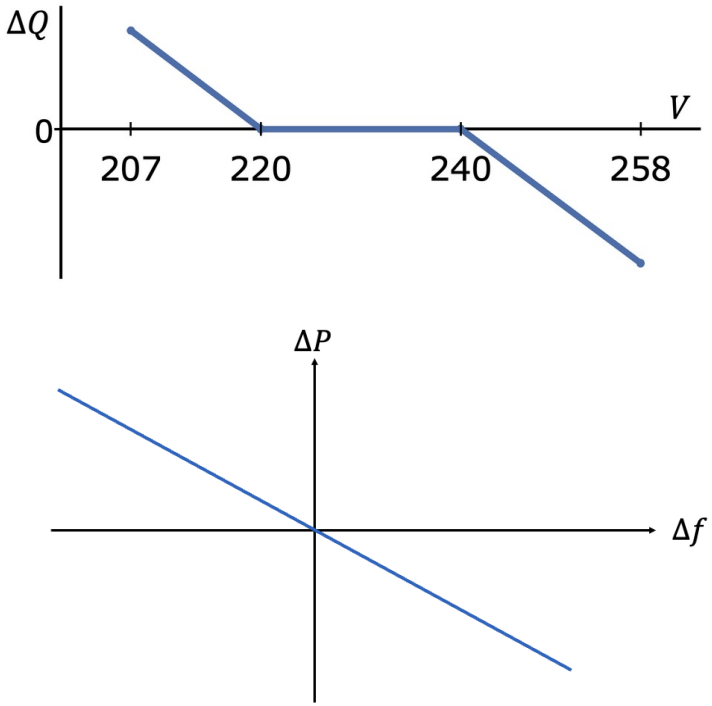
This was a compelling idea, but solar and battery inverters already contain the power electronics needed to provide this function.

Volt–VAR control uses the existing inverter capability to automatically provide many of the services Varentec promised, though not as optimally.

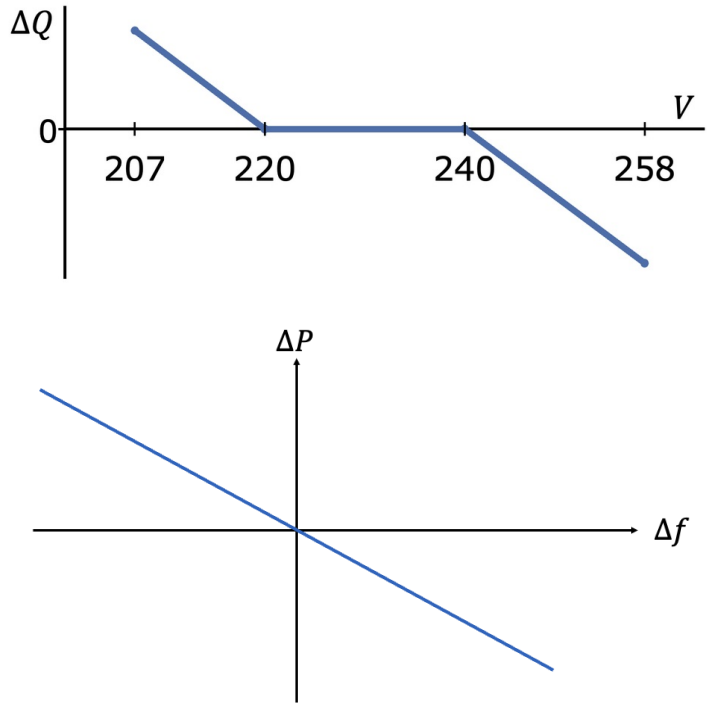
Thus, the overhead of buying and maintaining its inverters for reactive power optimization was not compelling for many utilities.

Varentec was acquired by Sentient Energy in 2021.

Simple, principled inverter-connection grid codes can support the stable grids of tomorrow





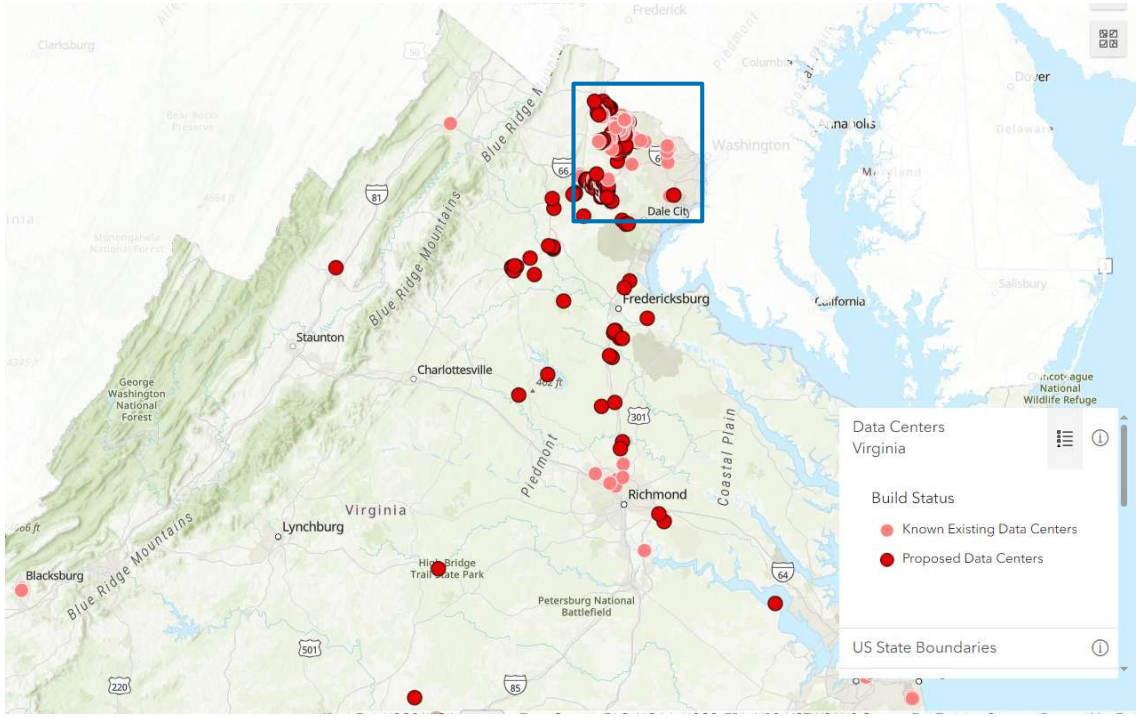


Voltage Control

ELEN90060 Power System Analysis

Keith Moffat

22 July 2026: A $230\,kV$ line fault triggered $3.8\,GW$ of data center load to disconnect from the grid.





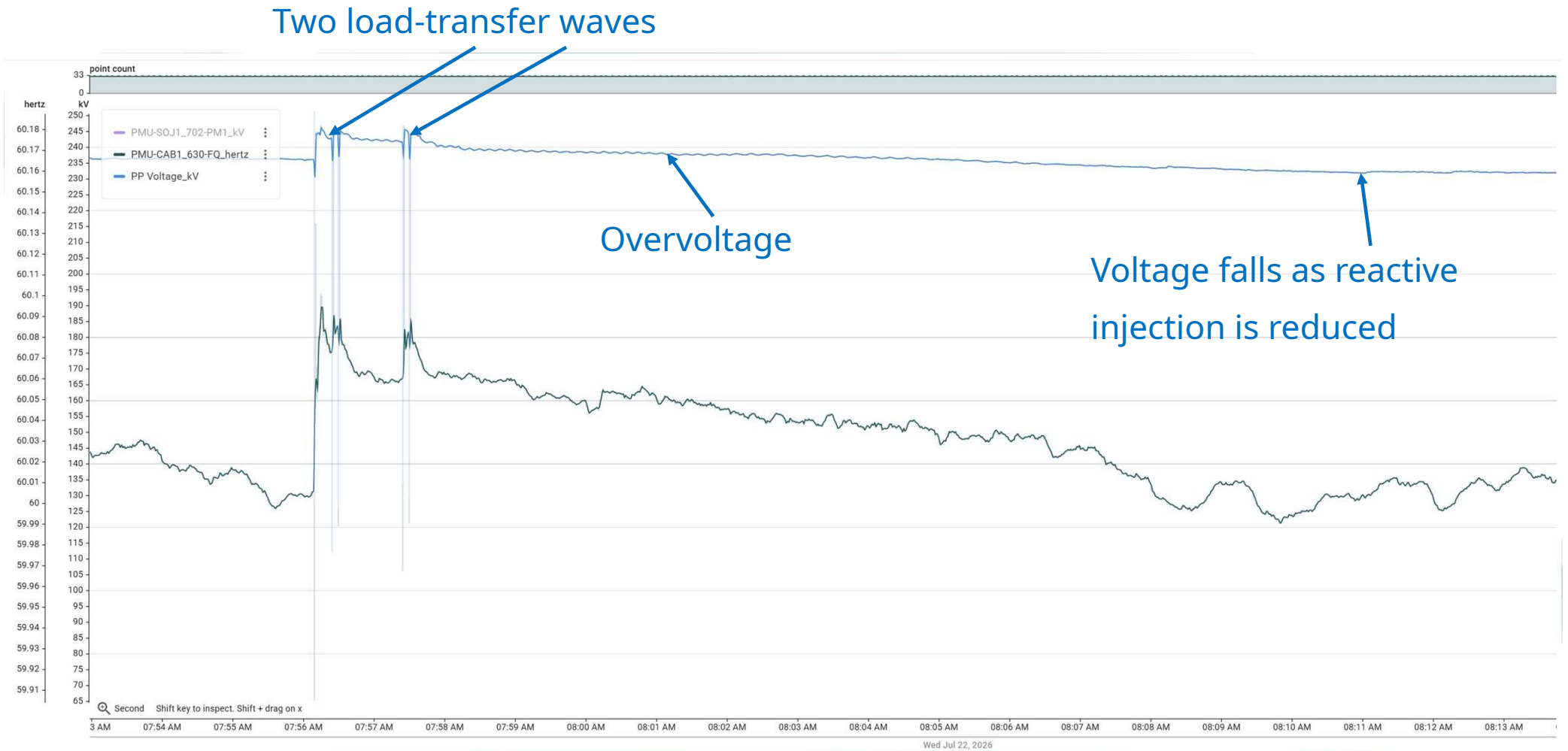
(Victoria's electricity demand typically ranges between 4.5 and $6.5\,GW$)

Why have you not heard of this event?

(The grid operators did a great job and Virginia is in the middle of a very large grid)

https://www.pjm.com/-/media/DotCom/committees-groups/committees/oc/2026/20260806/20260806-item-05---dominion-july-loss-of-load-event---presentation.pdf

https://www.dpr.com/projects/digital-realty-ground-up-data-center-building-m1



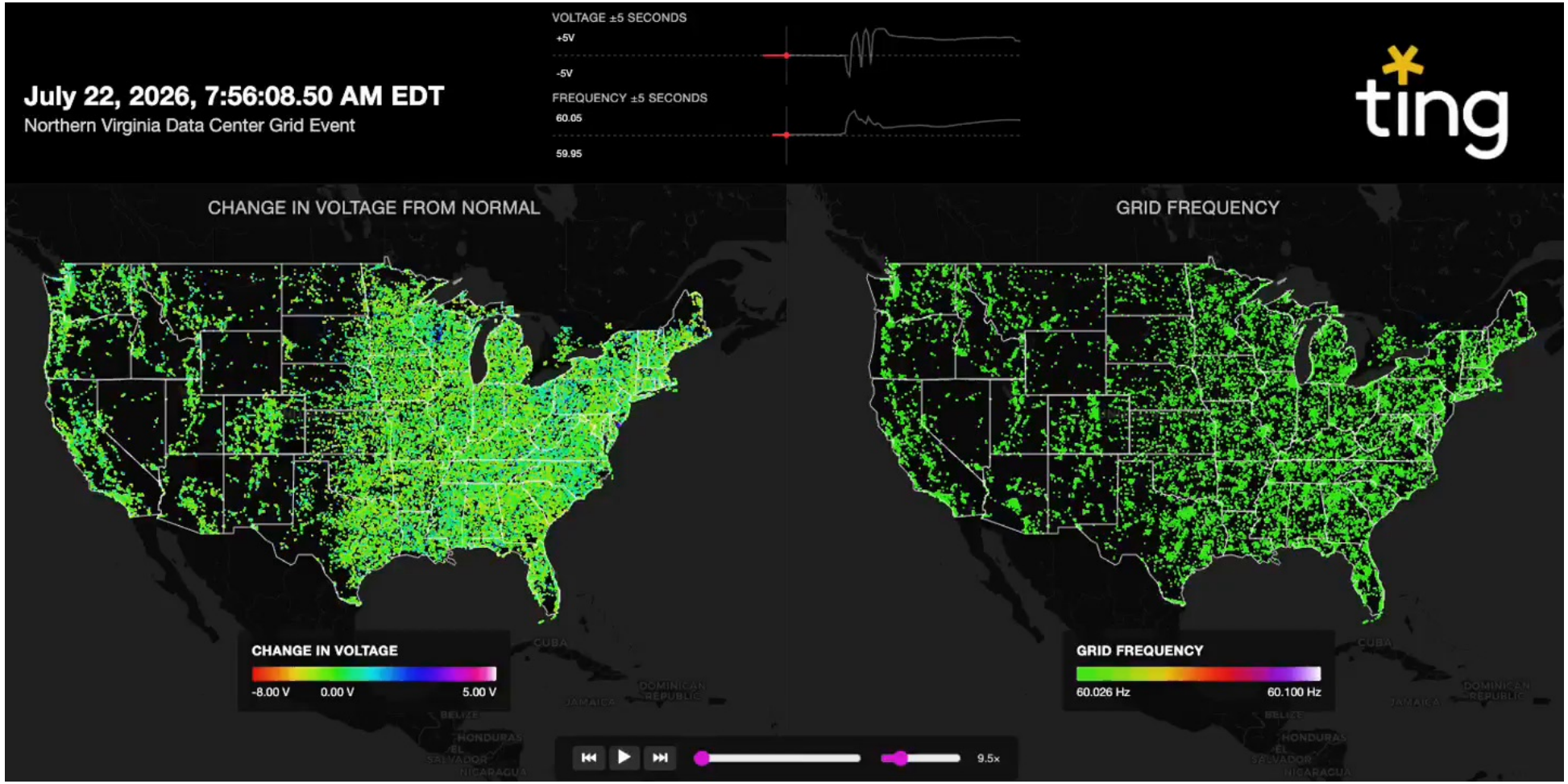
Capacitor banks trip off to stop injecting reactive power

Operators insert reactors to absorb reactive power

- **07:59:05:** Valley Reactor closed
- **07:59:55:** Dooms Reactor closed

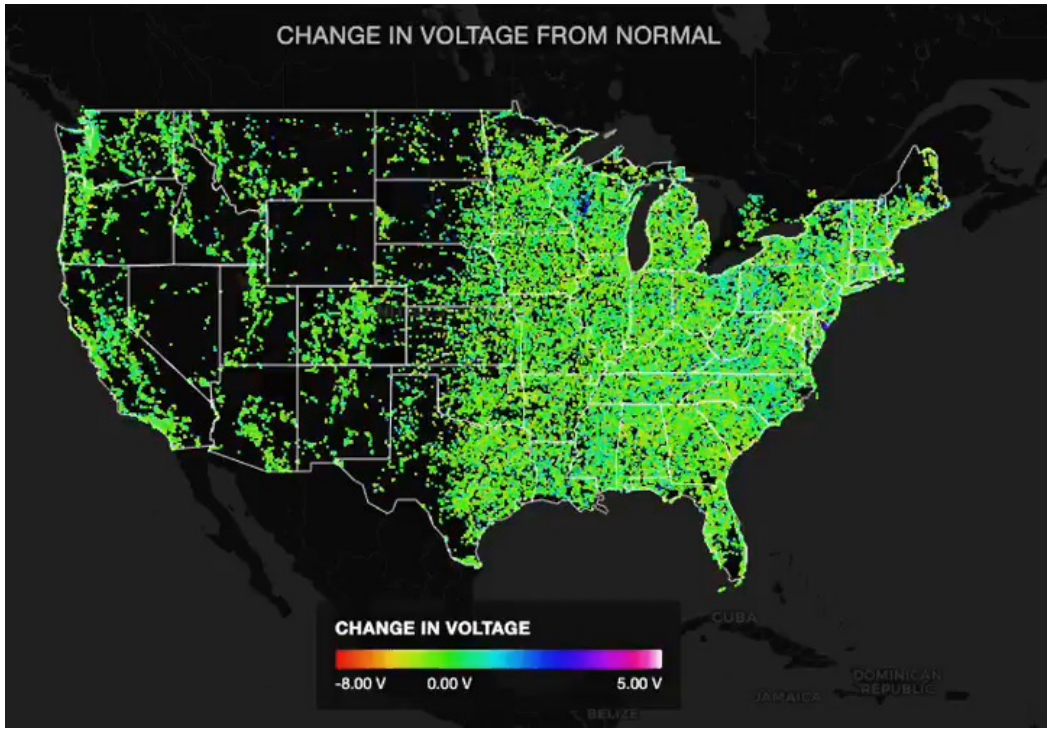
Generator AVRs absorb reactive power to maintain voltage

PJM / Dominion, 6 August 2026, p. 30



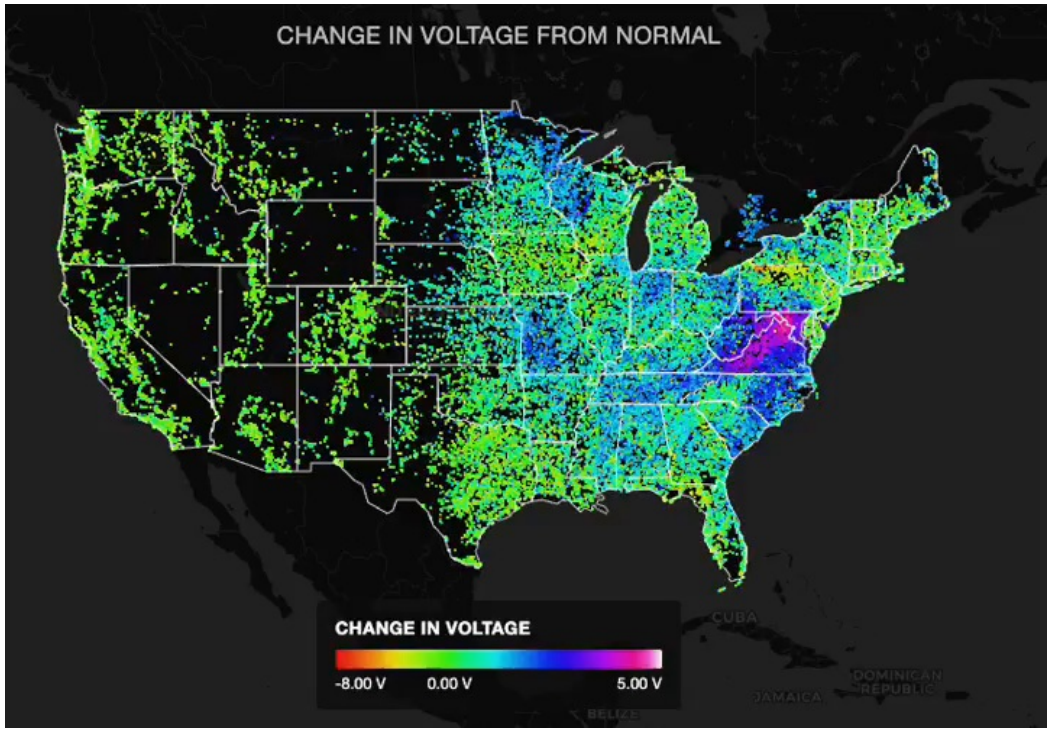
Source: Ting Labs / Bob Marshall. Northern Virginia event, 22 July 2026.

Before: 07:56:08.5 EDT



Source: Ting Labs / Bob Marshall. Northern Virginia event, 22 July 2026.

After: 07:56:17.1 EDT

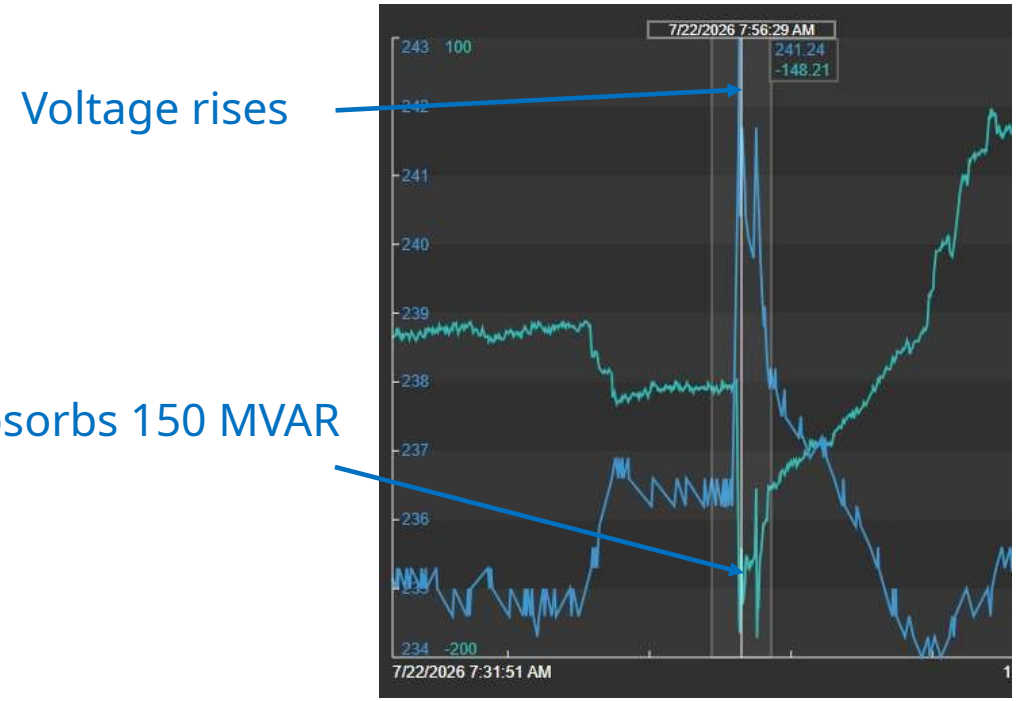


How did AVR-controlled generators respond to the voltage rise?

Nearby generator



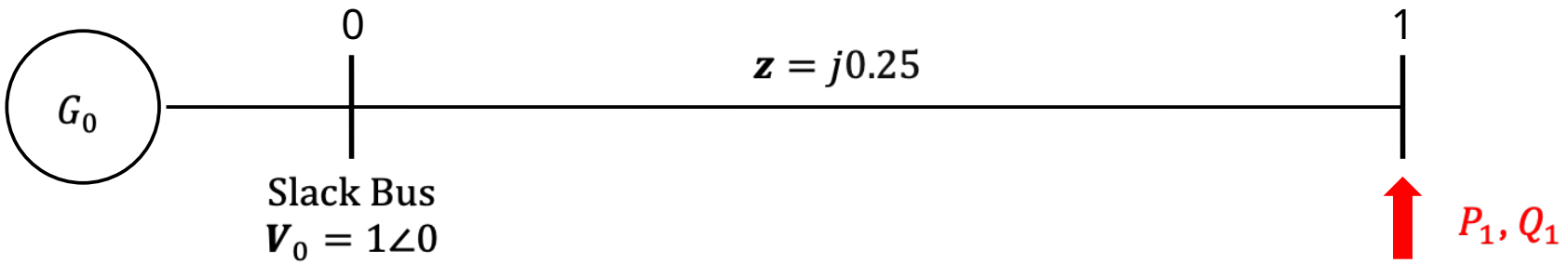
Distant generator



PJM / Dominion, 6 August 2026, p. 32

Exercises

Exercise: line reactive loss



A source supplies a load at bus 1 through a series reactance. Neglect shunts and resistance.

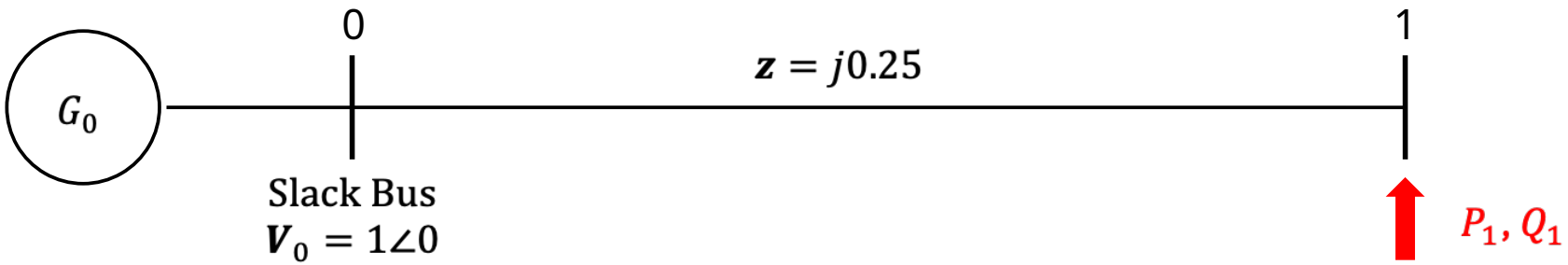
$$V_1 = 1\angle 0, \quad P_1 = -0.8, \quad Q_1 = -0.4$$

Line reactive power consumption: $Q_{\text{line}} = xI^2, \qquad I^2 = \frac{P_1^2 + Q_1^2}{V_1^2}$

Determine the source injections P_0 and Q_0 and the source's power factor. Is this greater or less than the load's power factor?

Is the source's power factor 1 when $Q_1 = 0$? If not, where does this reactive power go?

Exercise Solution: Line reactive loss



$$I^2 = 0.8^2 + 0.4^2 = 0.8$$

$$Q_{\text{line}} = 0.25(0.8) = 0.2 \text{ pu}$$

$$P_0 = 0.8, \qquad Q_0 = 0.4 + 0.2 = 0.6 \text{ pu}$$

$$\text{pf}_0 = \frac{0.8}{\sqrt{0.8^2 + 0.6^2}} = 0.8 \text{ lagging}$$

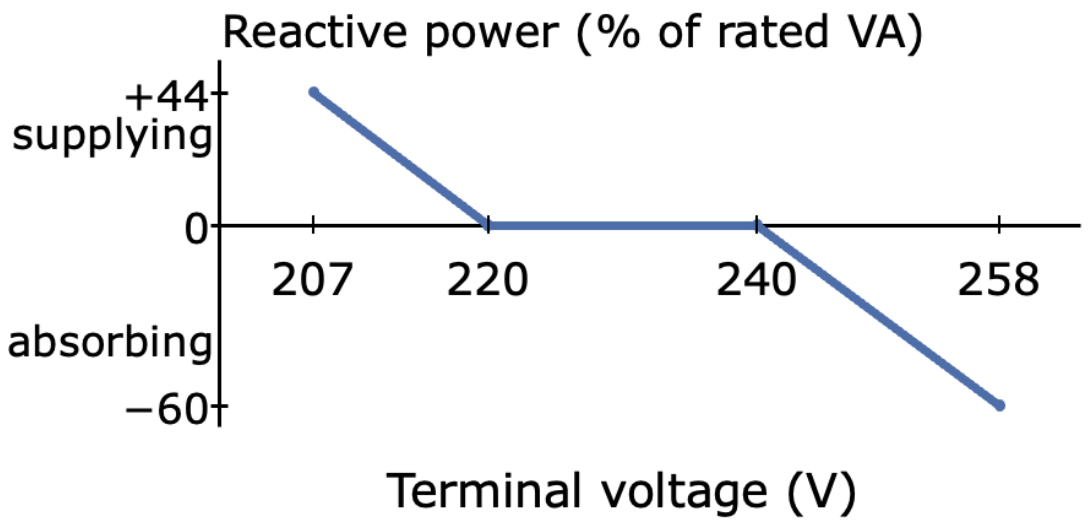
Less than the load PF= $\frac{0.8}{\sqrt{0.8^2+0.4^2}}$

When $Q_1 = 0$, the power factor is not 1 as it supplies reactive power to the line inductance

Exercise: fixed PF and Volt–VAR

An inverter rated $10\,\text{kVA}$ supplies $P = 8.2\,\text{kW}$

Compare fixed PF = 0.98 (absorbing) with Australia A Volt–VAR:



Fixed PF: $Q = -P\tan(\arccos \text{PF})$

Volt–VAR: $Q = -0.60 \times 10 \times \frac{V - 240}{258 - 240}\ \text{kVAR}$

max % absorbing

inverter rating

1. Initially, $V = 245\,V$. **Find Q for each control mode.**

2. Voltage later rises to $252\,V$, with P unchanged. **Find Q for each mode**. Which is better?

Exercise Solution:

1. Initially, $V = 245$

For set power factor: $Q = -8.2 \tan(\arccos 0.98) \approx -1.67\ \text{kVAr}$

For Volt-VAR: $Q = -0.60 \times 10 \times \frac{245 - 240}{258 - 240} \approx -1.67\ \text{kVAr}$

Both modes initially absorb approximately 1.67 kVAr.

2. After the voltage rise, $V = 252$

For set power factor: $Q \approx -1.67\ \text{kVAr}$

For Volt-VAR: $Q = -0.60 \times 10 \times \frac{252 - 240}{258 - 240} = -4\ \text{kVAr}$

Volt–VAR is better because it increases Q absorption as voltage rises, helping keep voltage down.

Exercise: using short-circuit current to determine voltage sensitivity

At Bus 1, a balanced three-phase bolted fault gives the following magnitudes:

$$E_{\text{th}} = 1.0\,\text{pu}, \qquad I_{\text{SC}} = 10\,\text{pu}, \qquad \frac{X_{\text{th}}}{R_{\text{th}}} = \frac{4}{3}$$

Find the voltage-magnitude sensitivities $\frac{\partial V_1}{\partial P_1}$, $\frac{\partial V_1}{\partial Q_1}$ at the operating point:

$$P_1^\circ = Q_1^\circ = 0, \qquad V_1^\circ = E_{\text{th}} = 1.0\,\text{pu}$$

Exercise solution:

$$Z_{\text{th}} = \frac{V_{\text{th}}}{I_{\text{SC}}} = 0.1$$

$$\frac{x_{\text{th}}}{r_{\text{th}}} = \frac{4}{3} \quad \Rightarrow \quad r_{\text{th}} = 0.06, \qquad x_{\text{th}} = 0.08$$

At the stated operating point:

$$\Delta V_1 \approx r_{\text{th}} \Delta P_1 + x_{\text{th}} \Delta Q_1 = 0.06\,\Delta P_1 + 0.08\,\Delta Q_1$$

$$\frac{\partial V_1}{\partial P_1} \approx 0.06, \qquad \frac{\partial V_1}{\partial Q_1} \approx 0.08$$

Exercise: feeder voltage control

An 11 kVA inverter initially delivers 10 kW.

At 258 V, Volt–VAR requests 6 kVAr absorption.

Can the inverter absorb the requested reactive power without reducing its P?

What about if the inverter also adheres to the Volt-Watt control curve?

Exercise Solution:

Volt–VAR alone

$$S = \sqrt{10^2 + 6^2} = 11.66\,kVA > 11\,kVA$$

No. At 10 kW, the maximum absorption is $\sqrt{11^2 - 10^2} = 4.58\,kVAR$.

With Volt–Watt at 258 V

$$P \leq \left(\frac{3}{7}\right) \times 11 = 4.71\,kW$$

$$S = \sqrt{4.71^2 + 6^2} = 7.63\,kVA < 11\,kVA$$

Yes. Volt–Watt curtails active power enough to absorb $6\,kVAR$.