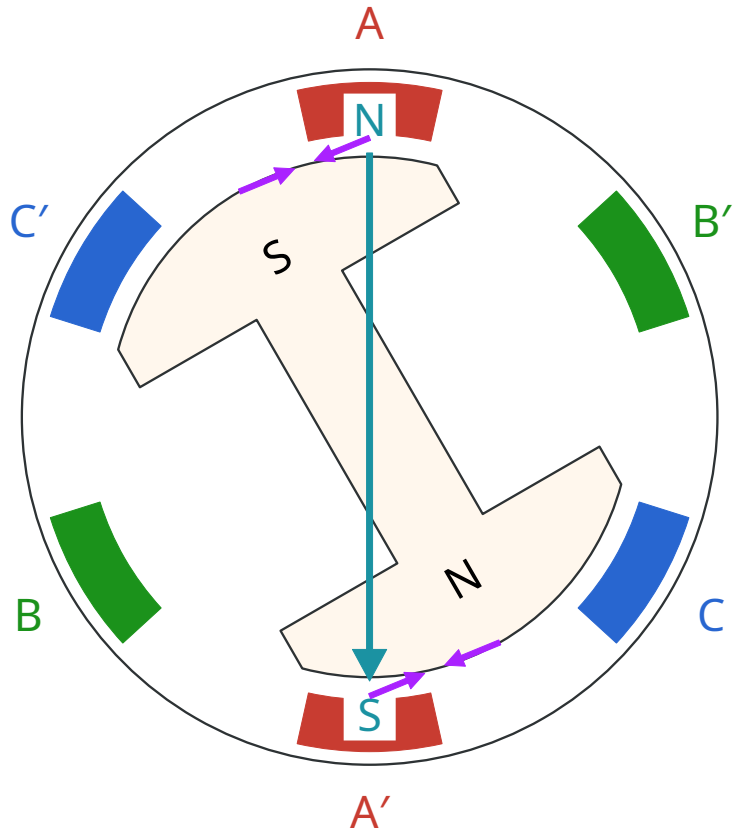
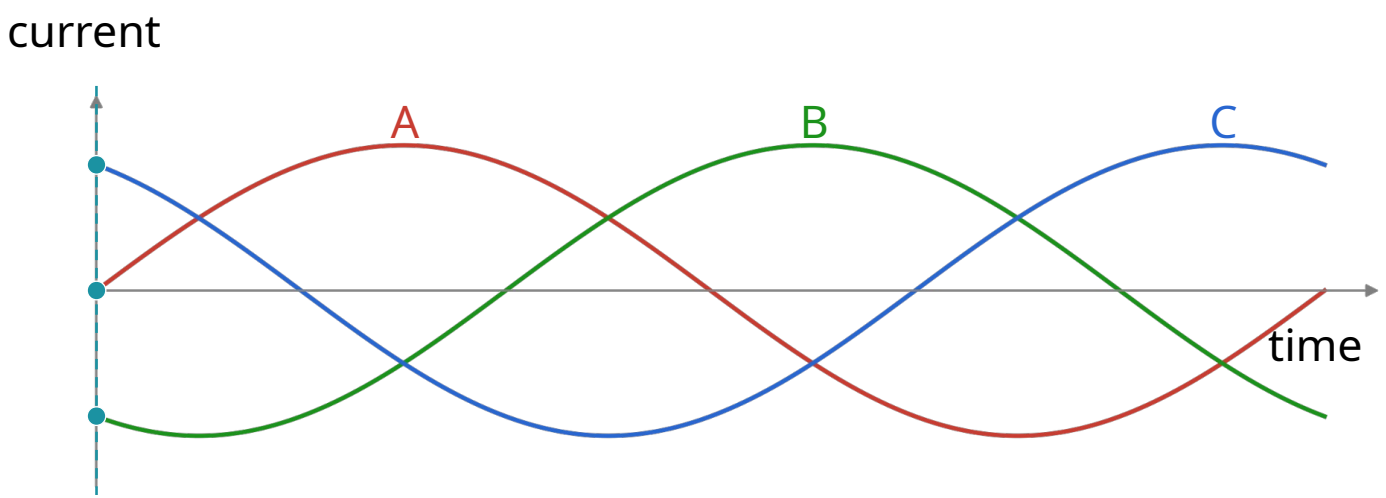
Power System Dynamics Fundamentals

ELEN90060 Power System Analysis

Keith Moffat

Synchronous machine: the turbine drives the rotor magnetic pole, pulling the stator magnetic pole.





History Intermezzo

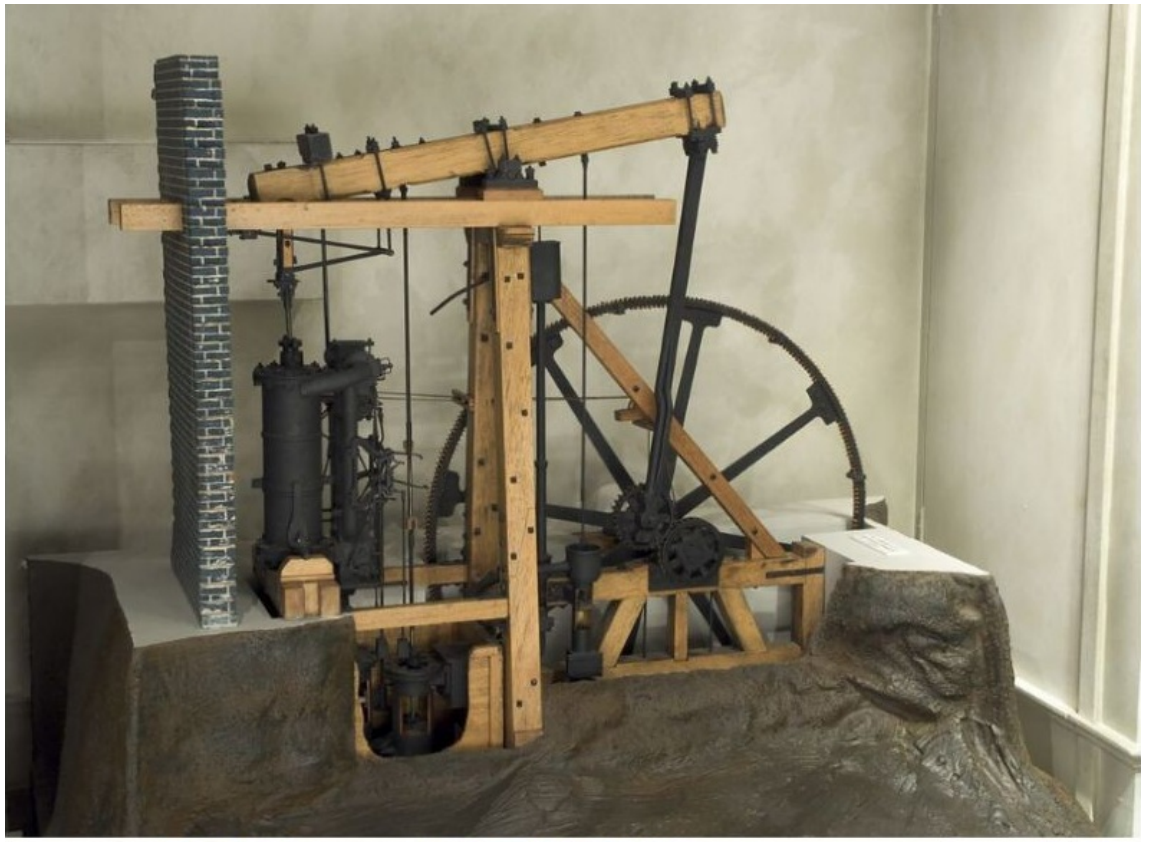
The grid’s stabilizing principle predates electricity

Steam engines had to hold their speed as the mechanical load changed.

1788: Watt fitted a centrifugal governor to a rotative steam engine.

The underlying problem was already power balance:

input power = load power.



commons.wikimedia.org/wiki/File:Thinktank_Birmingham_-_object_1950S00022(1).jpg

Watt’s governor made the engine self-regulating



commons.wikimedia.org/wiki/File:Boulton_and_Watt_centrifugal_governor-MJ.jpg

Load increases

↓

shaft speed falls

↓

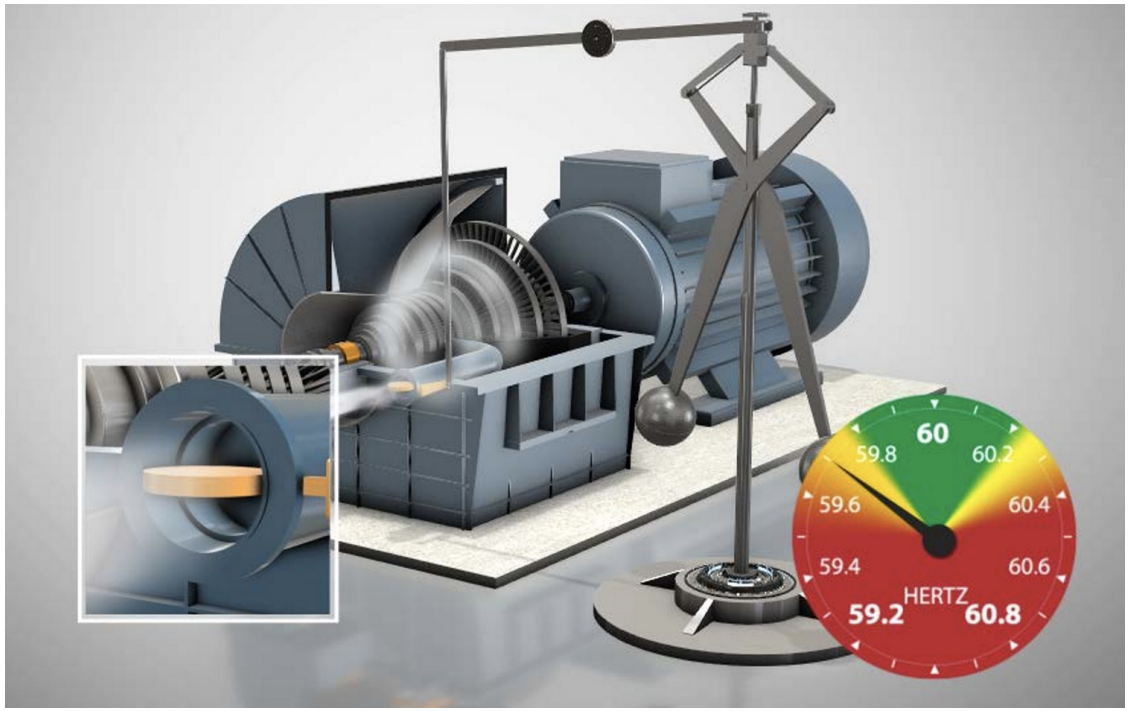
governor opens the steam valve

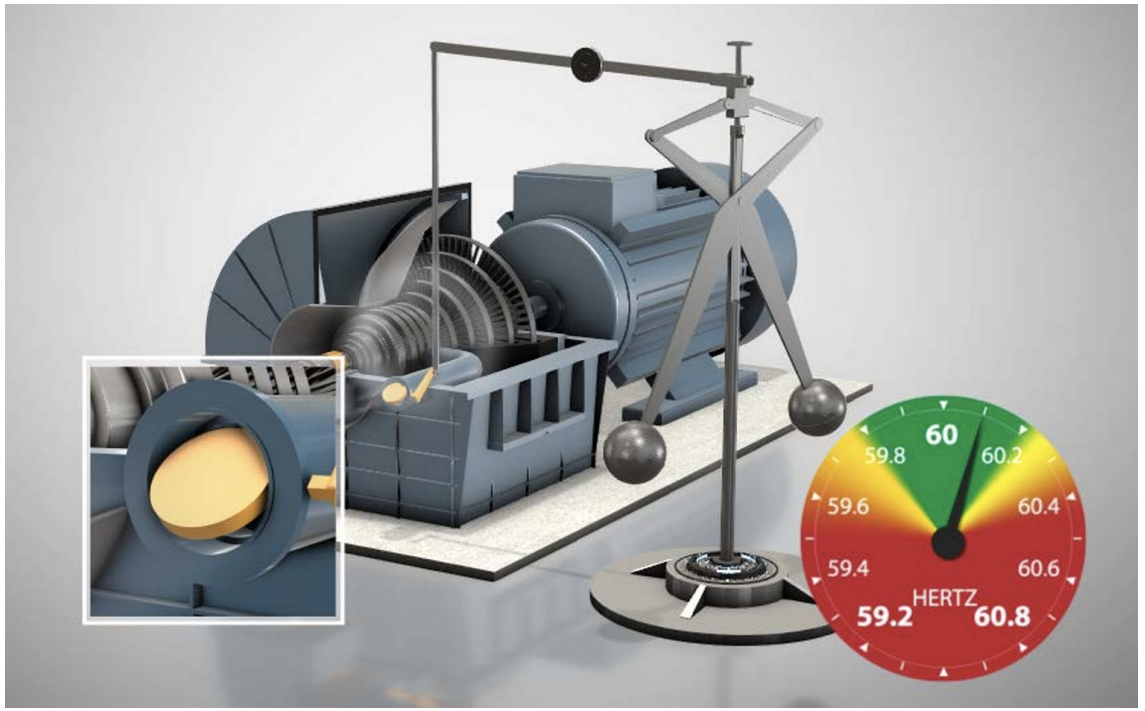
↓

mechanical power rises

Local measurement. Local feedback. No operator.

Droop control is still the backbone of power system stability today





NREL: Beyond the Spin https://docs.nlr.gov/docs/fy20osti/73856.pdf

Maxwell asked whether the feedback would settle



commons.wikimedia.org/wiki/File:James_Clerk_Maxwell_big.jpg

1868: On Governors

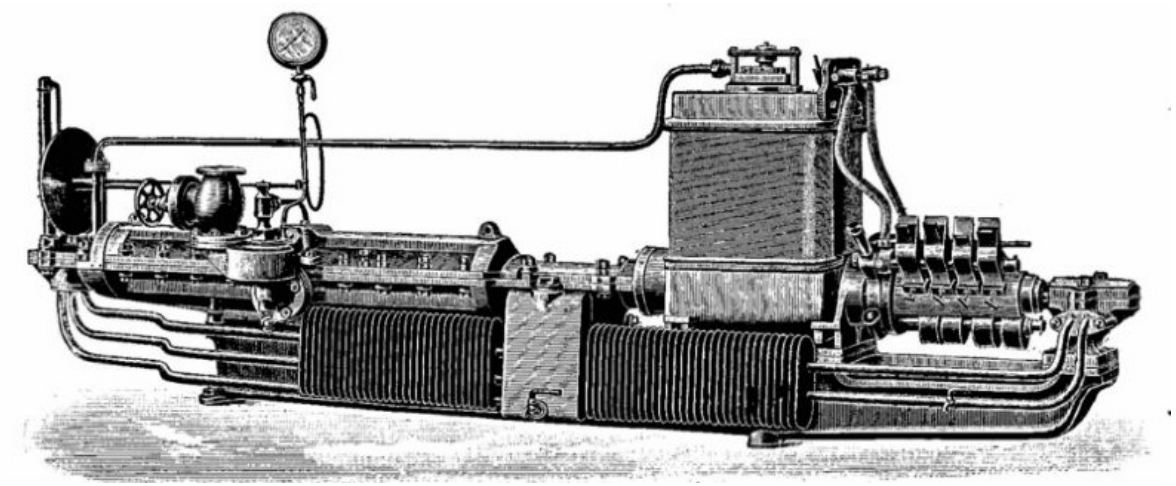
Feedback could produce:

- decay to equilibrium
- sustained “hunting”
- growing oscillations

Stability depended on the roots of the governing differential equation.

→ The birth of the field of control theory

The turbine made shaft speed an electrical variable



commons.wikimedia.org/wiki/File:Parson's_Compound_Steam_Turbine_-_1887_...png

1884: Parsons' first turbine-generator

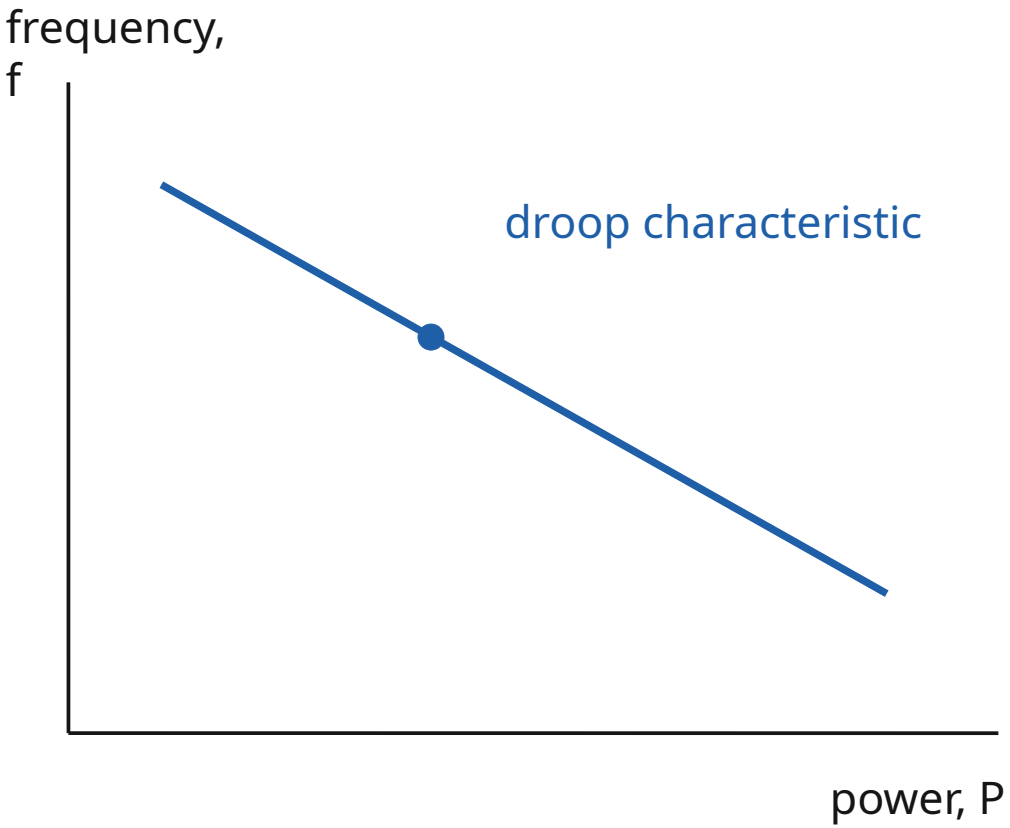
Mechanical speed now determined electrical frequency.

$$f = \frac{p}{4\pi}\,\omega_m$$

p = number of poles

Governor speed control became frequency control.

Droop makes generators share a change in load



$$\Delta P_m = -\frac{1}{R}\,\Delta f$$

Frequency falls → generator produces more power

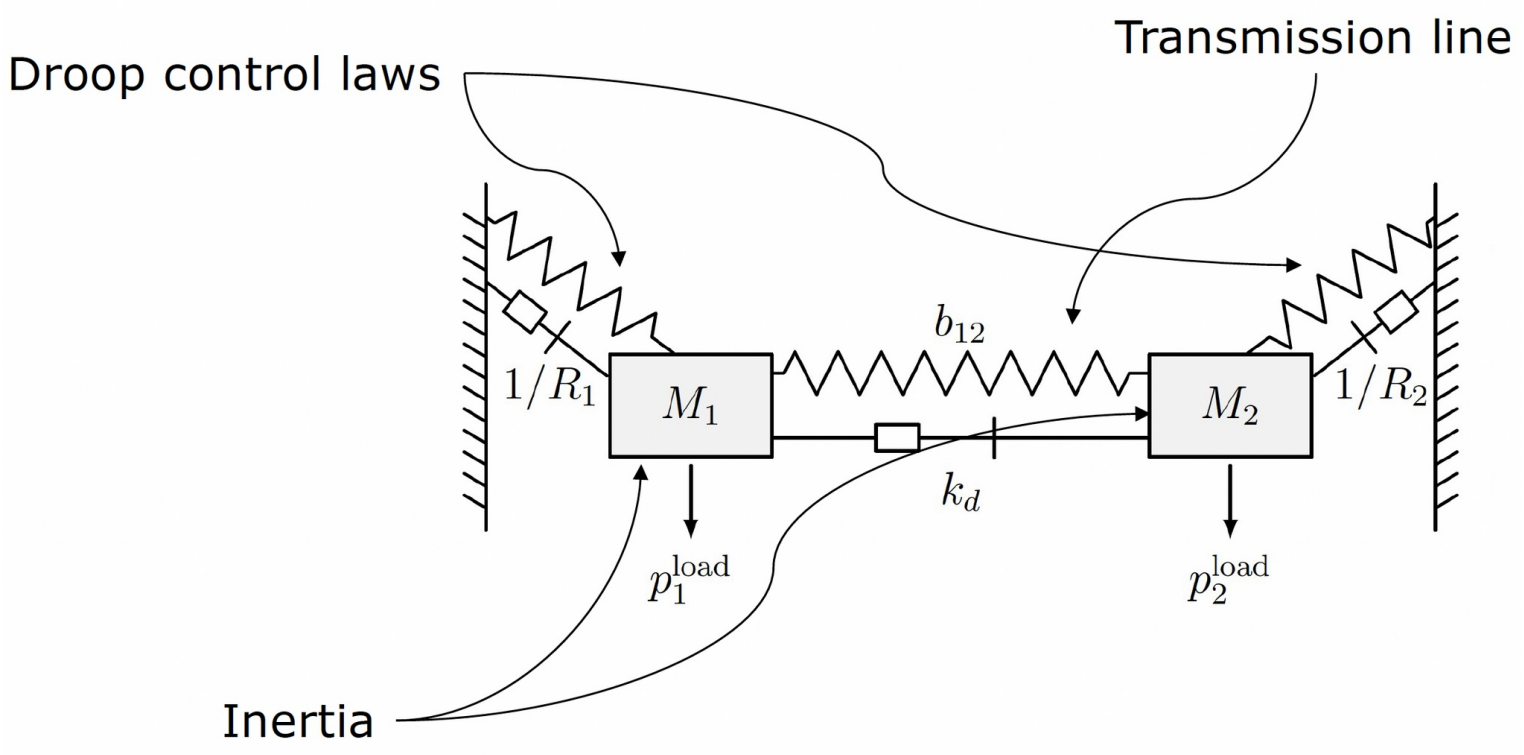
Frequency rises → generator produces less power

Droop leaves a small steady frequency error.

Secondary control later restores nominal frequency.

A Power Flow Analogy

A mechanical power flow analogy: Position <-> frequency



Exercise: Load connection, separate networks



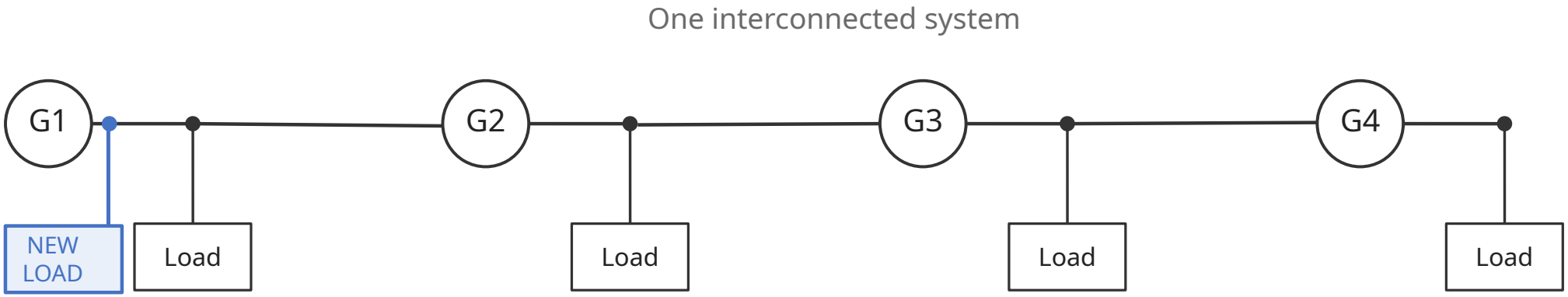
A large load is suddenly connected to Network 1. All generators are droop-controlled.

What happens at Generator 1 immediately after the load is connected?

Is G1's steady state frequency lower or higher than before?

Are the other generators affected by the load connected to Network 1?

Exercise: Load connection, connected network



A large load is suddenly connected close to Generator 1. All generators are droop-controlled.

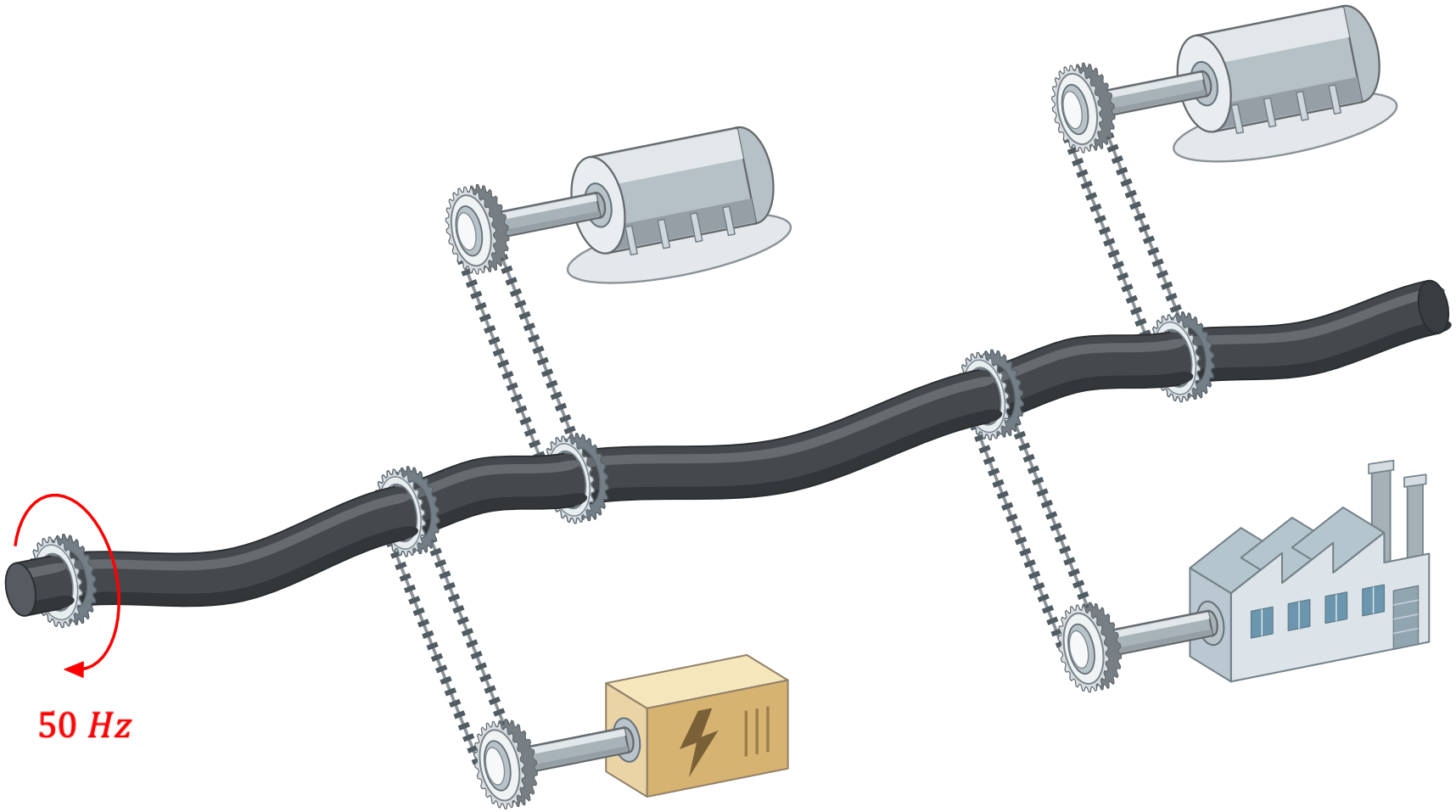
What happens at Generator 1 immediately after the load is connected?

Then what happens? Do all generators have the same frequency in the transient?

Do all generators have the same frequency at steady state?

At steady state, which generator supplies the increase in load?

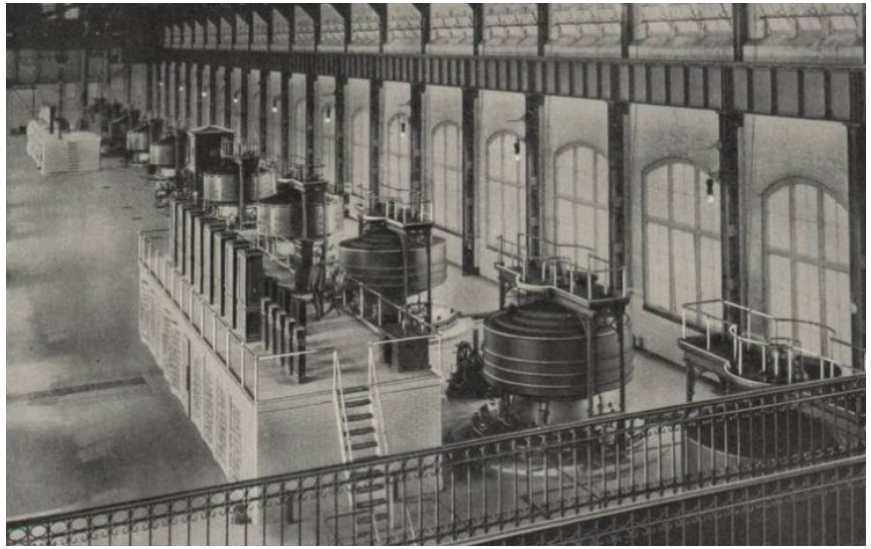
Another power flow/synchronization analogy

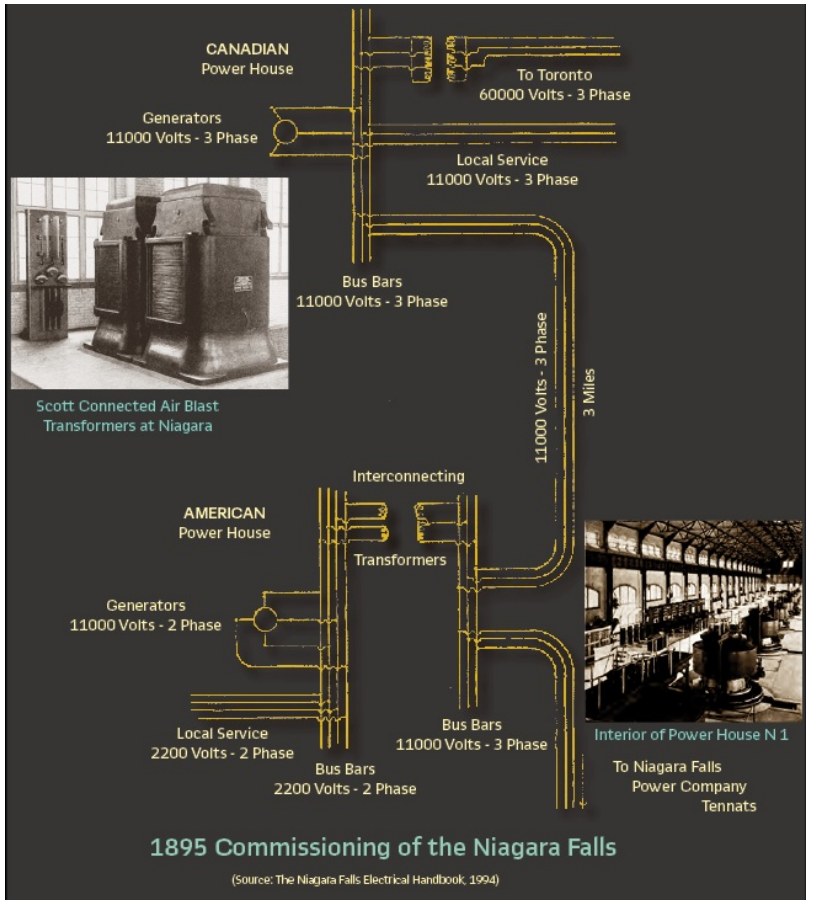


Adapted from https://www.nrel.gov/docs/fy20osti/73856.pdf

Engineers discovered that connecting generators in parallel worked well, even over large distances.

These parallel-connected generators were the first electric grids.





https://www.pacw.org/the-history-of-ground-fault-detection-the-first-power-current-transmissions-at-the-end-of-the-19th-century-part-2

The two systems are constantly in operation in parallel through this connection, and no trouble is experienced from pumping or in controlling the distribution of load.

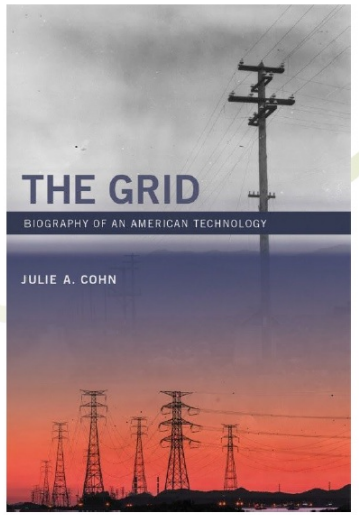
H. W. Buck, “The Electrical Plant of the Canadian Niagara Power Company,” *Electrical Age*, vol. 37, pp. 171–177, September 1906.

It all worked … but no one knew why



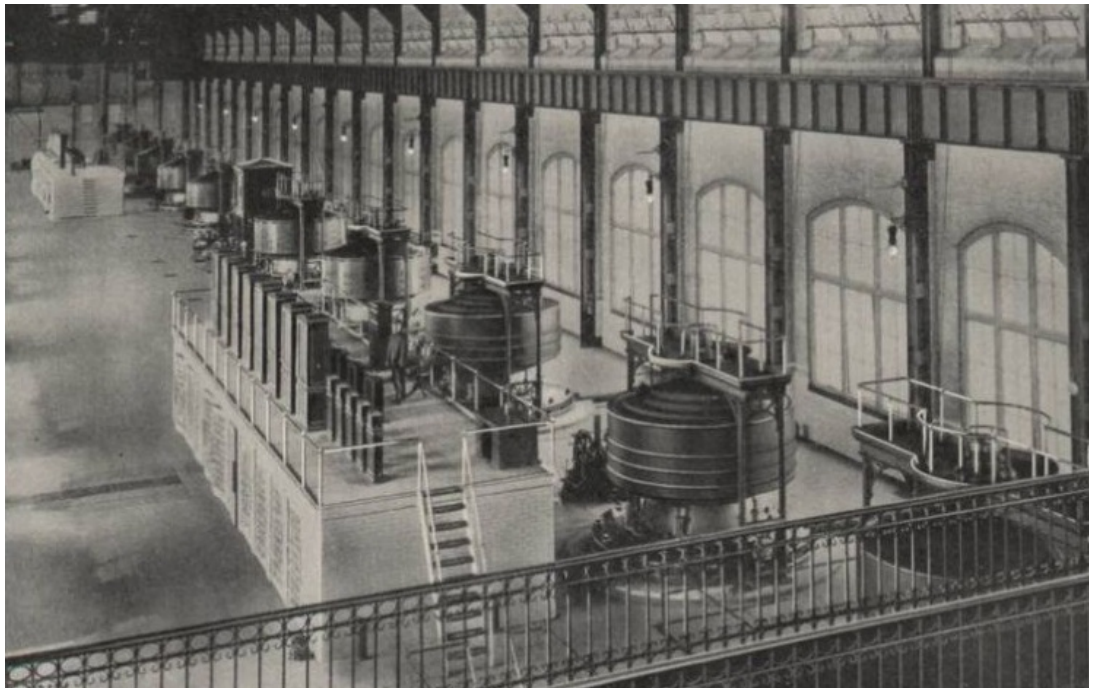
Joint meeting of Institution of Electrical Engineers
& American Institute of Electrical Engineers
(St. Louis, 9/14/1904)

"Notably, the attendees at the St. Louis meeting displayed a great deal of curiosity about the experiences of presenters who had operated stations in parallel—a requisite of interconnection."



Slide: Prof. Sairaj Dhople, history from Dr. Julie Cohn

Frequency +droop control coordinates generators without external communication.



Load increases

↓

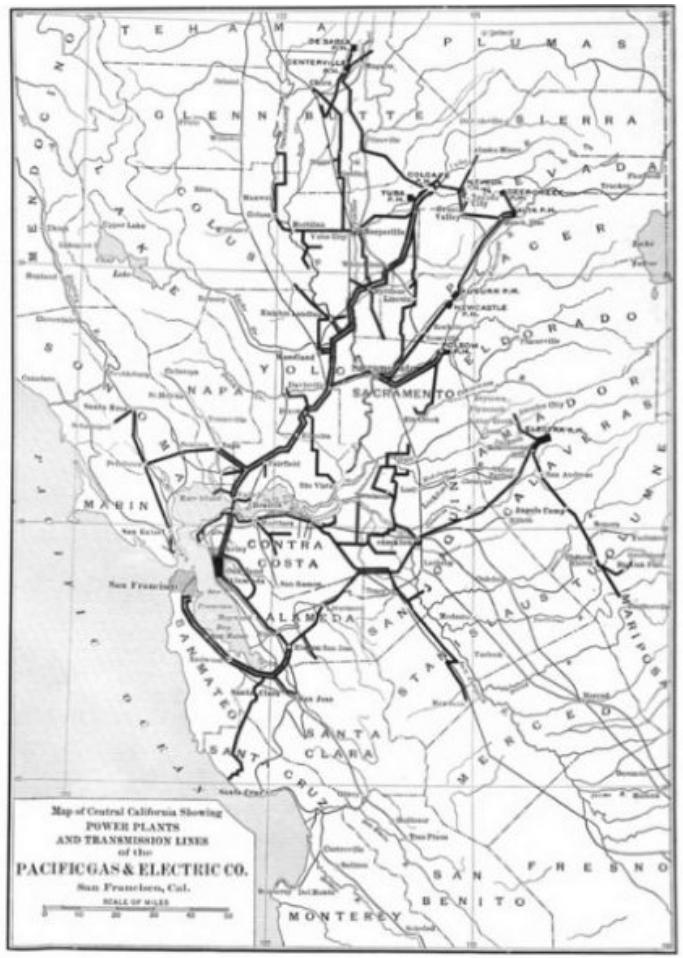
frequency falls

↓

every governor increases power according to its droop

The shared frequency tells every generator whether to work harder or less hard.

Interconnection turned many machines into one system



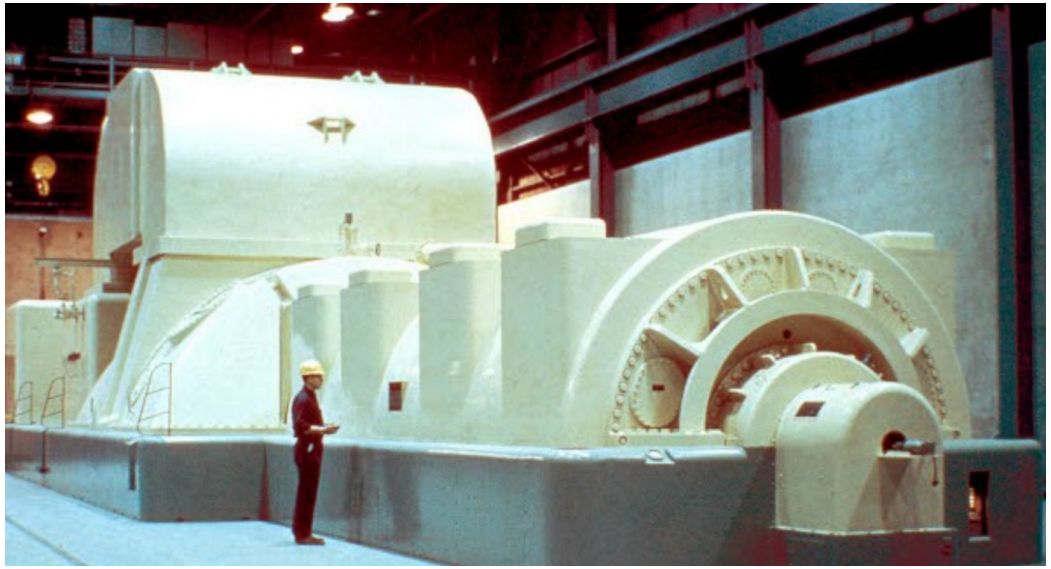
1880s	Alternators were operated in parallel.
1890s	AC stations and transmission formed the first grids.
later	Regional synchronous grids connected many stations.

Parallel droop response shared load changes across the interconnected machines.

Inertia buys time for the governor to respond

$$E_k = \frac{1}{2} J \omega_m^2$$

$$H = \frac{E_k}{S_{base}}$$



Stored kinetic energy initially supplies the power imbalance.

Inertia limits how quickly frequency changes.

Droop then arrests the frequency deviation.

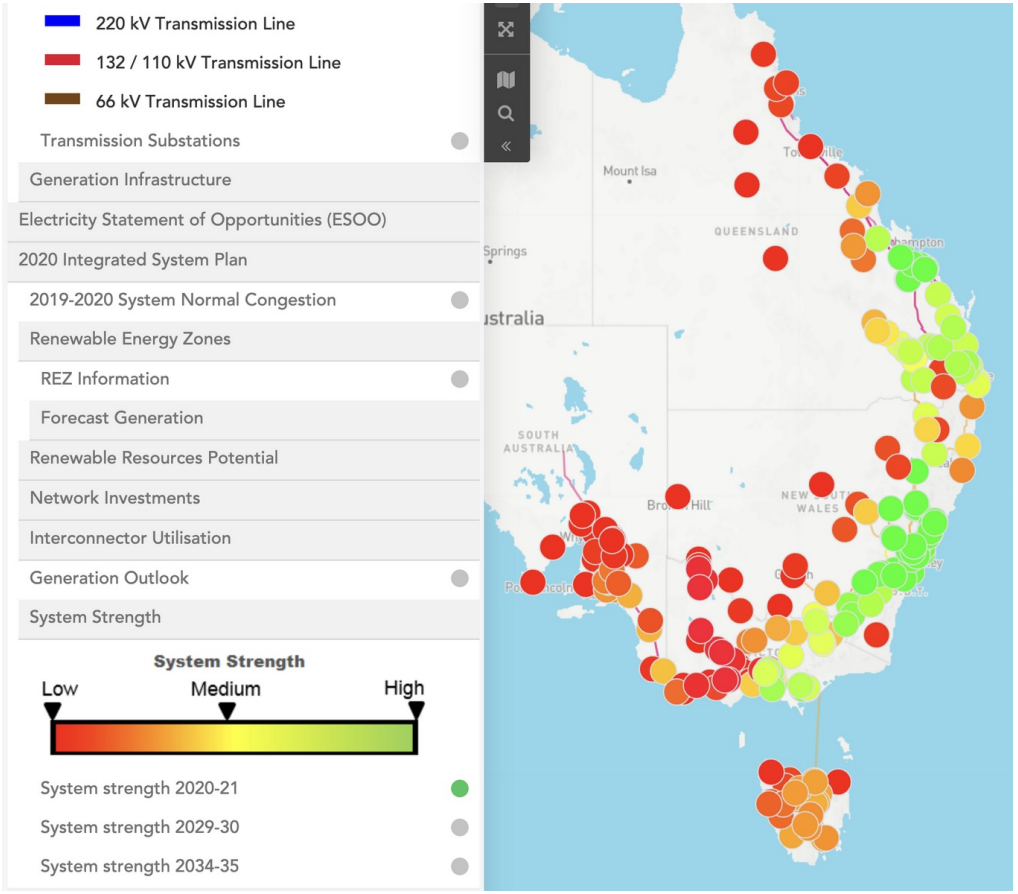


Droop remains the backbone of primary frequency control

then		now
mechanical governors		digital turbine governors
synchronous generators	→	batteries and grid-forming inverters
rotating inertia		synthetic or physical inertia

Same principle: change active power in response to frequency.

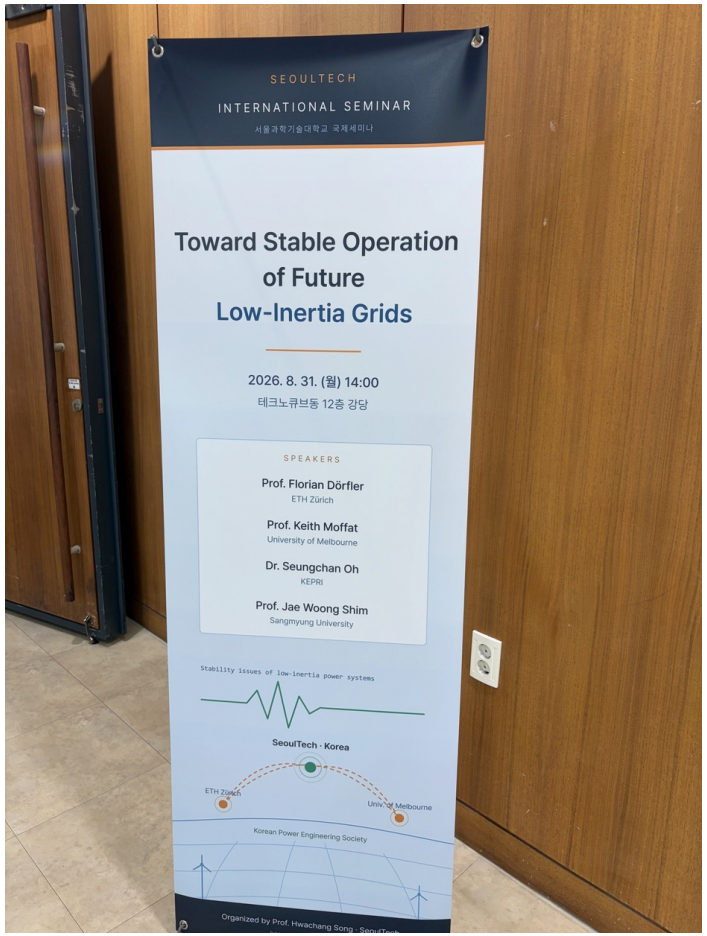
“Grid Strength”



https://www.aemo.com.au/aemo/apps/visualisations/map.html

A useful concept or industry jargon?

“Grid Strength”





End of History Intermezzo

References

A. J. Wood, B. F. Wollenberg,“Power generation, operation and control”, Wiley, 1996

P. Kundur, “Power system stability and control”, New York, USA, McGraw-Hill, 1993

P. Sauer *et al.*, ”Power system dynamics and stability”, Second edition, IEEE Wiley, 2018

A. Gomez-Exposito, A. J. Conejo, and C. Canizares, “Electric Energy Systems – Analysis and Operation”, CRC Press, 2009

M. H. L. Bollen, F. Hassan, “Integration of Distributed Generation in the Power System”, Wiley, 2011

Agenda

Fundamentals of system dynamics and stability

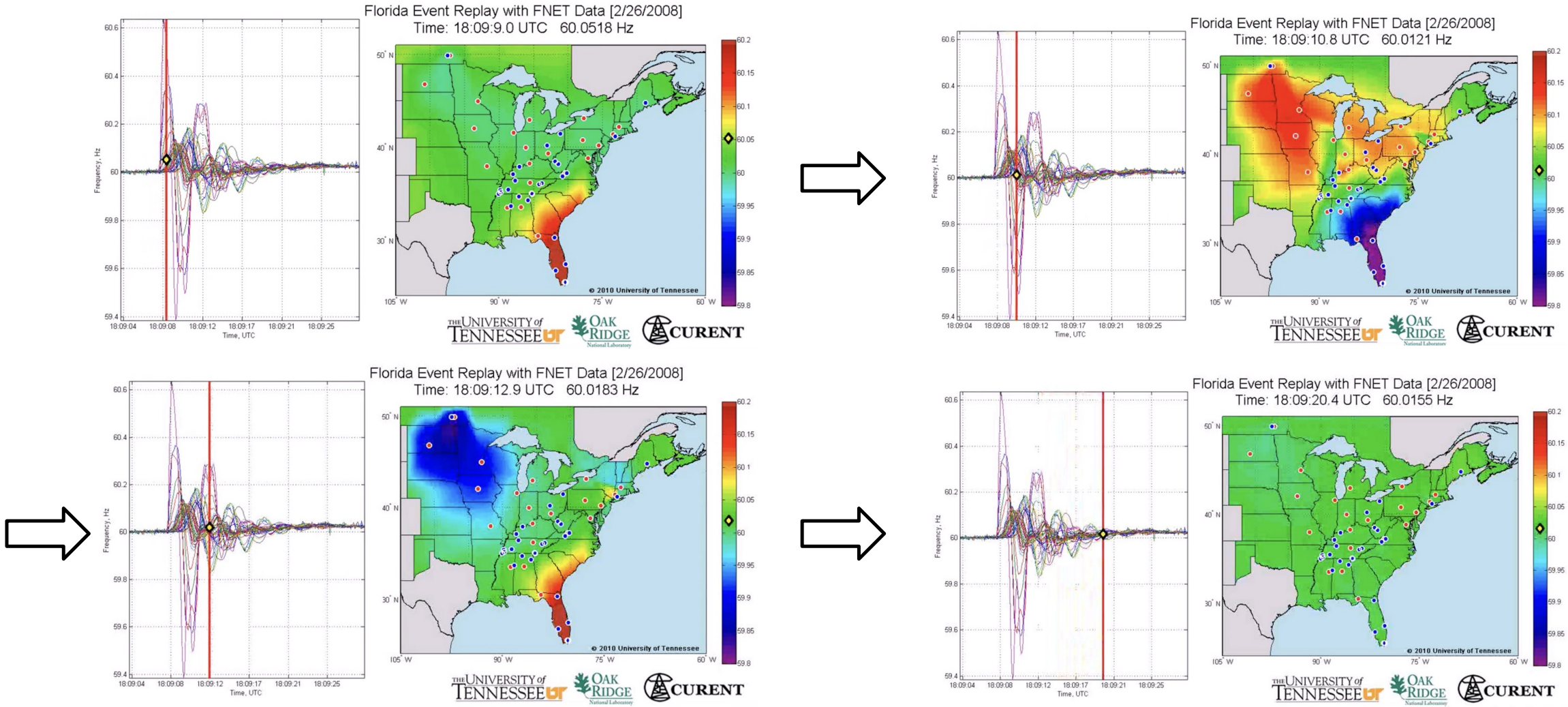
Overall frequency control mechanisms in power systems

Inertia and system inertia

Swing equation and active power dynamics of synchronous machines

Load modelling in frequency control analysis

Active power generation control (primary frequency control)



https://www.youtube.com/watch?v=bdBB4byrZ6U

FUNDAMENTALS OF SYSTEM DYNAMICS AND STABILITY

Power system stability and dynamics

"Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a system disturbance, with most system variables bounded so that practically the entire system remains intact"

What does this mean in practice?

Power system dynamics can be modelled by differential equations of the form:

We can define a system trajectory for the state variables X

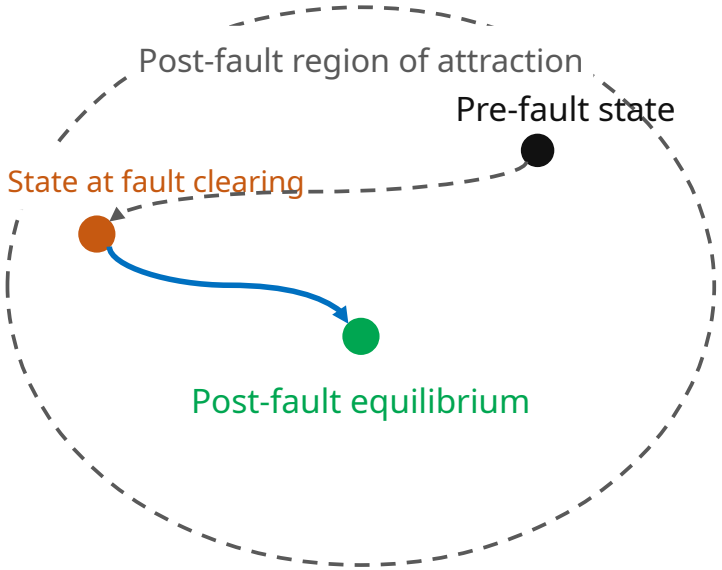
$$\dot{X} = f(X)$$

$$X = [x_1(t), x_2(t), x_3(t), \ldots . x_n(t)]$$

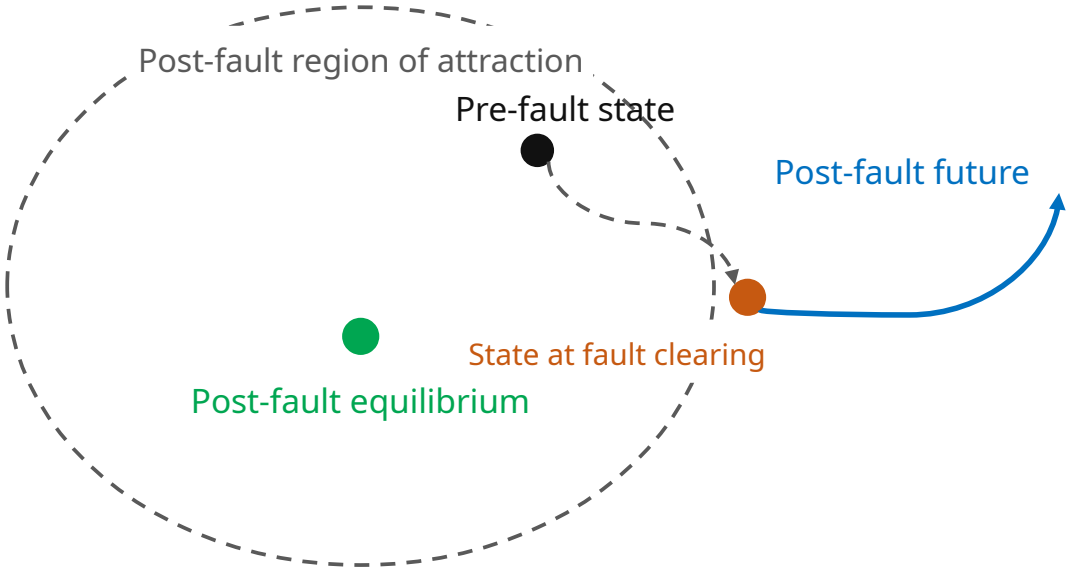
Partial derivatives of the system variables are zero at the equilibrium point:

$$X(t) = \hat{X} = constant$$

The state at fault clearing must lie inside the region of attraction



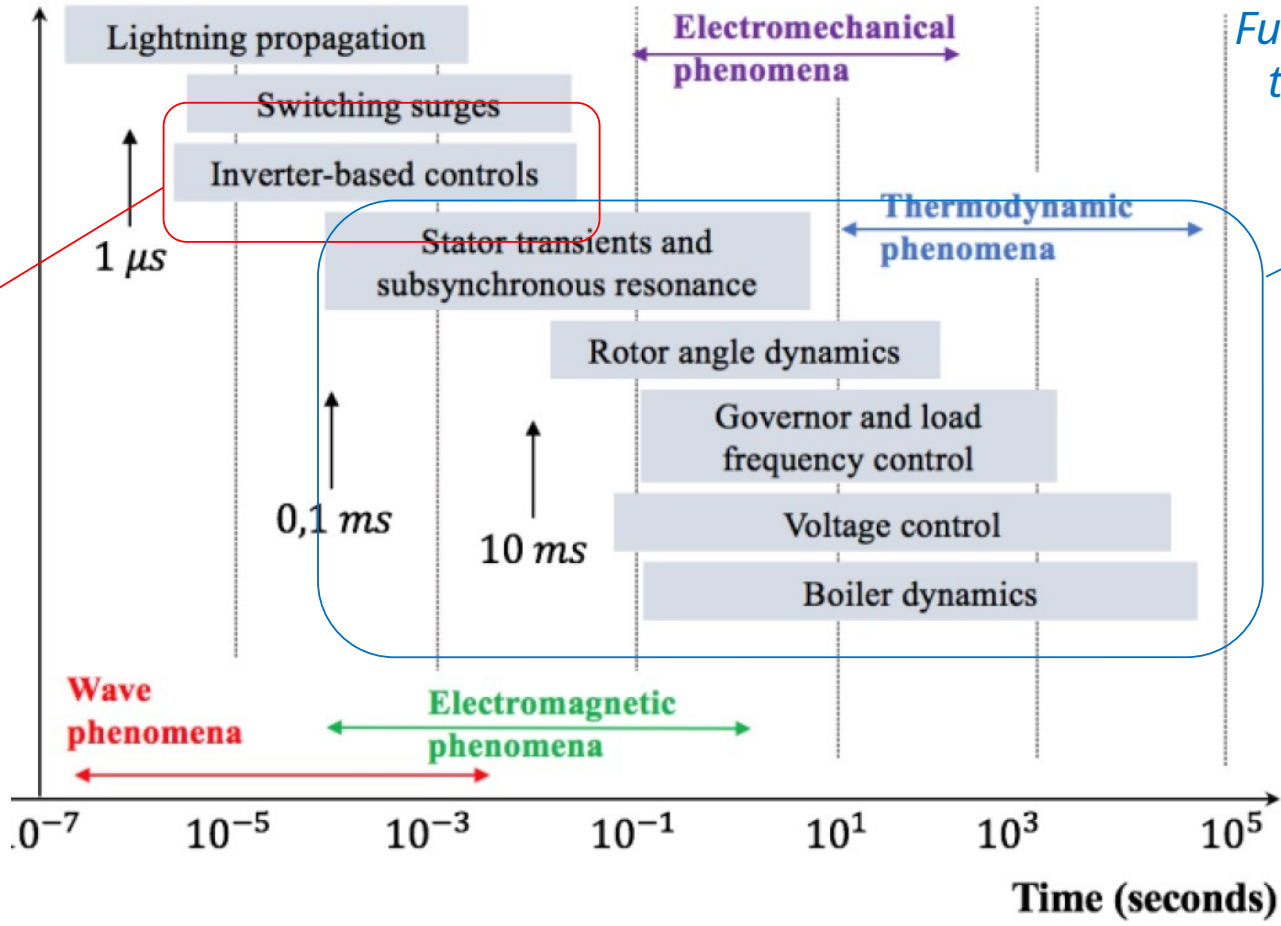
Stable



Unstable

Power system times scales

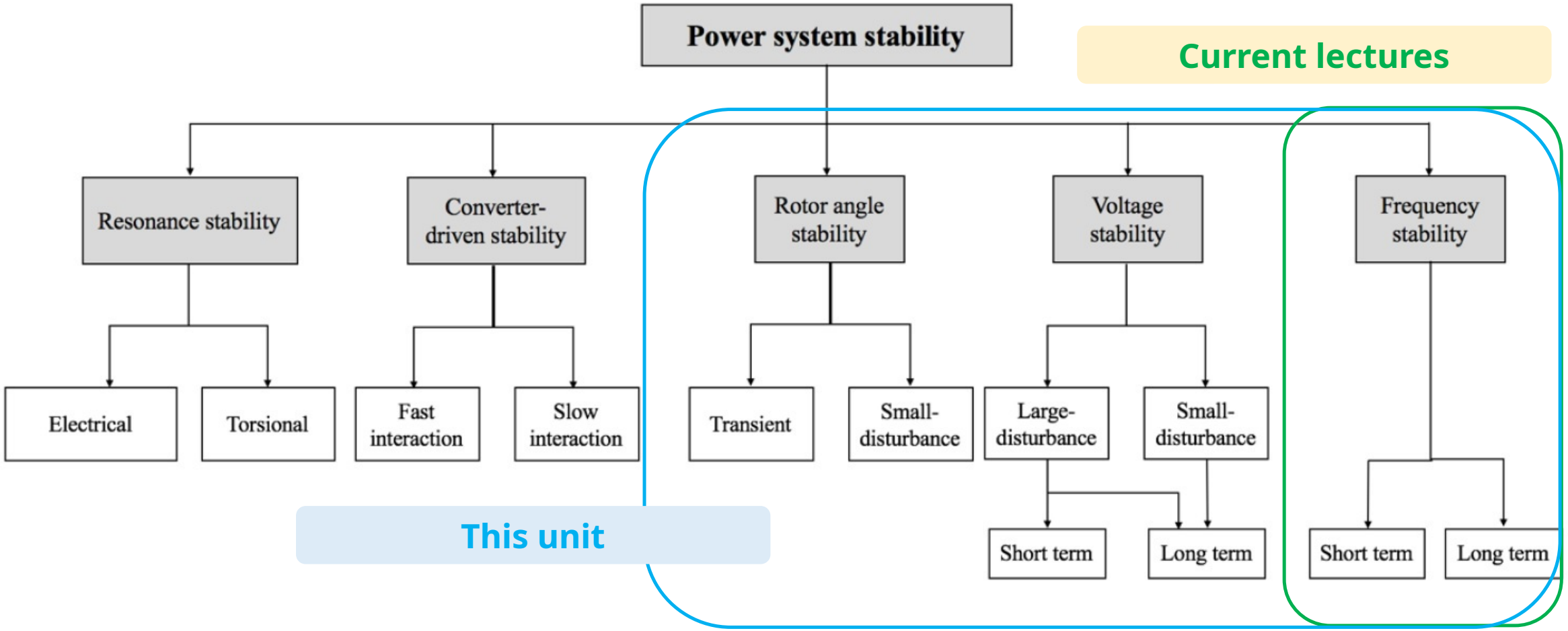
Growing interest: performance and stability of (the controls of) ***inverter-based resources***



Fundamentally associated with the physics of ***synchronous machines***

Power system variables are influenced by both physical characteristics and controls!

Power system stability classification



IEEE Power System Dynamic Performance Committee, "Task Force on stability definitions and characterization of dynamic behaviour in systems with high penetration of power electronic interfaced technologies,", 2020.

OVERALL FREQUENCY CONTROL MECHANISMS IN POWER SYSTEMS

Why do we need control in power systems?

A power system must continually meet the changing demand for active and reactive power

System frequency and voltage are indicators of system-wide power balance:

$$\sum_k P_{G_k} = \sum_i P_{D_i} + P_grid \Rightarrow f$$

$$\sum_j Q_{G_j} = \sum_i Q_{D_i} + Q_grid \Rightarrow V$$

Spatial balance in active and reactive power

$$P_k^{injected} = \sum_{i \in N_k} V_k \cdot V_i \cdot \left[G_{ki} \cdot \cos\theta_{ki} + B_{ki} \cdot \sin\theta_{ki} \right]$$

$$Q_k^{injected} = \sum_{i \in N_k} V_k \cdot V_i \cdot \left[G_{ki} \cdot \sin\theta_{ki} - B_{ki} \cdot \cos\theta_{ki} \right]$$

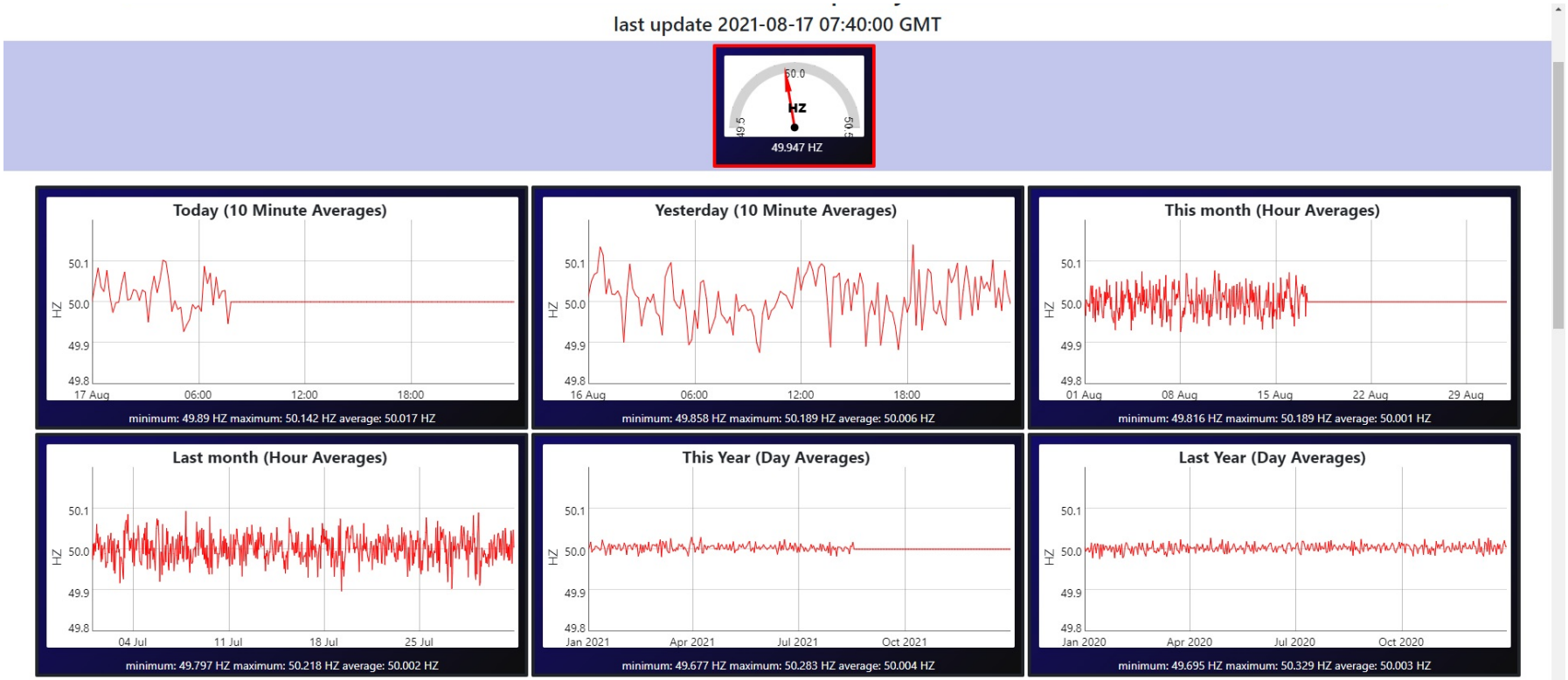
with $\theta_{ki} = \theta_k - \theta_i$

The power flow equations describe balance of active and reactive power at each bus

This is true at all times!

We will focus on active power for now

How variable is the frequency?



https://gridwatch.co.uk/frequency

Can we maintain system balance without control?

Kinetic energy and frequency

Energy balance over a time interval is an integral equation:

$$\frac{dK}{dt} = P_{\text{mechanical}} - P_{\text{electrical}} = P_{\text{m}} - P_{\text{e}}$$

$$K_2 = K_1 + \int_{t_1}^{t_2} (P_{\text{m}}(t) - P_{\text{e}}(t))dt$$

Thus, the frequency requires integrating the power imbalance over time:

$$f_2 = f_1 \sqrt{1 + \frac{1}{K_1} \int_{t_1}^{t_2} \big(P_{\text{m}}(t) - P_{\text{e}}(t)\big)\text{dt}}$$

For constant ΔP, the integral is $\Delta P \Delta t$.

The next slide uses $K \propto f^2$ to obtain f_2 algebraically, without solving an integral.

Kinetic energy and frequency

Relation between two operating points

$$K = \frac{1}{2} J\omega^2 \quad and \quad \omega \propto f$$

$$\frac{K_1}{K_2} = \frac{{f_1}^2}{{f_2}^2}$$

Formula for the new frequency

$$K_2 = K_1 + (P_m - P_e)\Delta t \quad \rightarrow \quad f_2 = f_1 \sqrt{1 + \frac{(P_m - P_e)\Delta t}{K_1}}$$

(no integral required!)

Worked Example

Initial state

$$f_1 = 50\ Hz$$
$$K_1 = 20\ GJ$$

Power deficit

$$1\ GW\ for\ 0.2\ s$$

Energy removed $= 0.2\ GJ$

Frequency after 0.2 s

$$f_2 = 49.75\ Hz$$

ROCOF from the Kinetic-energy-frequency relation

The time-derivative of the energy relation $K = K_0 \frac{f^2}{f_0^2}$ is

$$\frac{dK}{dt} = \frac{2K_0 f}{{f_0}^2}\frac{df}{dt} = \frac{2K}{f}\frac{df}{dt}$$

Solving for ROCOF from power imbalance

$$\frac{dK}{dt} = P_m - P_e$$

$$ROCOF : \frac{df}{dt} = \frac{f}{2K}(P_m - P_e) = \frac{f_0^2}{2Hf}\frac{(P_m - P_e)}{S_{base}}$$

Same Worked Example

Find the initial $ROCOF$

$$f = 50\ Hz$$
$$K = 20\ GJ$$
$$P_m - P_e = -1GW$$
$$ROCOF = -1.25\ Hz/s$$

or, for $S_{base} = 10GW$,

$$f_0 = 50\ Hz$$
$$H = \frac{K_0}{S_{base}} = 2\ s$$
$$\frac{(P_m - P_e)}{S_{base}} = -0.10\ pu$$
$$ROCOF = -1.25\ Hz/s$$

Exercise 1: A non-controlled generator

A generator is running at no load at f = 50 Hz

Kinetic energy of the generator and turbine: K1 = 200 MJ

A 10 MW load is suddenly connected

Calculate the new frequency after 1 second

Exercise 1 Solution

In 1 second, the load "consumes" 10 MW x 1s = 10 MJ

As a result, the new kinetic energy of the generator will be K2 = 190 MJ and the generator will slow down

In fact, for a generic rotating mass, the kinetic energy K is proportional to the square of the angular speed ω, which is in turn proportional to the frequency f

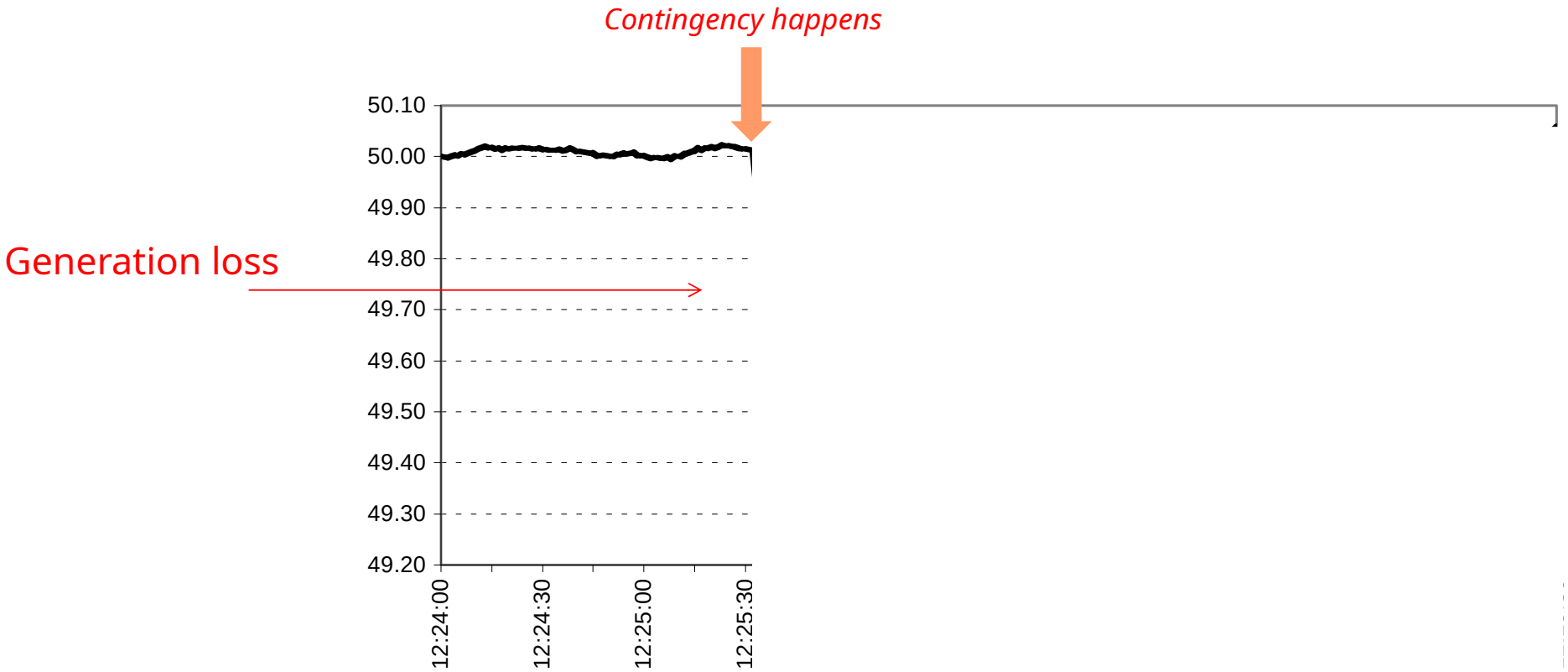
Therefore:

$$\frac{K_1}{K_2} = \frac{200}{190} = \frac{f_1^2}{f_2^2} = \frac{50^2}{f_2^2} \Rightarrow f_2 = 48.73\ Hz$$

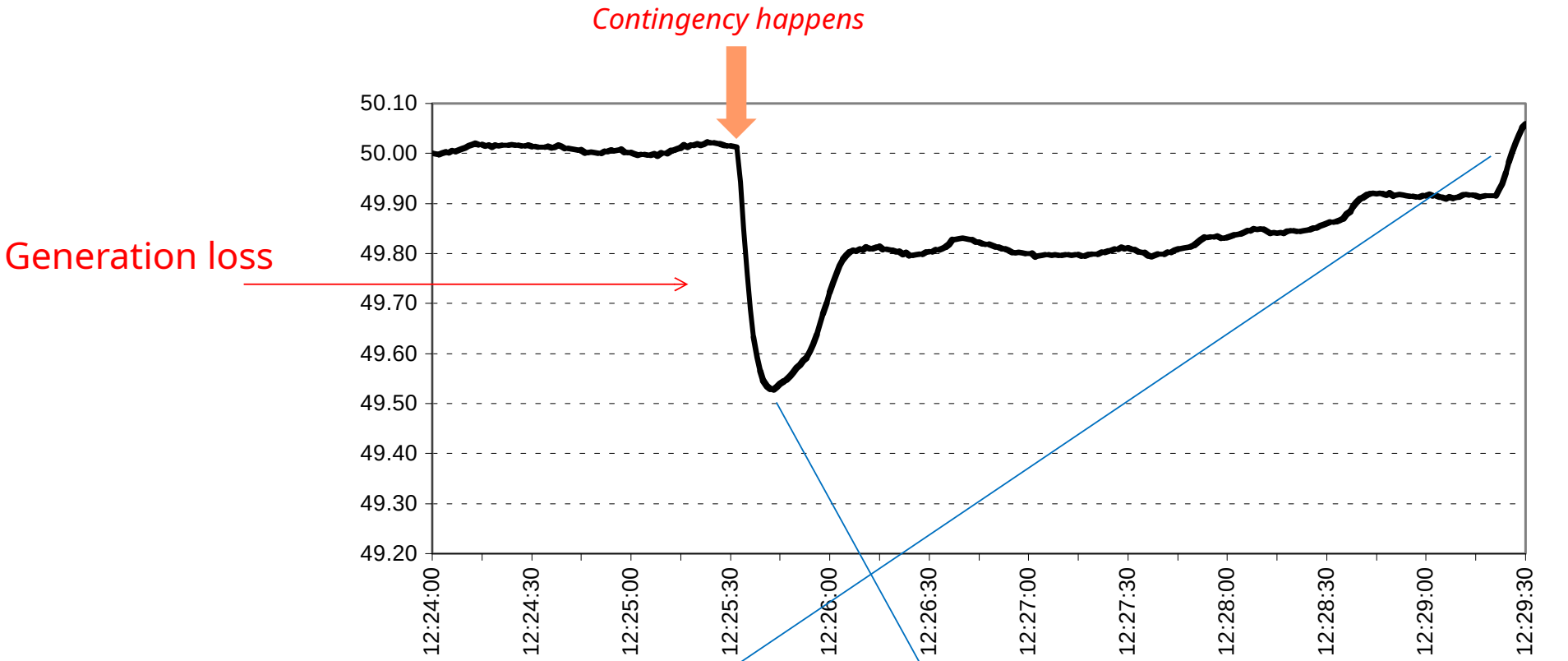
Is it ok to have such a low frequency?

We need control on generating units!

Frequency behaviour following a large loss of generation in Great Britain

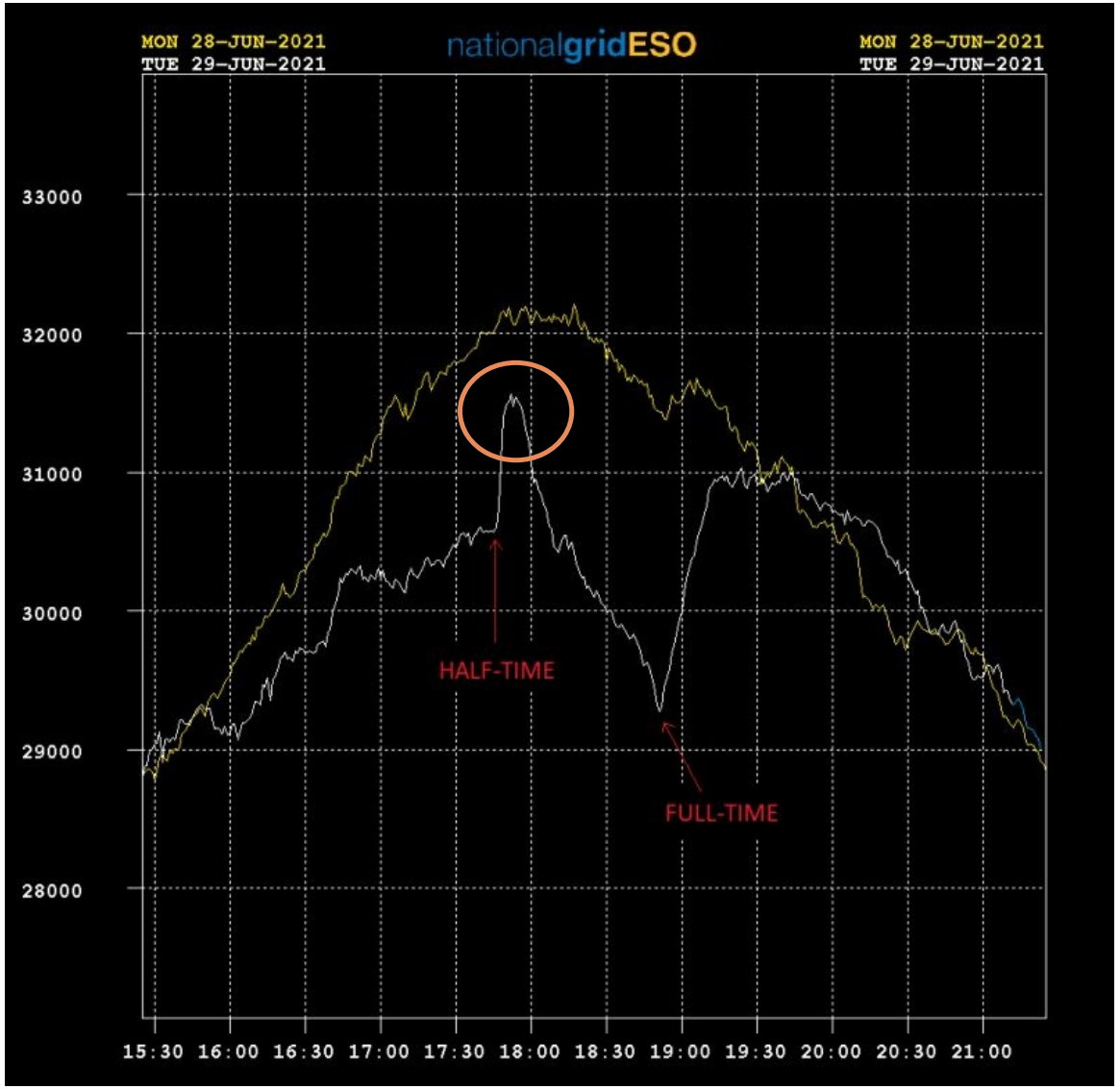


Frequency behaviour following a large loss of generation in Great Britain



How do we limit the frequency drop and ensure frequency recovery to the nominal value (e.g., 50 Hz)?

Example: England-Germany football match at Euro 2021



Source: National Grid

Example:
Euro 2021 demand spike in Great Britain

- During the Euro 2021 football match between England and Germany, the GB power system was supplying 30,500 MW
- Assume the stored kinetic energy was roughly 150,000 MJ
- Estimate the initial Rate Of Change Of Frequency (ROCOF) due to a sudden load increment of 1,000 MW from simultaneous kettle usage at half time of the match

$$KE \propto \omega^2 \propto f^2 \Rightarrow KE = KE^0 \left(\frac{f}{f^0}\right)^2$$

- Assuming all the excess power is supplied from kinetic energy:

$$\Delta P_{el} = -\frac{dKE}{dt} = -KE^0 \frac{d\left(\frac{f}{f^0}\right)^2}{dt} = -KE^0 \frac{d\left(\frac{f^0 + \Delta f}{f^0}\right)^2}{dt} \cong -2\frac{KE^0}{f^0}\frac{d\Delta f}{dt}$$

$$\frac{d\Delta f}{dt} = -f^0 \frac{\Delta P_{el}}{2 \cdot KE^0} = *** \text{ Hz/s}$$

Example:
Euro 2021 demand spike in Great Britain

- During the Euro 2021 football match between England and Germany, the GB power system was supplying 30,500 MW
- Assume the stored kinetic energy was roughly 150,000 MJ
- Estimate the initial Rate Of Change Of Frequency (ROCOF) due to a sudden load increment of 1,000 MW from simultaneous kettle usage at half time of the match

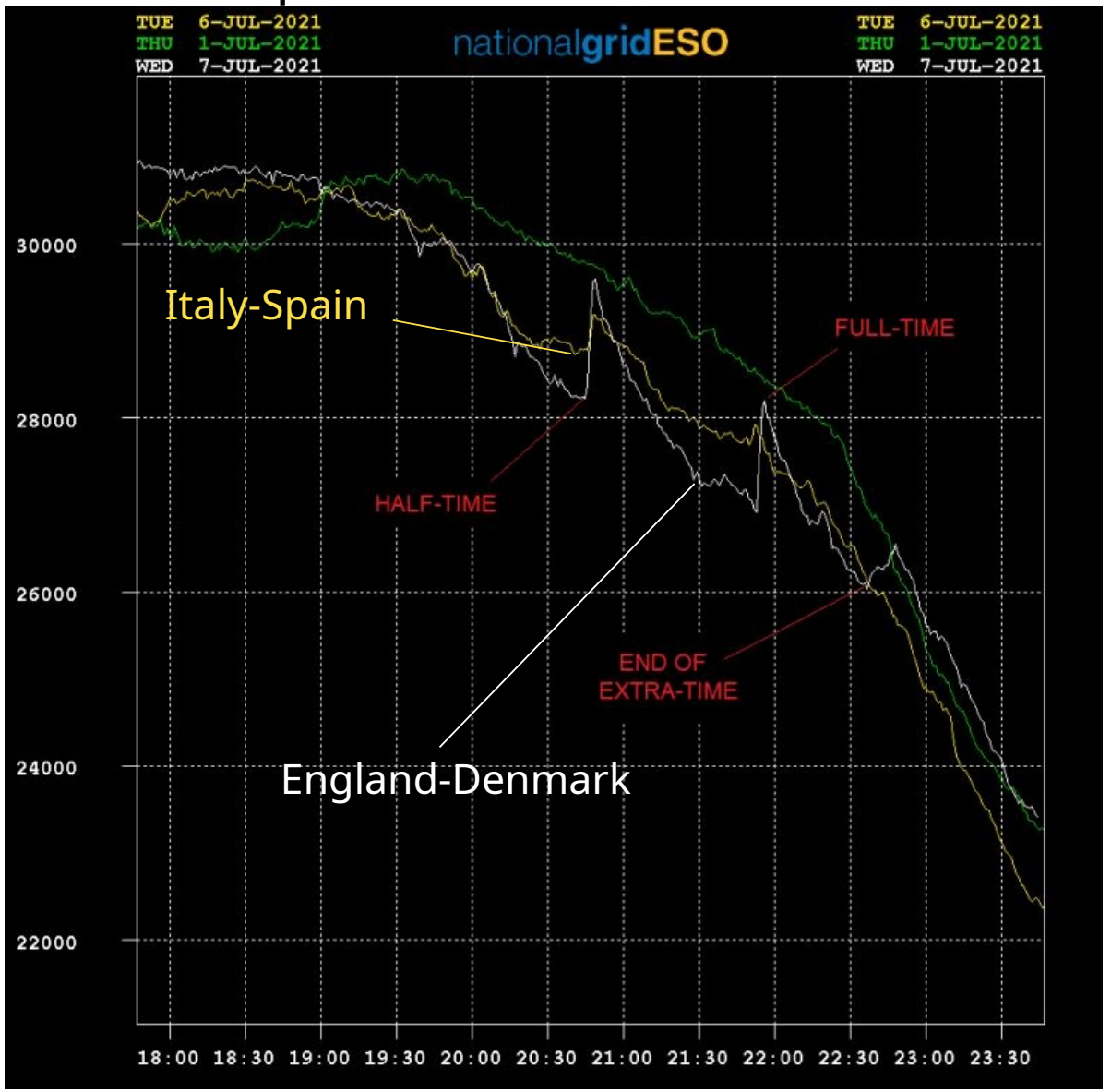
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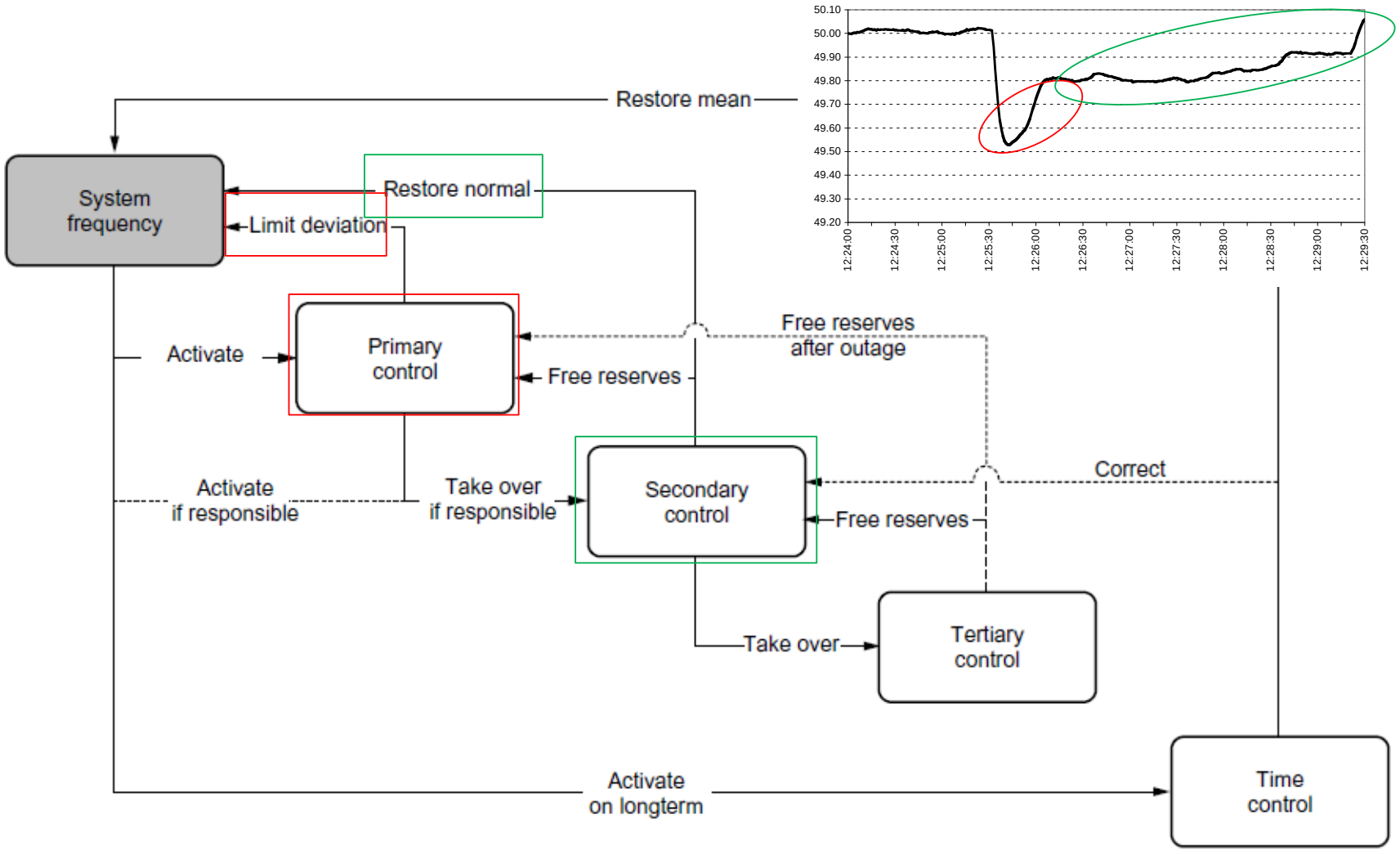
$$\frac{d\Delta f}{dt} = -f^0 \frac{\Delta P_{el}}{2 \cdot KE^0} = -50\frac{1000}{2 \cdot 150000} = -0.167 \text{ Hz/s}$$

Example: Euro 2021 semi-finals



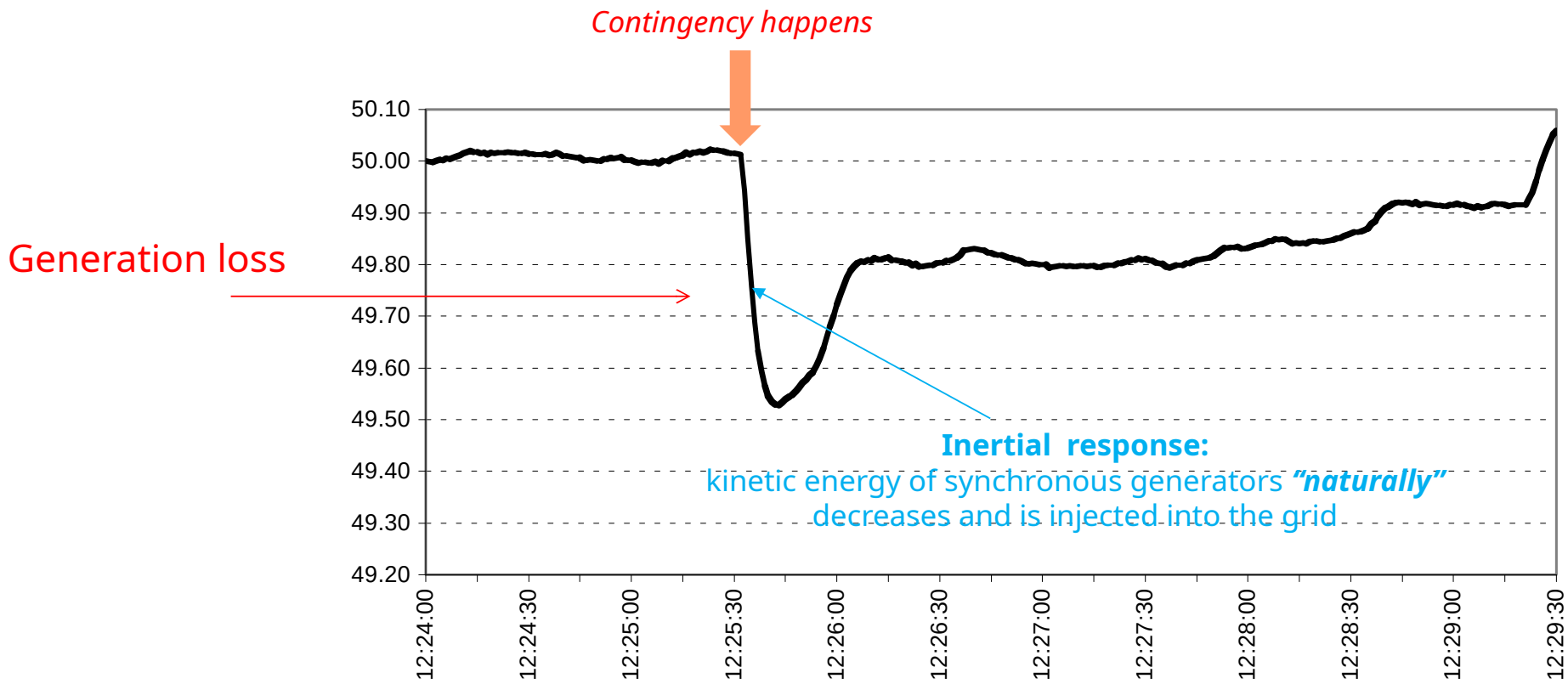
Source: National Grid

Frequency control mechanisms



Example from the European interconnected system; Source: Gomez-Exposito et al.

Maintaining frequency stability

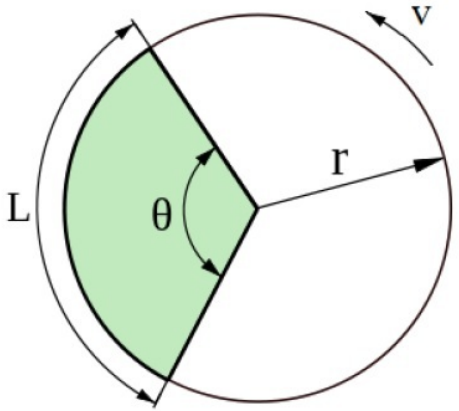


We need to understand the link between active power and frequency dynamics

INERTIA AND SYSTEM INERTIA

What is inertia and system inertia?

Inertia is a physical characteristic which refers to the **kinetic energy stored** in the **rotational masses** of generators and motors



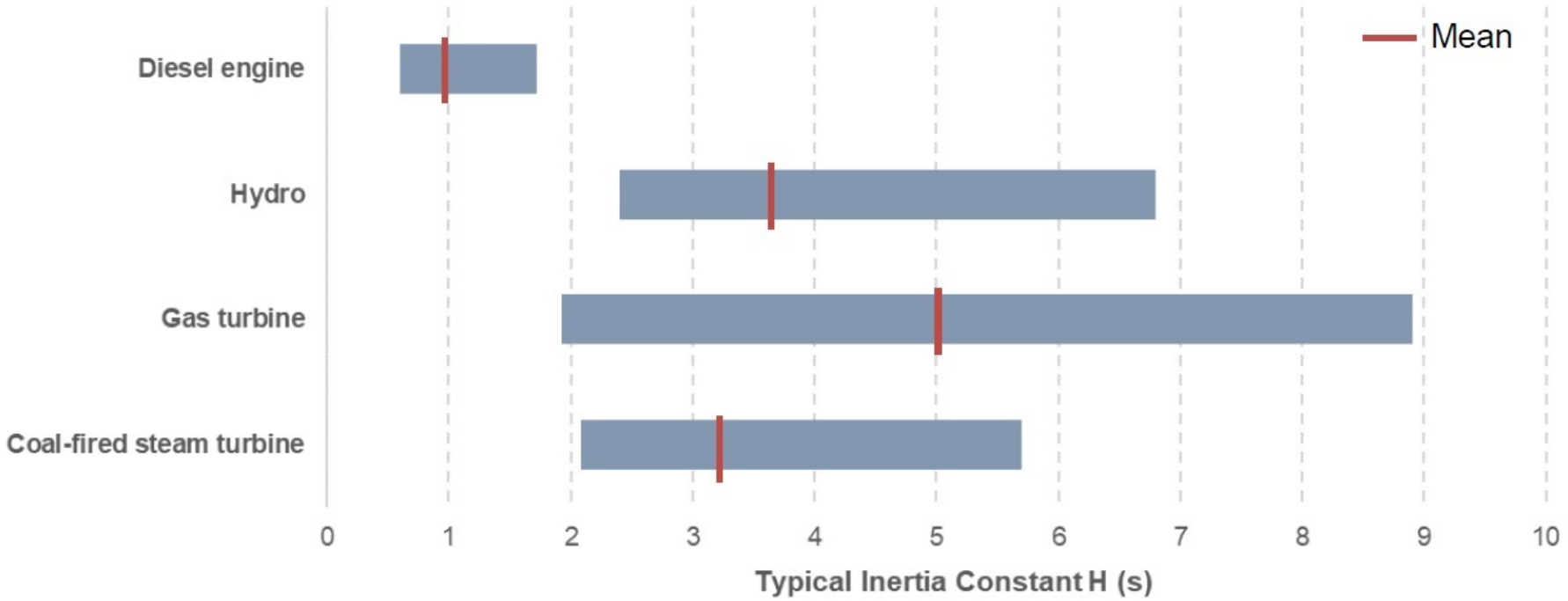
$$J = m \cdot r^2$$

$$E_{KE} = \frac{1}{2} \cdot J \cdot \omega^2 \ [\text{MWs}]$$

$$H = \frac{E_{KE}}{S_n} = \frac{1}{2}\frac{J \cdot \omega^2}{S_n} \ [\frac{\text{MWs}}{\text{MVA}}]$$

$$E_s = \sum_{i=1}^{N} E_{ke}^{gen,i} + \sum_{j=1}^{M} E_{ke}^{load,j}$$

Inertia constant: range of technologies

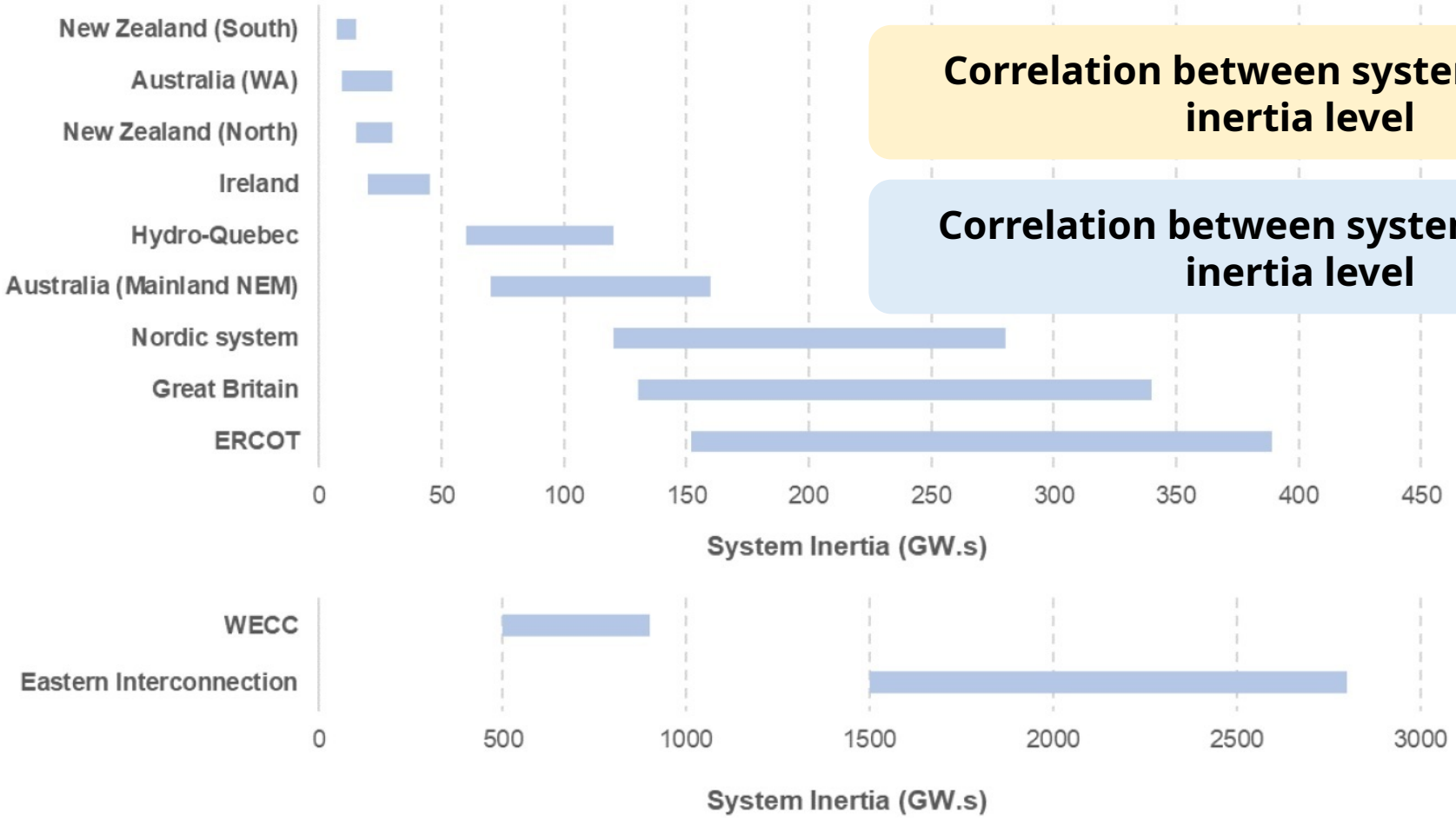


Renewable energy resources, mainly inverter-based units, *have no inherent inertia*

Increase in inverter-based units → Reduction in system inertia

J. Susanto, "Frequency control in low inertia power systems," *2020International Conference on Smart Grids and Energy Systems (SGES)*, Australia, 2020.

Inertia estimates for power systems around the world



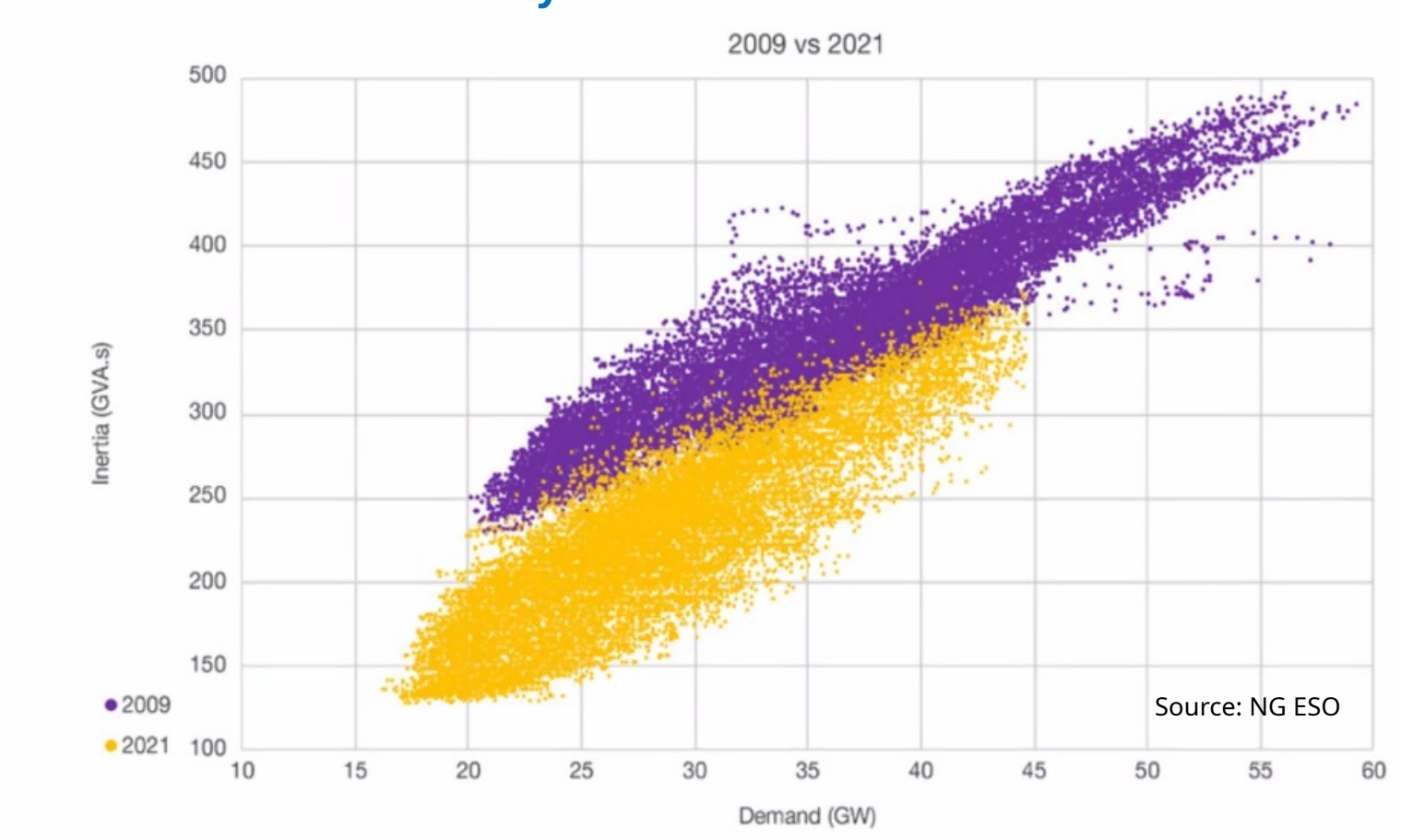
Correlation between system size and inertia level

Correlation between system load and inertia level

J. Susanto, "Frequency control in low inertia power systems," *2020International Conference on Smart Grids and Energy Systems (SGES)*, Australia, 2020.

Inertia vs operational demand

Reduced system inertia in GB 2009-2021



What do you think happened in those 10+ years?

Exercise 2

Calculate the system kinetic energy and the system inertia (in GWs and in MWs/Hz) in the following cases:

1. Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=1: all operate at full load
2. Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=0.85: all operate at full load
3. Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=0.85: all operate at 50% capacity
4. Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=0.85: five plants operate at full load and five of them are offline
5. Ten CCGT plants, rated 500 MW capacity, H=6s, nominal pf=0.85: all plants operate at 50% capacity, and 3 GW of wind installed capacity, H=3s, rated pf=1, Type IV generator
   - *connected via power electronics interface*

Exercise 2 solution

Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=1: all operate at full load

$$KE = \sum H_g \overline{S_g} = 10 \cdot 6 \cdot \frac{500}{1} = 30{,}000 \text{ [MWs]} = 30 \text{ [GWs]}$$

$$System\ inertia = \frac{KE}{f_0} = \frac{30{,}000}{50} = 600 \left[\frac{\text{MWs}}{\text{Hz}}\right]$$

Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=0.85: all operate at full load

$$KE = \sum H_g \overline{S_g} = 10 \cdot 6 \cdot \frac{500}{0.85} = 35295 \text{ [MWs]} = 35.295 \text{ [GWs]}$$

$$System\ inertia = \frac{KE}{f_0} = \frac{35295}{50} = 706 \left[\frac{\text{MWs}}{\text{Hz}}\right]$$

Exercise 2 solution

Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=0.85: all operate at 50% output

$$KE = \sum H_g \overline{S_g} = 10 \cdot 6 \cdot \frac{500}{0.85} = 35295[\text{MWs}] = 35.295[\text{GWs}]$$

$$System\ inertia = \frac{KE}{f_0} = \frac{35295}{50} = 706 \left[\frac{\text{MWs}}{\text{Hz}}\right]$$

Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=0.85: five plants operate at full load and five of them are offline

$$KE = \sum H_g \overline{S_g} = 5 \cdot 6 \cdot \frac{500}{0.85} = 17647\ [\text{MWs}] = 17.647\ [\text{GWs}]$$

$$System\ inertia = \frac{KE}{f_0} = \frac{17647}{50} = 353 \left[\frac{\text{MWs}}{\text{Hz}}\right]$$

Exercise 2 solution

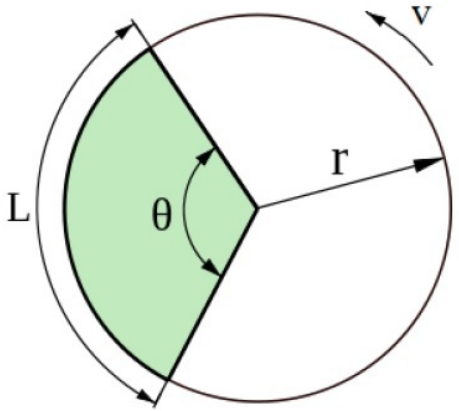
Ten CCGT plants, 500 MW rated capacity, H=6s, nominal pf=0.85: all operate at 50% output and 3 GW of Type IV wind installed capacity with H=3s, rated pf=1

$$KE = \sum H_g \overline{S_g} = 10 \cdot 6 \cdot \frac{500}{0.85} = 35295[\text{MWs}] = 35.295[\text{GWs}]$$

$$System\ inertia = \frac{KE}{f_0} = \frac{35295}{50} = 706\ \left[\frac{\text{MWs}}{\text{Hz}}\right]$$

What is inertia and system inertia?

Inertia is a physical characteristic which refers to the **kinetic energy stored** in the **rotational masses** of generators and motors



$$J = m \cdot r^2$$

$$E_{KE} = \frac{1}{2} \cdot J \cdot \omega^2 \ [\text{MWs}]$$

$$H = \frac{E_{KE}}{S_n} = \frac{1}{2}\frac{J \cdot \omega^2}{S_n} \ [\frac{\text{MWs}}{\text{MVA}}]$$

$$E_s = \sum_{i=1}^{N} E_{ke}^{gen,i} + \sum_{j=1}^{M} E_{ke}^{load,j}$$

Calculation of system inertia *by* ***summing up individual generator components*** *may be* ***notaccurate***

System Inertia Monitoring
ARENA Project

Australian Government
Australian Renewable Energy Agency

ARENA 10 YEARS

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Reactive Technologies System Inertia Measurement Demonstration



$1.43m
Funded by ARENA

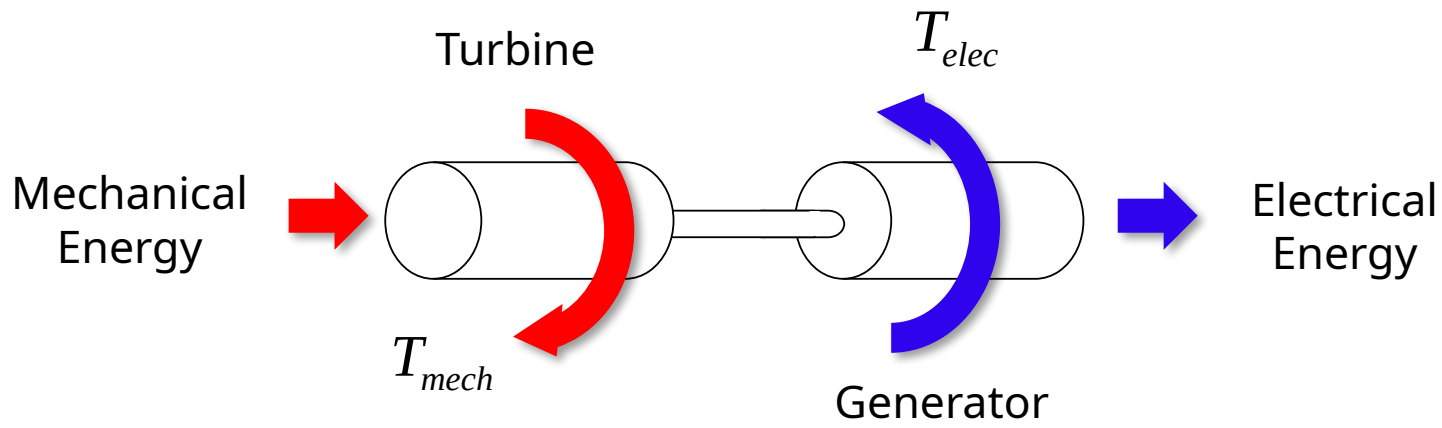
$3.07m
Total project cost

Where is inertia hiding?

B. Moya, *et al.*, “**Techno-Economic Assessment of Inertia Measurements**: A Case Study for the Australian National Electricity Market”, *Electric Power System Research, 2025*

SWING EQUATION AND ACTIVE POWER DYNAMICS OF SYNCHRONOUS MACHINES

Active power imbalance and generator dynamics



T_m: Mechanical torque applied by turbine
T_e: Electrical reaction torque developed by generator (electromagnetic torque)
T_a : Accelerating torque
J: Moment of inertia of the turbine/generator set
δ_m : Position of the rotor
ω_m: Rotor mechanical speed [rad/s]

$$J\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e$$

$$J\frac{d\omega_m}{dt} = T_a = T_m - T_e$$

Demand-supply balance: steady-state operation

In the case of **steady state** operation, the mechanical torque of a synchronous machine equals the electromagnetic torque:

$$T_m = T_e$$

- In steady state, the rotor rotates at constant speed and the angle increases linearly with time

$$J\frac{d^2\delta_m}{dt^2} = 0$$

$$\frac{d\delta_m}{dt} = constant = \omega_m^{syn}$$

J: Moment of inertia [kg·m^2]
δ_m: rotor mechanical angle [rad]
ω_m: rotor mechanical speed [rad/s]

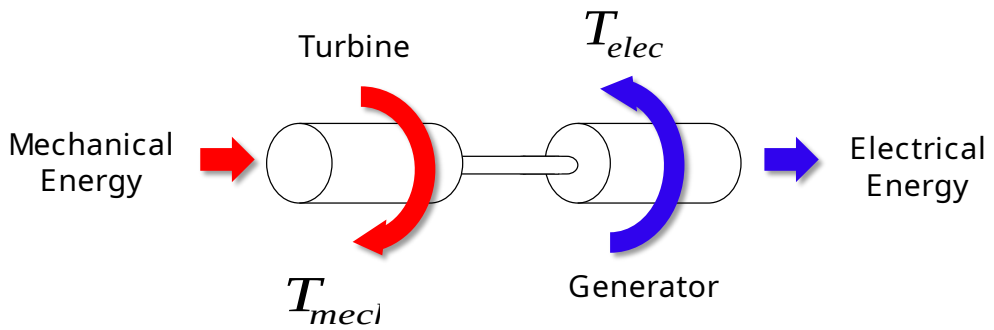
Synchronous generator dynamics

Turbine-generators contain kinetic energy due to rotating masses

When there is a sudden load increase:

- The electrical torque increases instantaneously to supply the load
- The electrical torque increase is met initially by release of this kinetic energy
- The mechanical torque initially remains constant

From Newton's 2nd law (F=m•a) the acceleration a is negative



$$J \cdot \alpha = T_m - T_e$$

The swing equation

Remember that $E_{KE} = \frac{1}{2} J\omega^2 [\text{MWs}]$ and $H = \frac{E_{KE}}{S_n} = \frac{1}{2}\frac{J\omega^2}{S_n}$ $[\frac{\text{MWs}}{\text{MVA}}]$

By using the expression for *H*, the generator's equation can be rewritten as:

$$J\frac{d\omega}{dt} = T_m - T_e \quad \Longrightarrow \quad \frac{2H \times S_B}{\omega_0^2}\frac{d\omega}{dt} = T_m - T_e$$

After some mathematical manipulation, the equation can be formulated in **per unit** as:

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = T_m - T_e$$

The swing equation

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = T_m - T_e$$

This is the famous "**swing equation**" representing the *equation of motion of synchronous machines*

The name comes from the fact that it represents swings in rotor angle δ following generic disturbances

In general, a "damping" component proportional to $\frac{d\delta}{dt}$ could also be added in the model

The swing equation

The swing equation can be decomposed into the two equations:

$$\frac{d\Delta\omega}{dt} = \frac{\omega_0}{2H}(T_m - T_e)$$

$$\frac{d\delta}{dt} = \Delta\omega$$

This is what we'll use more in the rest of these lectures as we are particularly interested in controlling the **frequency**

We'll use more the angle swing equation later, when we'll study rotor angle stability

Synchronous generator dynamic model

- Power systems work considering inputs and constraints in terms of **power** (rather than torque)
- Therefore, it is desirable to write the *equations of motion in terms of power*, considering that:

$$P = \omega \cdot T$$

- It is easy to show that, for small disturbances so that the rotational speed remains around the nominal point, and neglecting higher order effects, it is possible to write:

$$\frac{d\Delta\omega}{dt} = \frac{\omega_0}{2H}(\Delta T_m - \Delta T_e), \qquad M = 2H$$

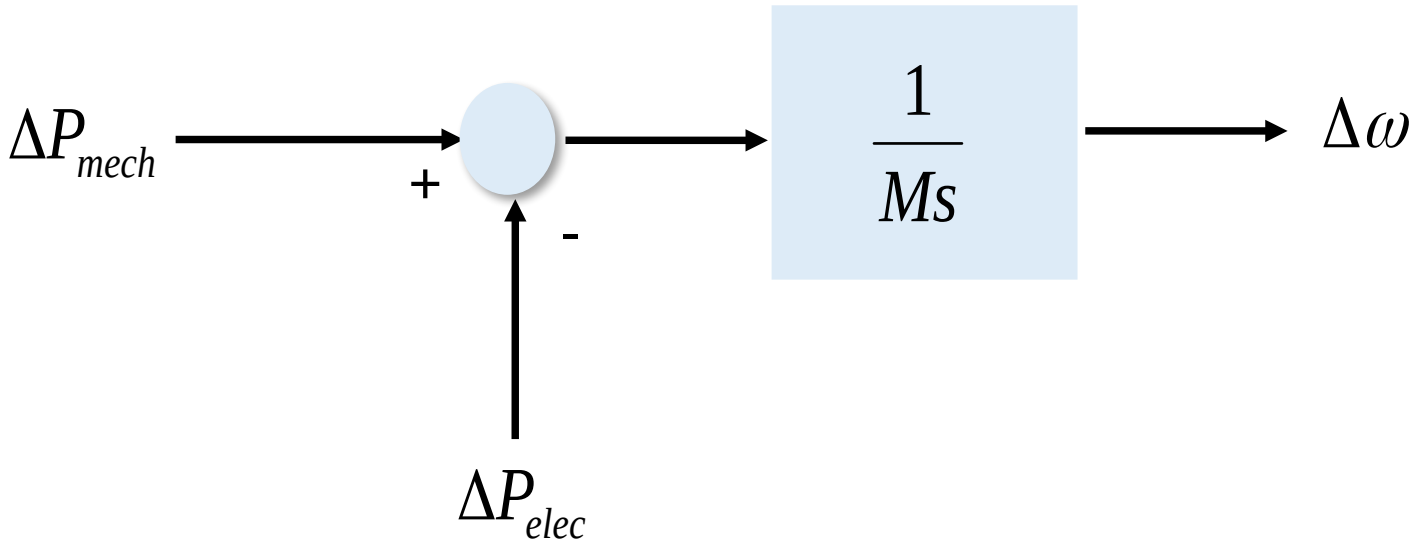
$$\boldsymbol{M\frac{d\Delta\omega}{dt} = (\Delta P_m - \Delta P_e)}$$

$$\Delta P_{mech} - \Delta P_{elec} = Ms\Delta\omega$$

See Wood and Wollenberg or Kundur for details

Synchronous generator dynamic model

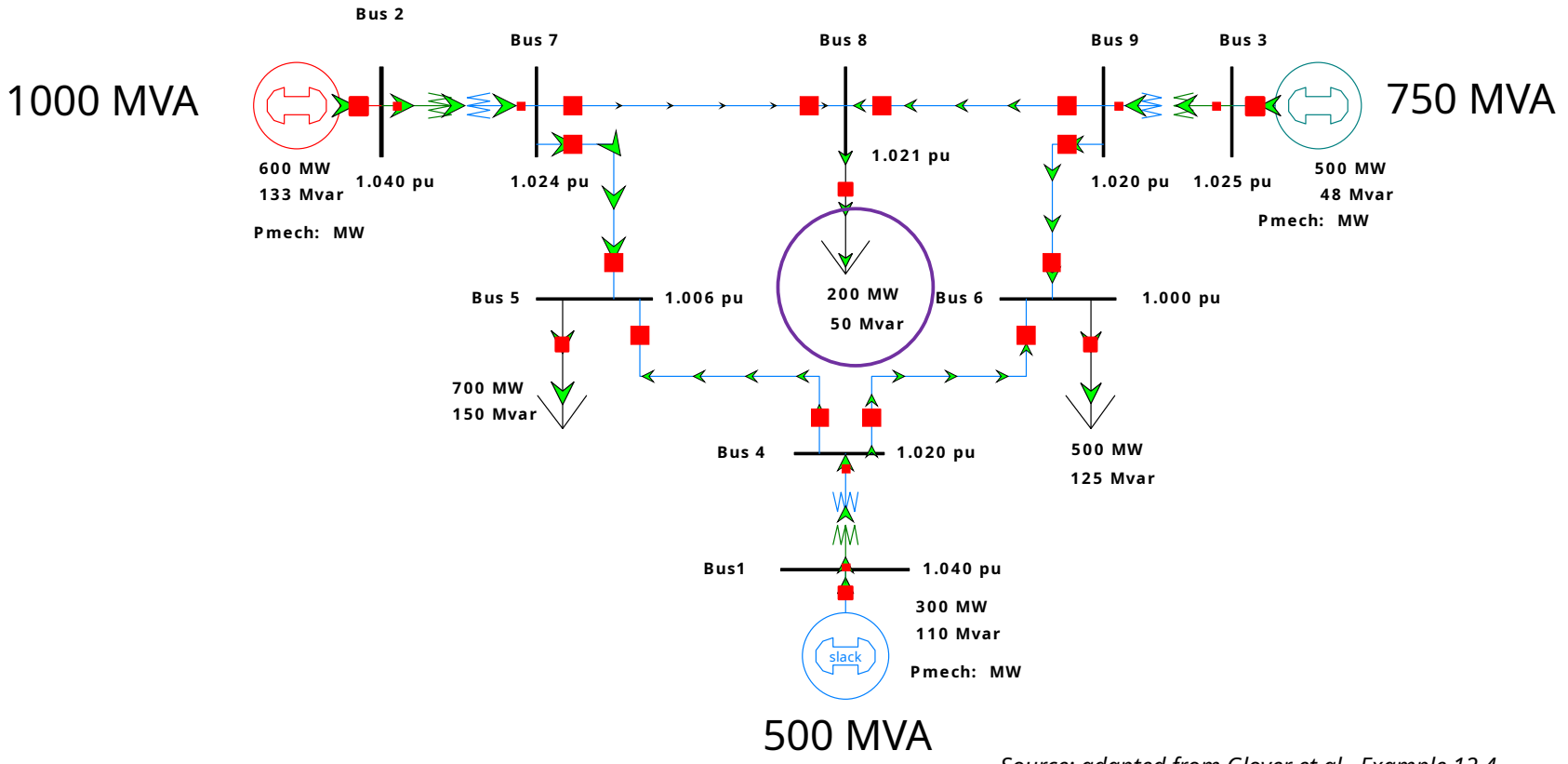
Block diagram of the relationship between mechanical and electrical power and speed change:



Note: this also represent the natural response model of an entire power system, where H (and M) represent the inertia contributions from all rotating machines (generators and loads) that are electromagnetically coupled to the grid

Example: load change with no automatic response

- For each and every generator: H=5s
- Total initial load = 1400 MW
- Sudden increase of 40 MW in load at bus 8 after 0.5s (200 MW => 240 MW)

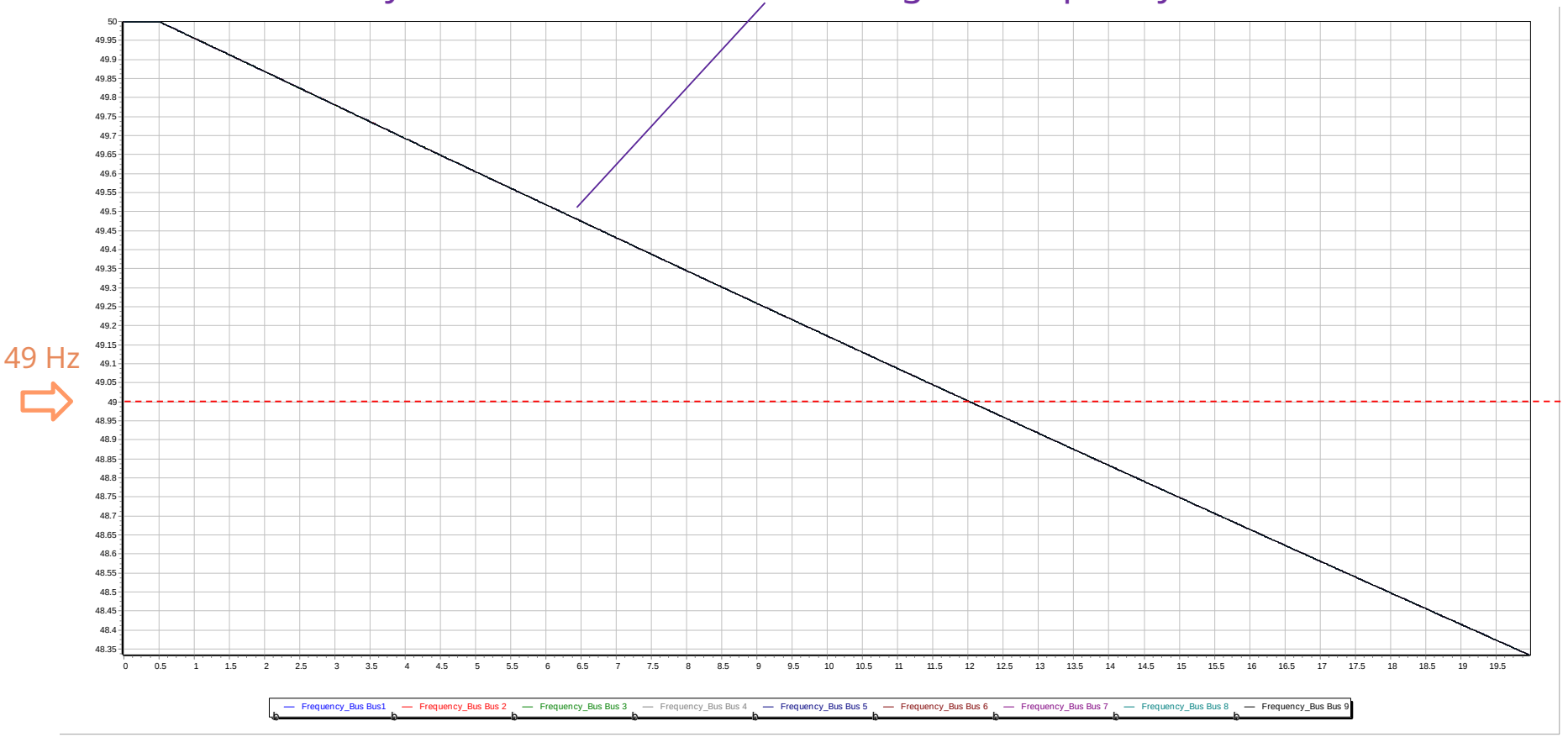


Source: adapted from Glover et al., Example 12.4

Example:
Frequency response: 200 MW load increase, bus 8

Can you estimate the rate of change of frequency?

49 Hz



Solution:
Frequency response: 200 MW load increase, bus 8

Capacity of machines:

- G2=1000 MVA
- G3 = 750 MVA
- G1 = 500 MVA

Total capacity = 2250 MVA

$$\frac{d\Delta f}{dt} = -f^0 \frac{\Delta P_{el}}{2 \cdot KE^0}$$

Total KE =

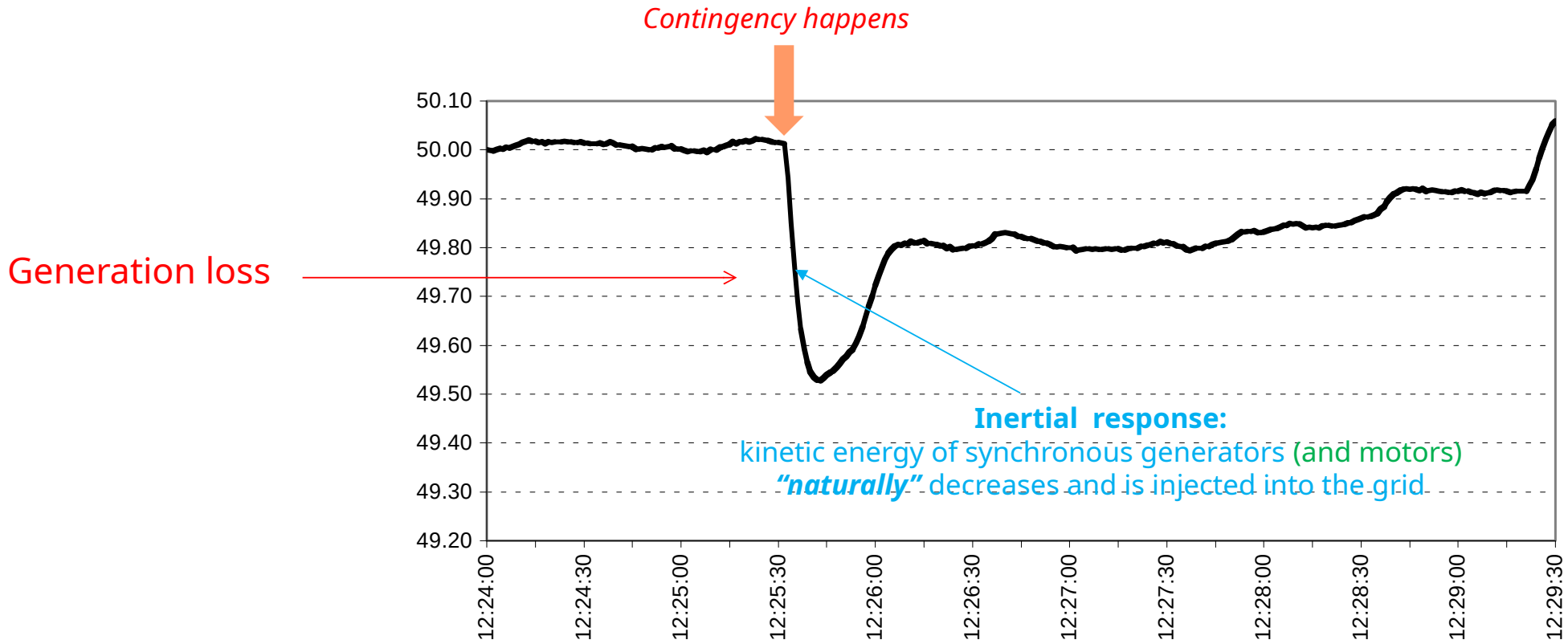
2250 MVA x 5 s = 11250 MWs

$\Delta P_{el} = 40\ MW$

ROCOF =- 50 x 40 / (2 x 11250) =- 0.089 Hz/s

LOAD MODELLING IN FREQUENCY CONTROL ANALYSIS

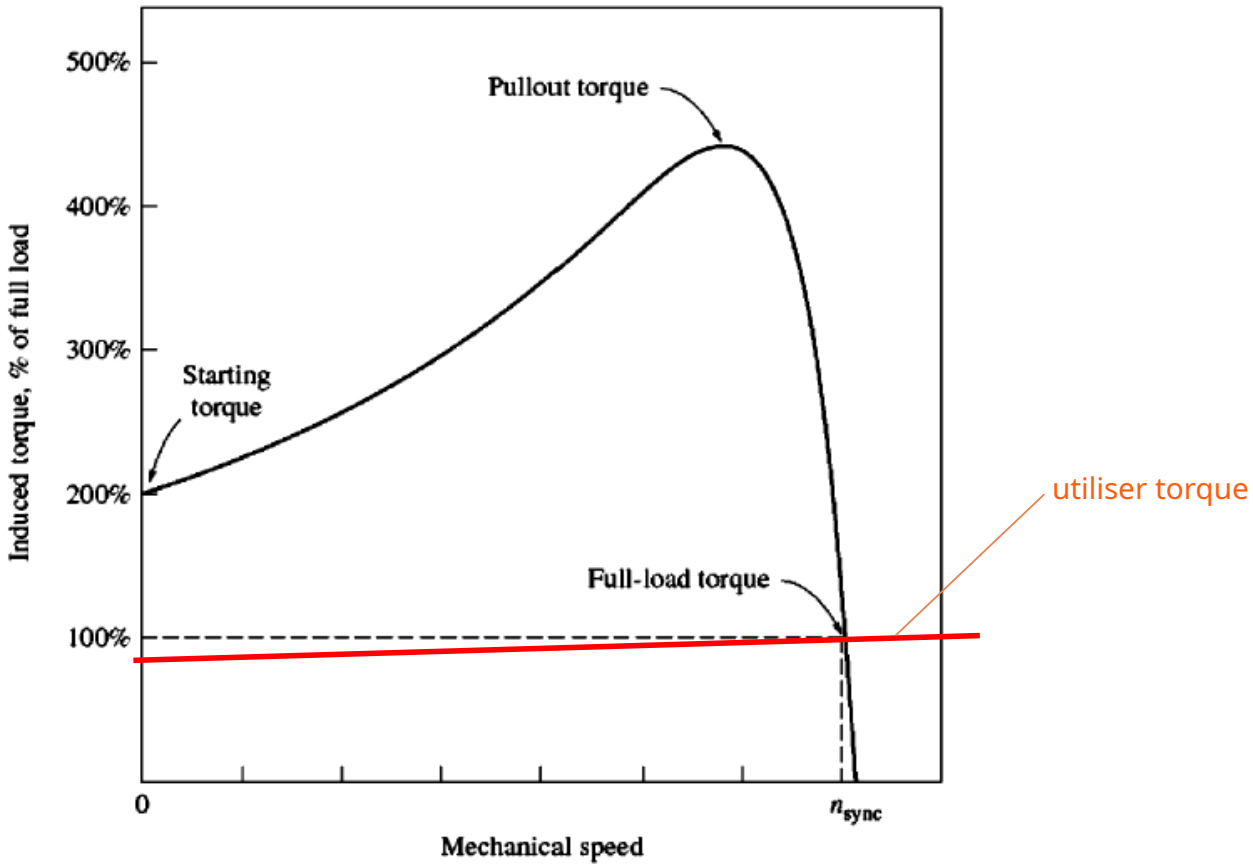
Maintaining frequency stability



Does load naturally contribute in any way to frequency control (besides inertial response)?

Example: induction motor

What happens to the power drawn by the motor if the frequency goes down?



Modelling the frequency dependence of loads

A model of the frequency dependence of the load is usually derived by multiplying an **exponential** (or alternatively a ZIP) load model by a factor that depends on the bus frequency

$$P(V,f) = \left[1 + k_{Pf}(f - f_0)\right] P_0 \left(\frac{V}{V_0}\right)^{n_P}$$

$$Q(V,f) = \left[1 + k_{Qf}(f - f_0)\right] Q_0 \left(\frac{V}{V_0}\right)^{n_Q}$$

Load damping factor

Typical values for k_{Pf} in **conventional power systems** are 1 to 2

A value of k_{Pf}=2 means that a 1% change in frequency would cause a 2% percent change in load

What does $D = 2$ mean in a 50 Hz system with a total load value of 1GW?

*D=2 means that the system load changes by 2% (ΔPLD=0.02*1GW=10MW) when there is a 1% change in system frequency (Δf =0.01*50Hz=0.5Hz)*

Load relief in the Australian power system





Load relief reduction:

- *From **1.5%** to **0.5%** in mainland*
- *From **1%** to **0%** in Tasmanian region*

What do you think may be the reason for this update?

What do you think is likely to happen in future grids?

Exercise 3

- Demand = 30 GW
- Nominal frequency = 60 Hz
- Load damping factor = 2 (= 2% of demand per % of frequency variation)
- What is the new demand level if, due to the sudden loss of a generator, the frequency drops by 0.3 Hz

Exercise 3

- Demand = 30 GW
- Nominal frequency = 60 Hz
- Load damping factor = 2 (= 2% of demand per % of frequency variation)
- What is the new demand level if, due to the sudden loss of a generator, the frequency drops by 0.3 Hz

Solution

$$\Delta f\% = 0.3/60 \cdot 100 = 0.5\%$$

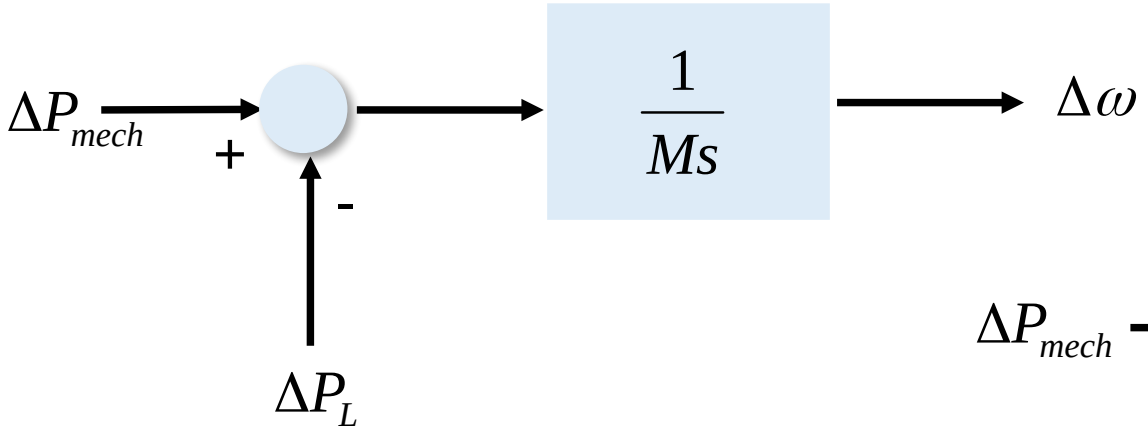
$$\Delta P = P0 \cdot kpf \cdot \Delta f\% = 30{,}000 \cdot 2/100 \cdot 0.5 = 300 \text{ MW}$$

$$P' = P0 - \Delta P = 29{,}700 \text{ MW}$$

Combined load-generator model

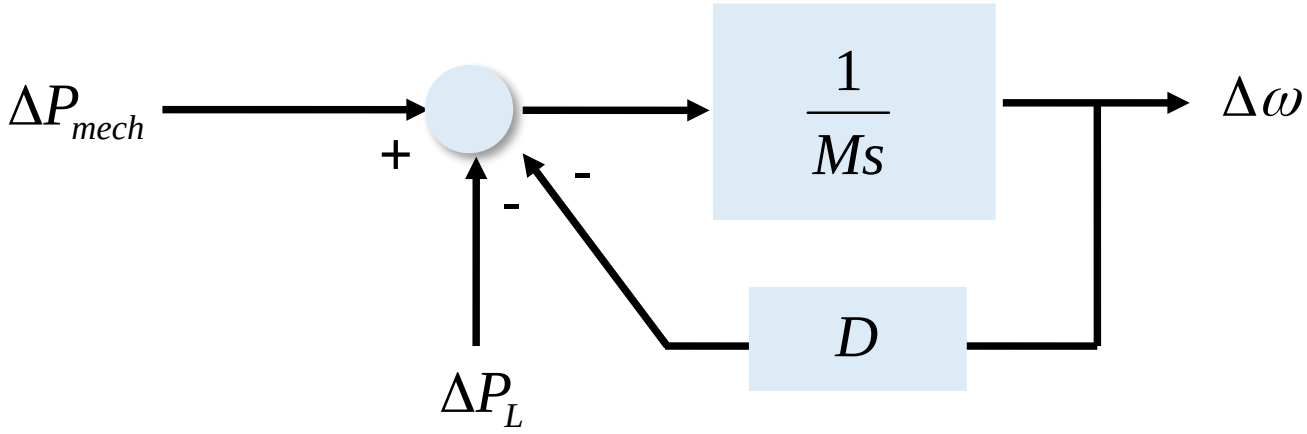
Without load relief modelling

$$\Delta P_{mech} - \Delta P_L = Ms\Delta\omega$$



With load relief modelling

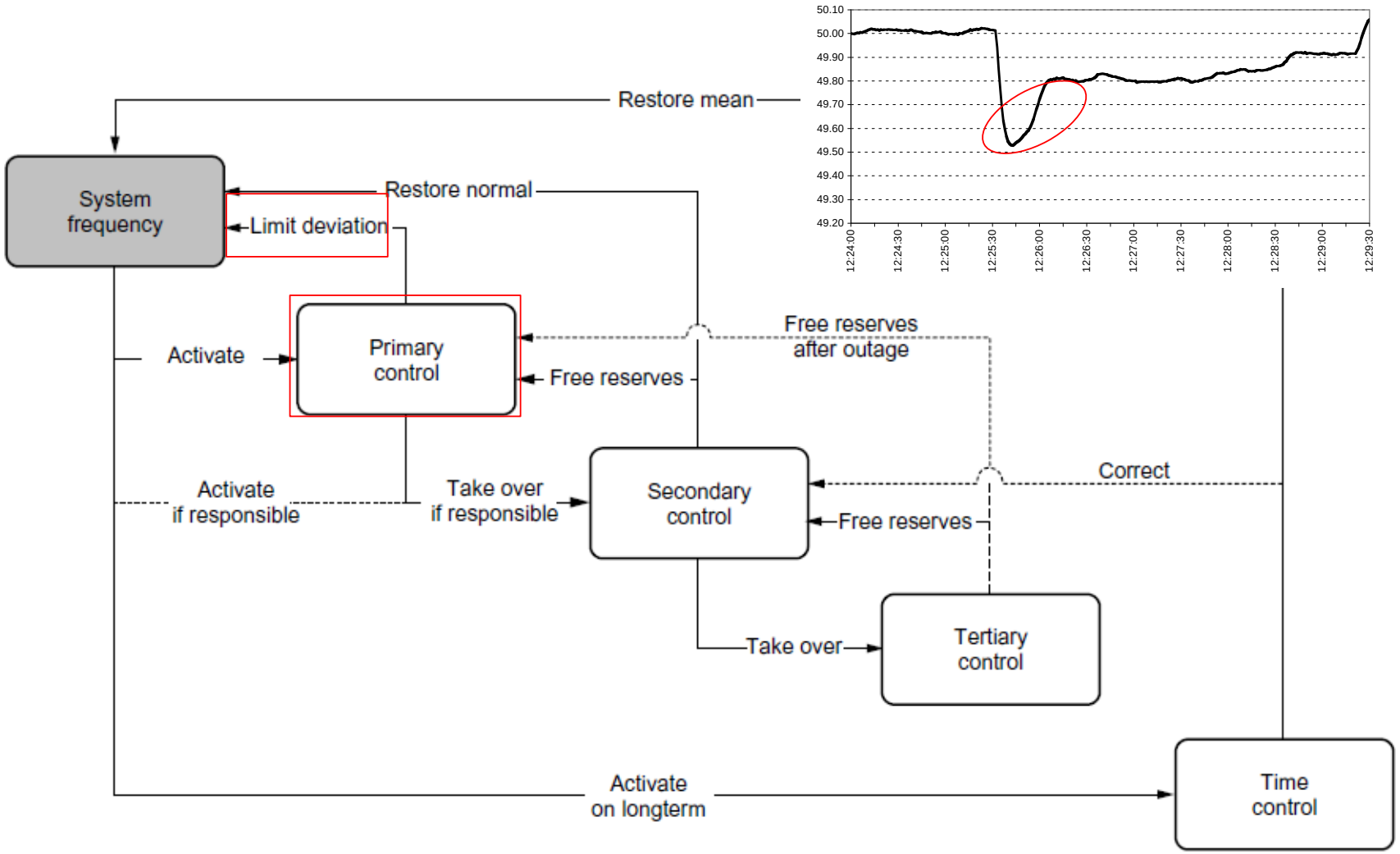
$$\Delta P_{mech} - \Delta P_L - D\Delta\omega = Ms\Delta\omega$$



Full system natural frequency response following active power imbalance

ACTIVE POWER GENERATION CONTROL (PRIMARY FREQUENCY CONTROL)

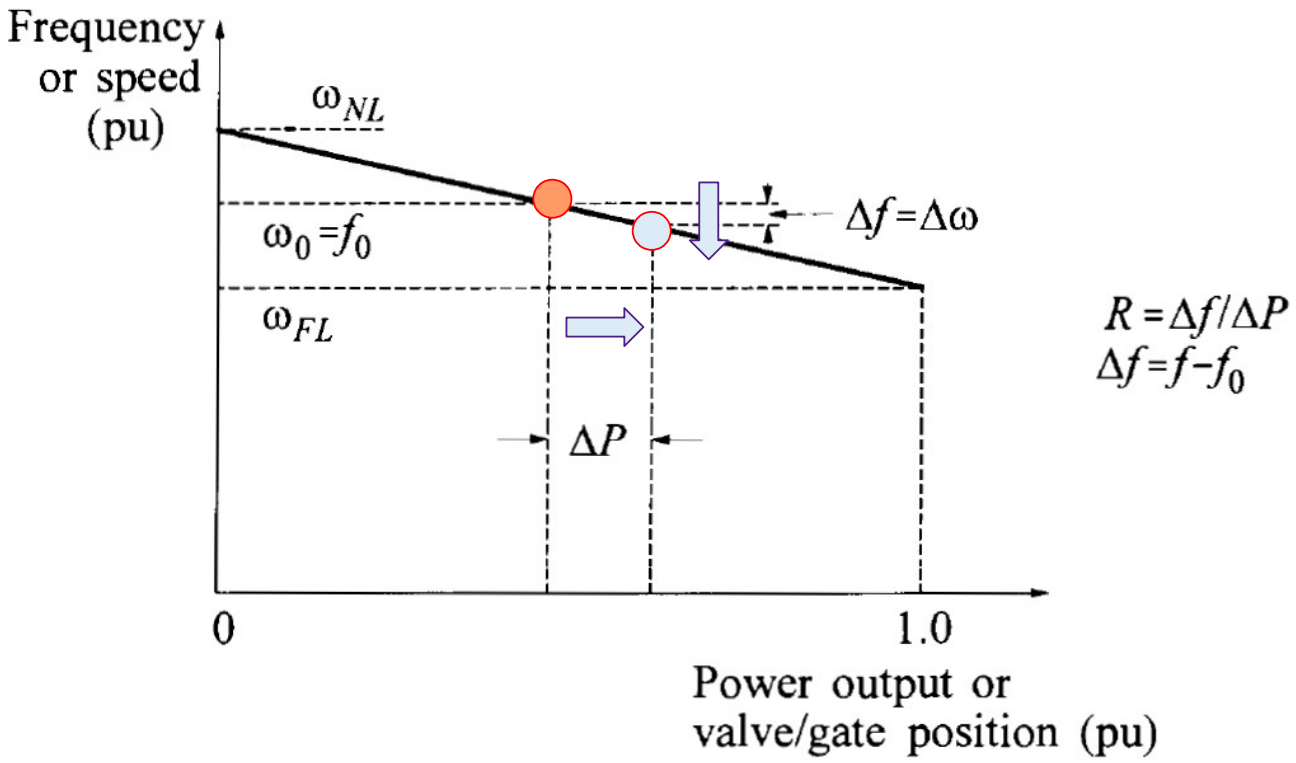
Frequency control mechanisms



Example from the European interconnected system; Source: Gomez-Exposito et al.

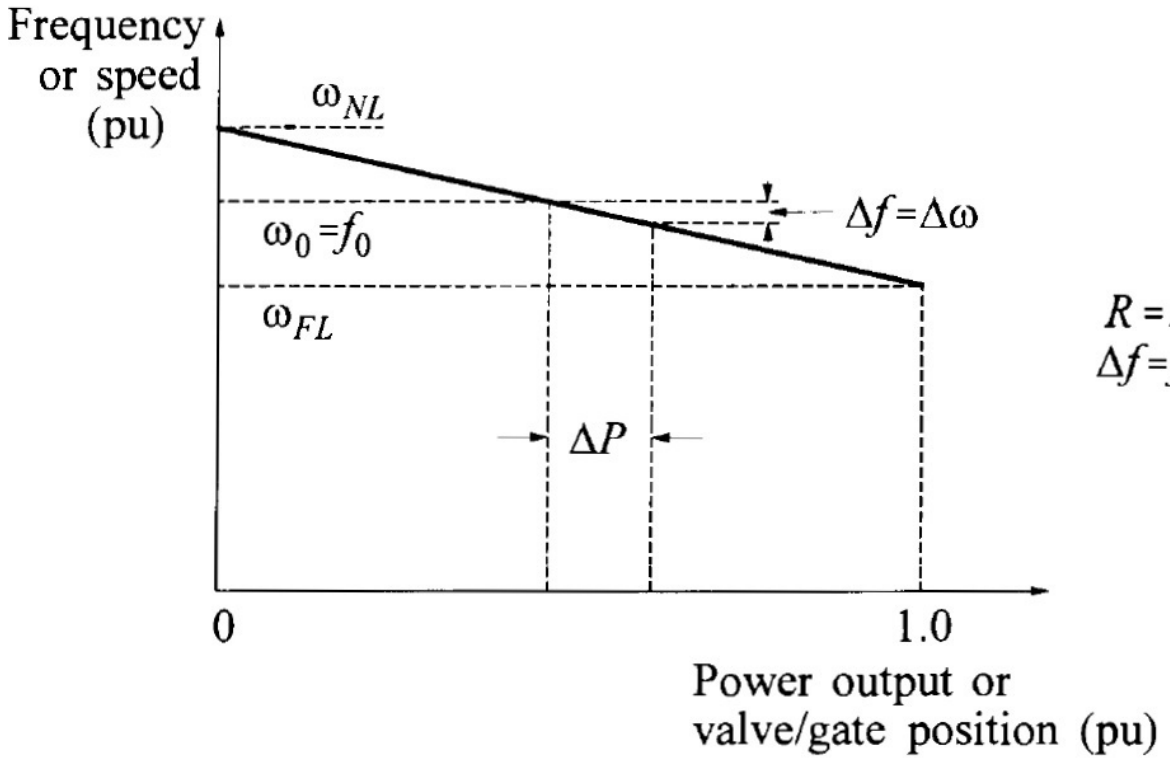
Speed-Droop Governor Control

One way to control the prime mover's speed, and subsequently frequency, following a disturbance is to equip the turbine with a "**governor**"



A. J. Wood, B. F. Wollenberg,"Power generation, operation and control", 1996

Speed-Droop Governor Control



$$R = \Delta f / \Delta P$$
$$\Delta f = f - f_0$$

$$\text{Percent } R = \frac{\text{percent speed or frequency change}}{\text{percent power output change}} \times 100$$

$$= \left(\frac{\omega_{NL} - \omega_{FL}}{\omega_0} \right) \times 100$$

ω_{NL} = steady-state speed at no load
ω_{FL} = steady-state speed at full load
ω_0 = nominal or rated speed

A. J. Wood, B. F. Wollenberg,"Power generation, operation and control", 1996

Speed-Droop Governor Control

The droop control results in changes in the turbine mechanical power

$$\Delta P_m = \Delta P_{ref} - \frac{1}{R}\Delta f$$

Where

ΔP_m	is change in turbine mechanical power
ΔP_{ref}	is change in reference power setting
R	is the speed regulation (frequency droop) constant
Δf	is change in frequency

Exercise 4

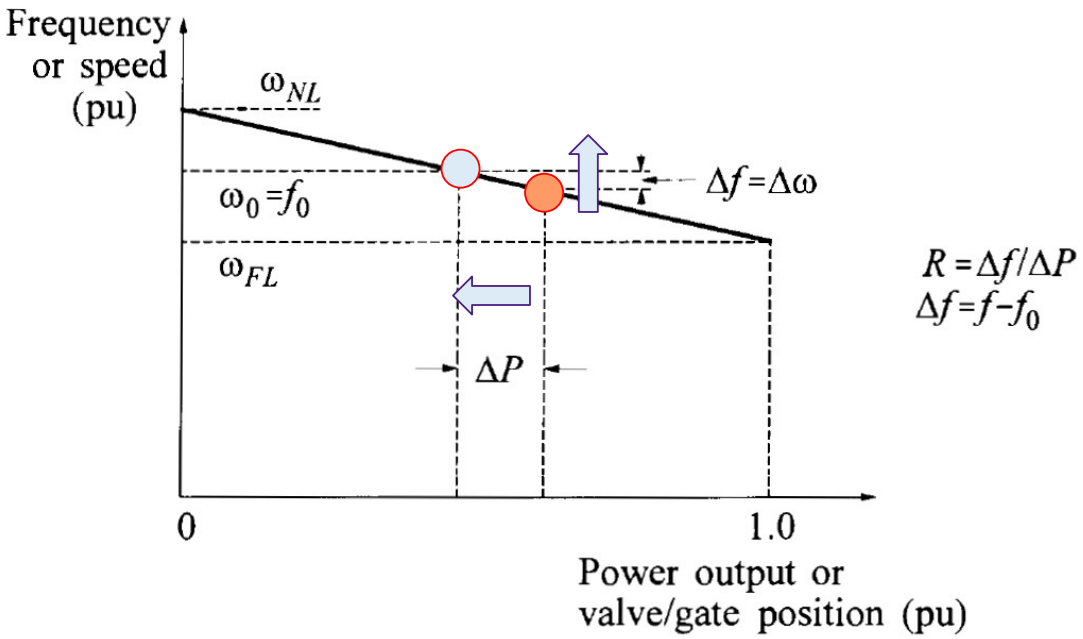
A 500 MVA, 50 Hz turbine-generator has a regulation constant R=0.05 p.u. (based on its own rating)

If the generator frequency increases by 0.01 Hz in steady-state, what is the decrease in turbine mechanical power output?

Assume a fixed reference power setting

Exercise 4 Solution

In this case, there is a change in frequency!



$$\Delta f_{\text{p.u.}} = \frac{\Delta f}{f_{base}} = \frac{0.01}{50} = 0.0002 \text{ p.u.}$$

$$\Delta P_m = \Delta P_{ref} - \frac{1}{R}\Delta f$$

Fixed reference power setting

$$\Delta P_{ref} = 0$$

$$\Delta P_{m\,\text{p.u.}} = -\frac{1}{0.05} \cdot 0.0002 = -0.004 \text{ p.u.}$$

$$\Delta P_m = \Delta P_{m\,\text{p.u.}} \cdot S_{base} = -0.004 \cdot 500 = -2 \text{ MW}$$

Exercise 5

A 500 MVA, 50 Hz turbine-generator has a regulation constant R=0.05 p.u. (based on its own rating)

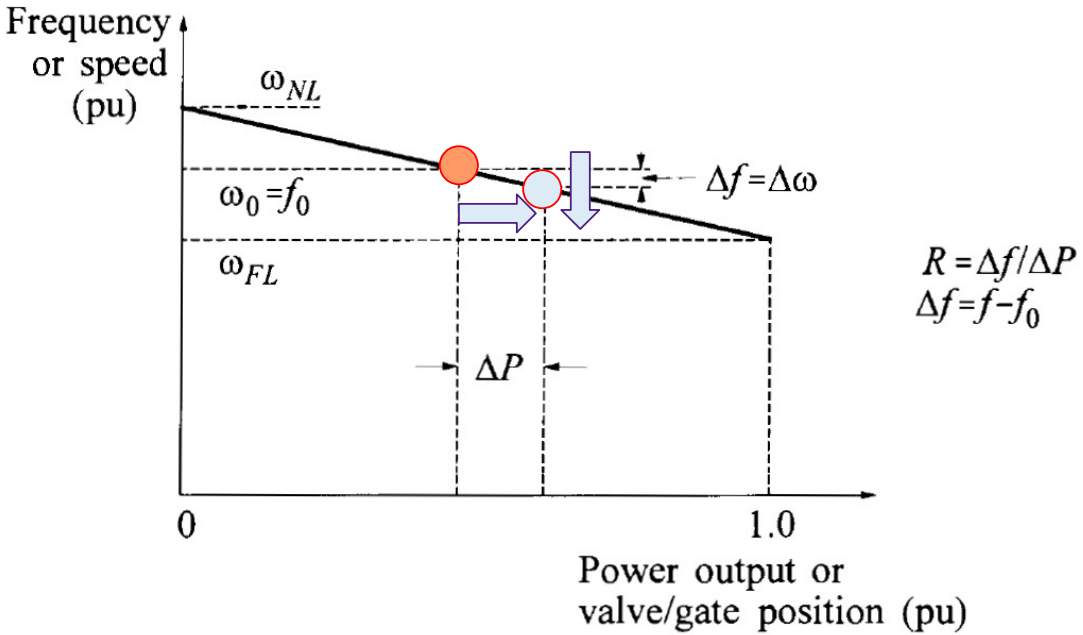
The generator is operating at half of its nominal capacity when the load that it is driving suddenly increases by 10 MW

What is the steady-state drop in frequency?

Assume a fixed reference power setting

Exercise 5 Solution

In this case, there is a change in system load (active power)!



$$\Delta P_m = \Delta P_{ref} - \frac{1}{R}\Delta f$$

$$\Delta P_{ref} = 0$$

Fixed reference power setting

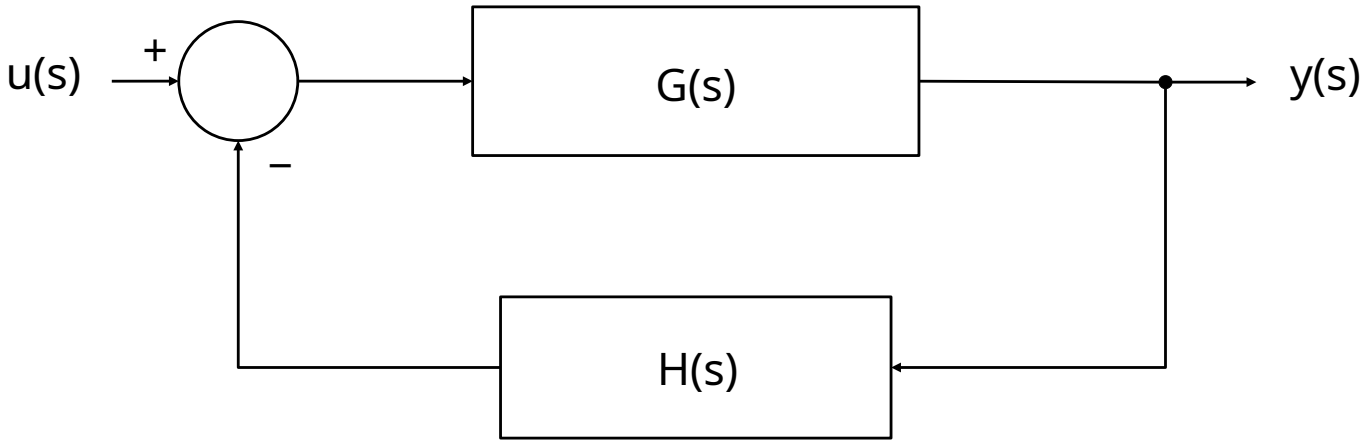
$$\Delta f_{\text{p.u.}} = -R_{\text{p.u.}} \cdot \Delta P_m \text{ p.u.} = -0.05 \cdot \frac{10}{500}$$

$$\Delta f = -0.001 \cdot 50 = -0.05 \text{ Hz}$$

Droop control is a feedback law imposed on the generator.

This feedback law changes the system dynamics.

Negative Feedback



Transfer function for negative feedback:

$$\frac{y(s)}{\mathrm{u}(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

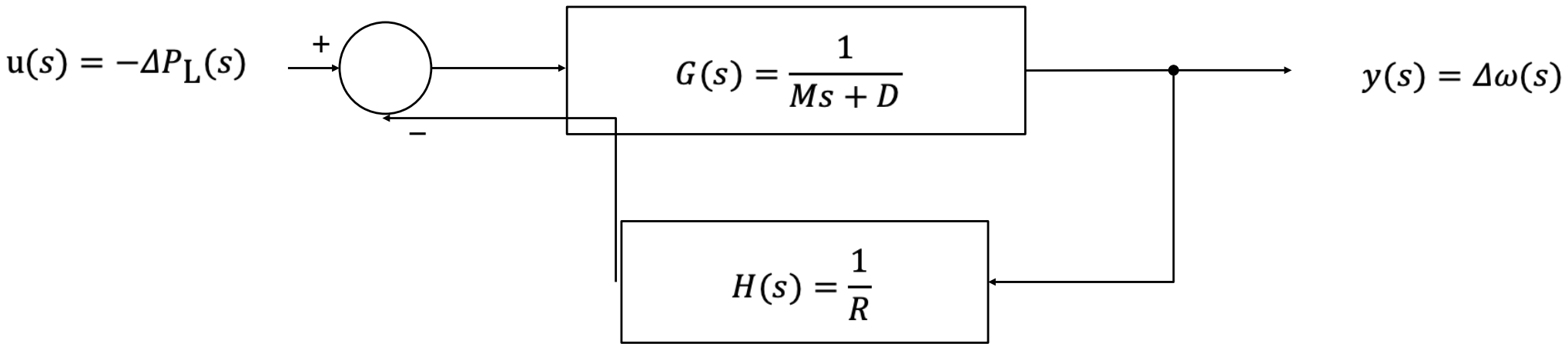
Derivation of that transfer function:

$$\mathrm{y}(s) = G(s)[\mathrm{u}(s) - H(s)y(s)]$$

$$[1 + G(s)H(s)]y(s) = G(s)\mathrm{u}(s)$$

Droop response to a load disturbance

Fixed power reference



Using the closed-loop transfer function formula:

$$\frac{\Delta\omega(s)}{\Delta P_L(s)} = -\frac{G(s)}{1 + G(s)H(s)} = -\frac{1}{Ms + D + \frac{1}{R}}$$

This is a low-pass-filter from the change in load to the change in frequency.

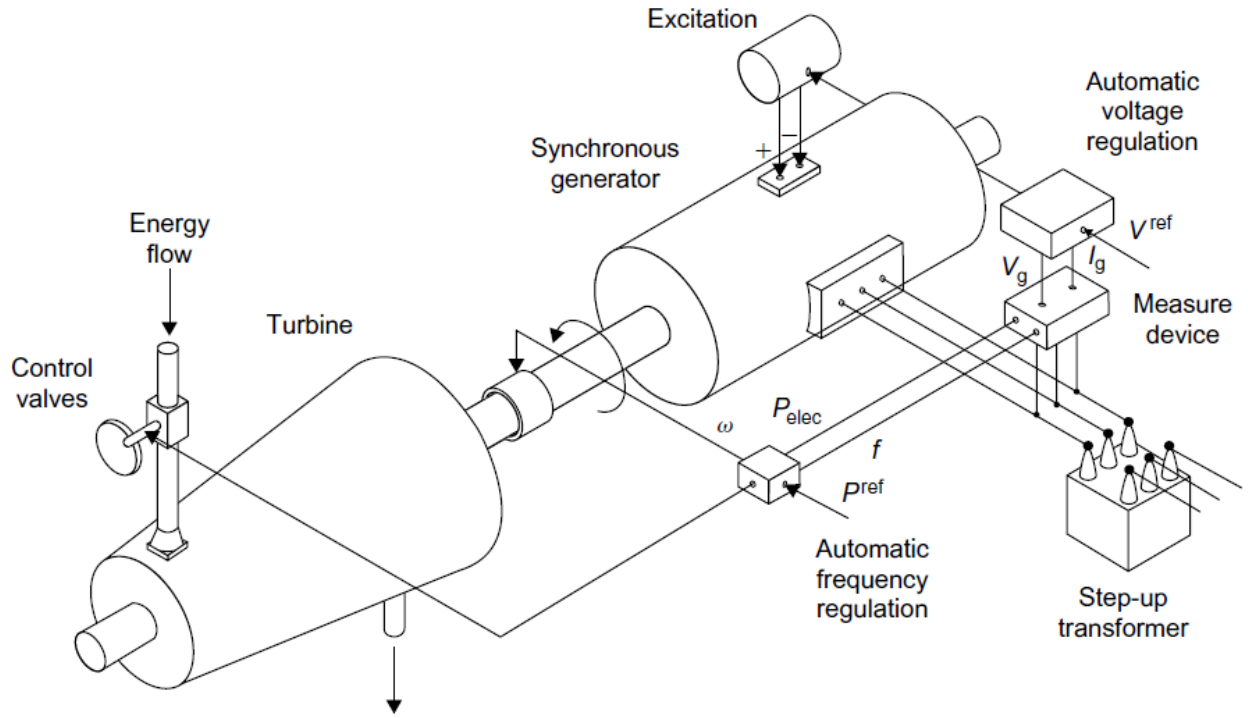
In the time-domain, low-pass-filters give exponentially-decaying behavior.

Have we considered all the dynamics?

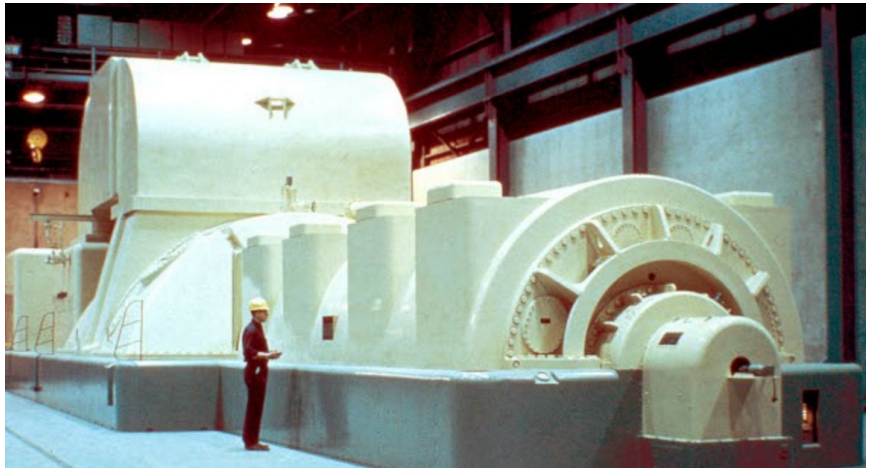
Not yet, we have just considered the swing equation.

Rotating machines also have prime-mover and governor (low-pass-filter) dynamics

Rotating Machines

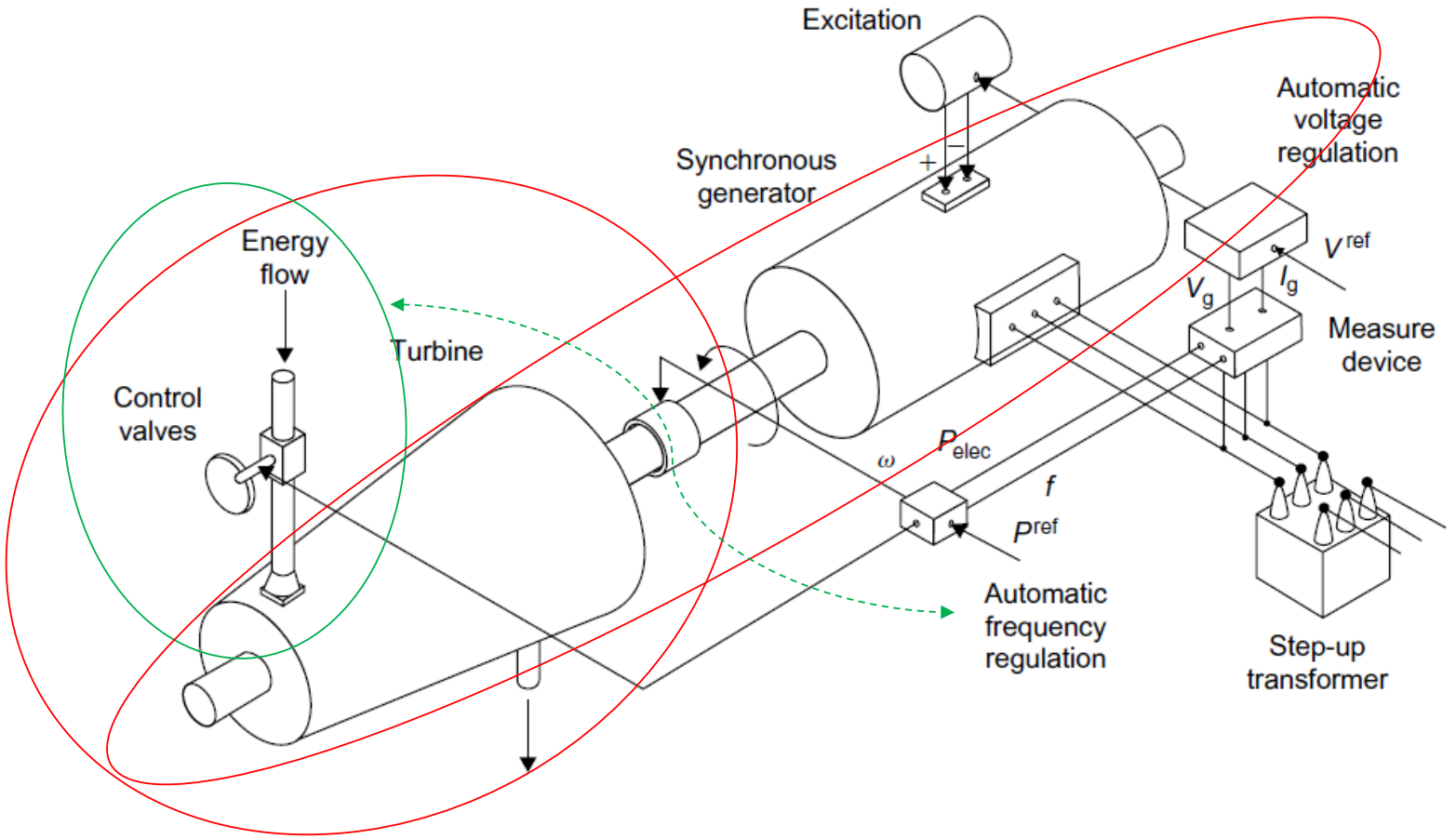


Source: Gomez-Exposito et al.





Active Power Generation Control



Source: Gomez-Exposito et al.

Prime Mover Modelling

The turbine mechanical power is essentially a function of the valve (steam and gas turbines) and gate (hydro turbines) positions, and is independent of frequency

- Steam turbine: control of the steam supply and boiler
- Gas turbine: control of the gas supply
- Hydro turbine: control of penstock (water flow)

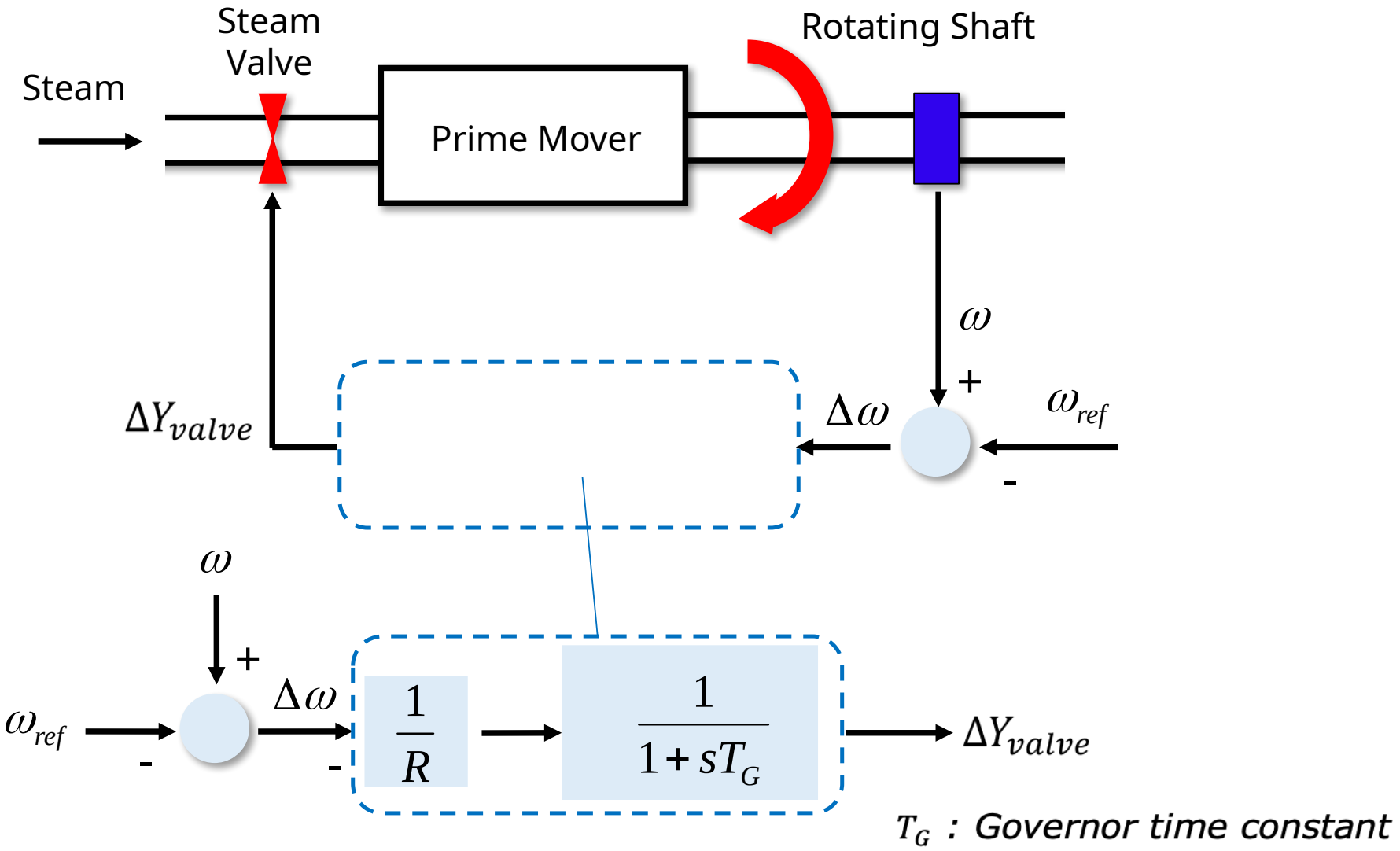
$$\Delta Y_{valve} \longrightarrow \frac{1}{1+sT_{CH}} \longrightarrow \Delta P_{mech}$$

T_{CH} is the charging time constant

ΔY_{valve} is the per unit change in valve position from nominal

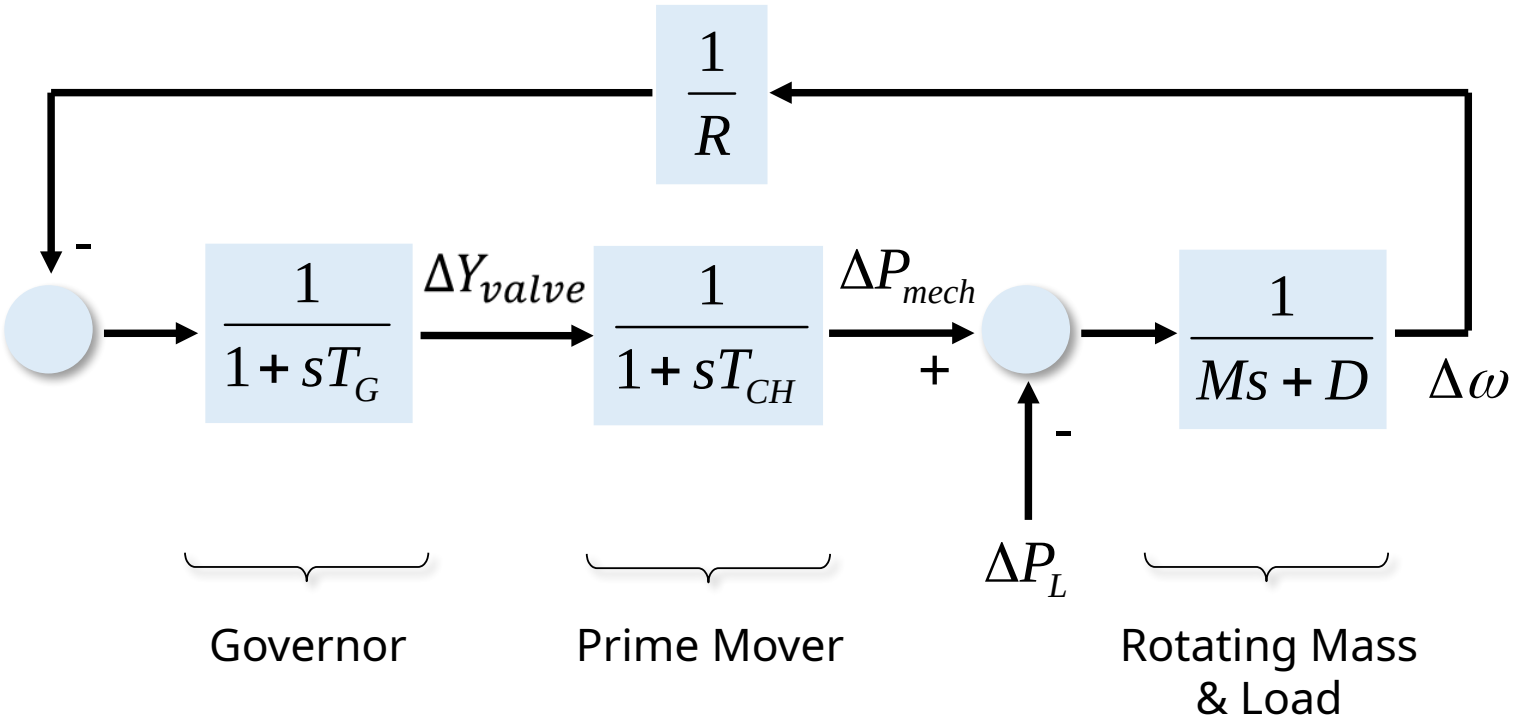
A. J. Wood, B. F. Wollenberg,“Power generation, operation and control”, 1996

Governor Model with Droop



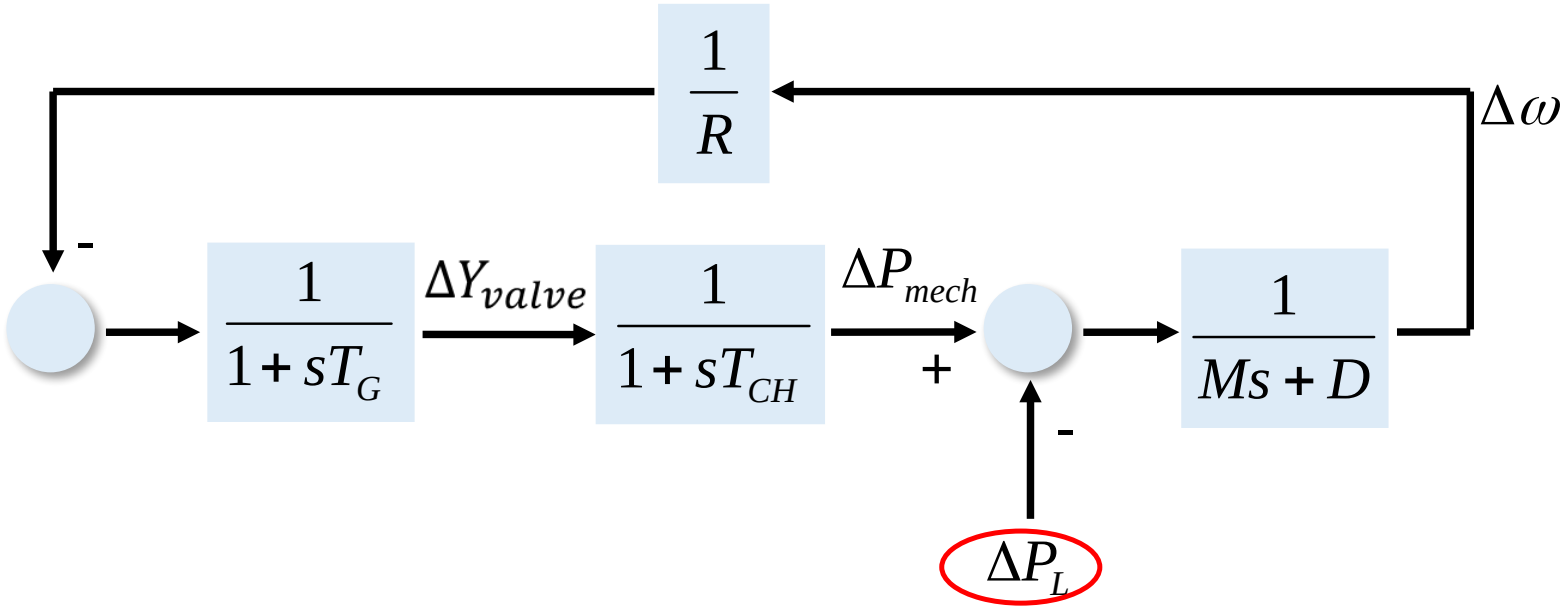
T_G : *Governor time constant*

Governor, Prime Mover, and Rotating Mass Model



A. J. Wood, B. F. Wollenberg,"Power generation, operation and control", 1996.

Response to load step change



$$\Delta P_L(s) = \frac{\Delta P_L}{s}$$

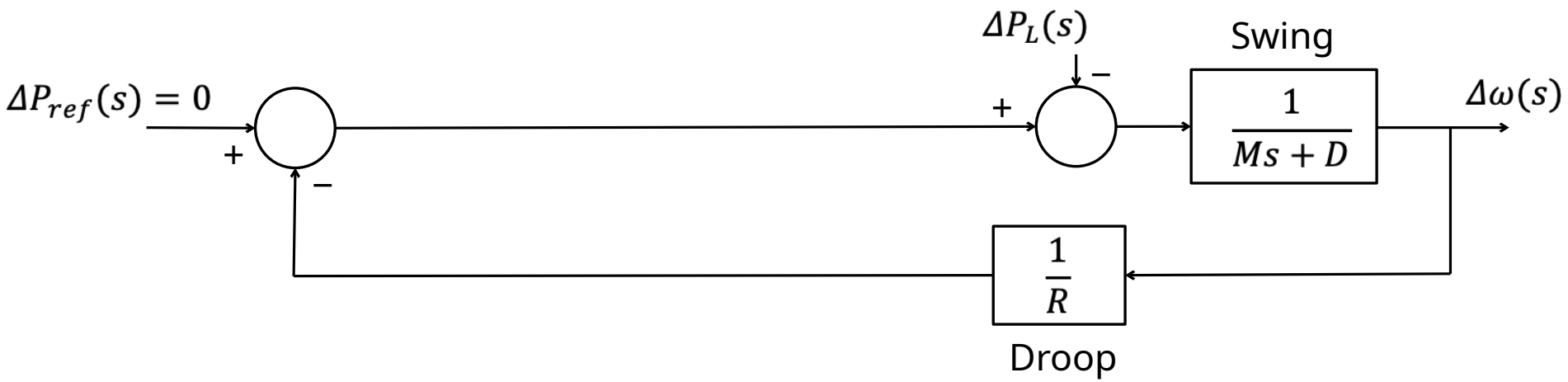
$$\Delta\omega(s) = \frac{\Delta P_L}{s}\left[\frac{\frac{-1}{Ms+D}}{1+\frac{1}{R}\left(\frac{1}{1+sT_G}\right)\left(\frac{1}{1+sT_{CH}}\right)\left(\frac{1}{Ms+D}\right)}\right]$$

$$\Delta\omega \text{ steady state} = \lim_{s\to 0}\left[s\Delta\omega(s)\right]$$

$$\Delta\omega = \frac{-\Delta P_L\left(\frac{1}{D}\right)}{1+\left(\frac{1}{R}\right)\left(\frac{1}{D}\right)} = \frac{-\Delta P_L}{\frac{1}{R}+D}$$

A. J. Wood, B. F. Wollenberg,"Power generation, operation and control", 1996.

Instantaneous droop: first-order response

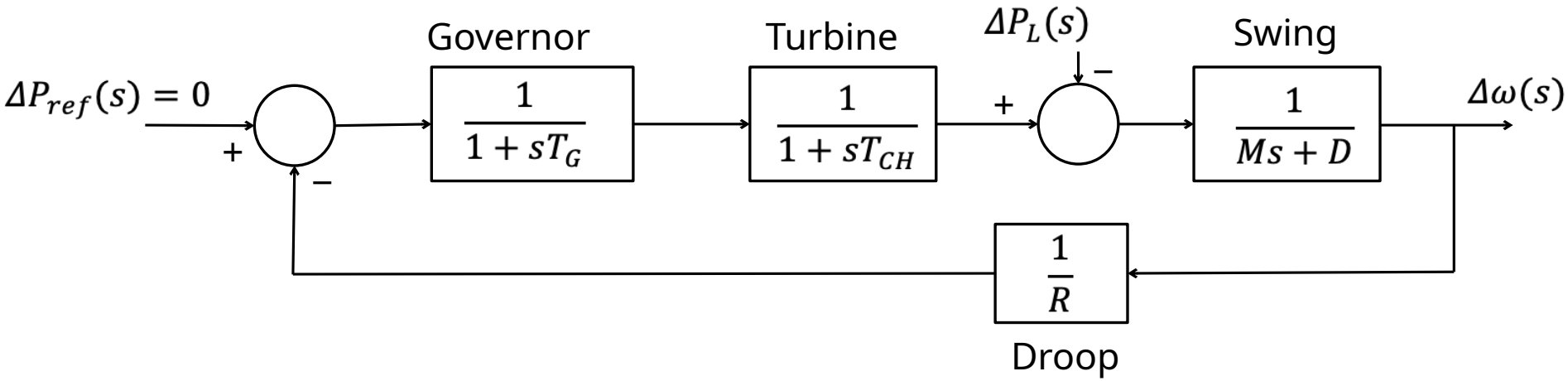


Closed-loop transfer function: $\frac{\Delta\omega(s)}{\Delta P_L(s)} = -\frac{1}{Ms+D+\frac{1}{R}}$

Time-domain step response: $\Delta\omega(t) = -\frac{P_0}{D+\frac{1}{R}}(1 - e^{-(D+1/R)t/M})$

→ frequency exponentially decays to its new steady value

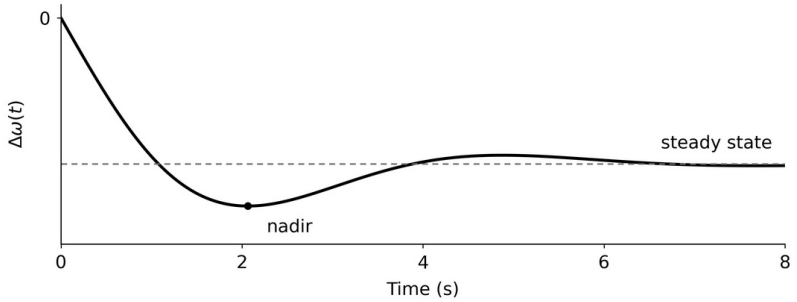
Considering governor and turbine dynamics

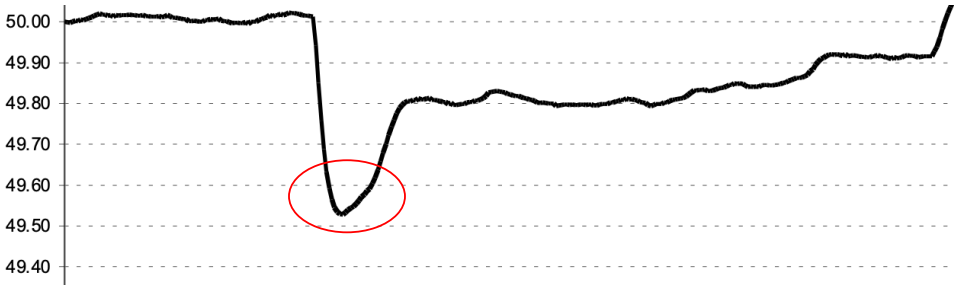


The closed-loop transfer function:

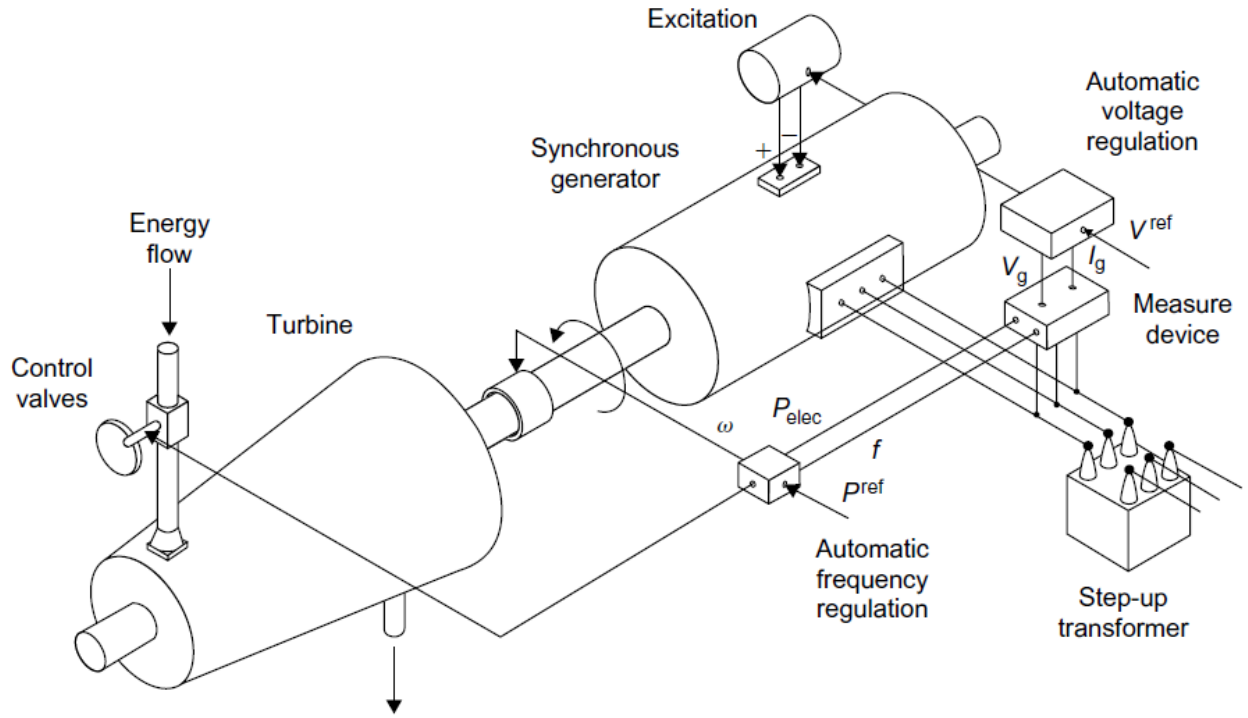
$$\frac{\Delta\omega(s)}{\Delta P_L(s)} = -\frac{(1+sT_G)(1+sT_{CH})}{(Ms+D)(1+sT_G)(1+sT_{CH})+\frac{1}{R}}$$

Third order transfer functions can have a nadir lower than the steady-state value

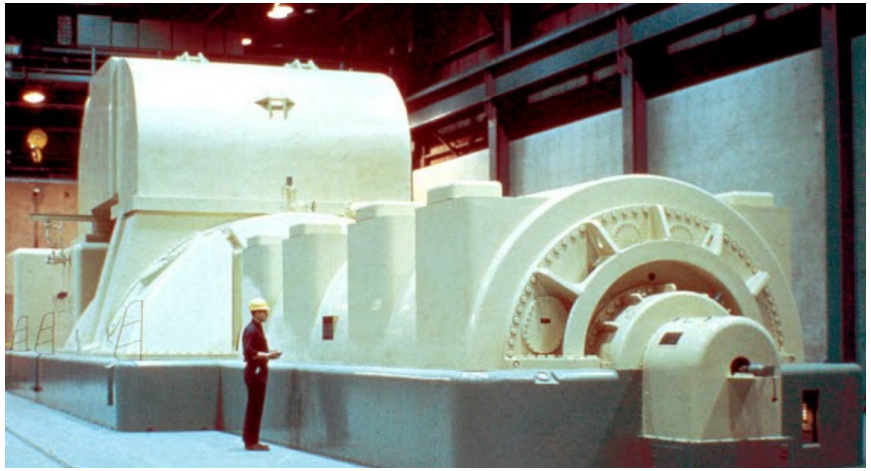




Rotating Machines

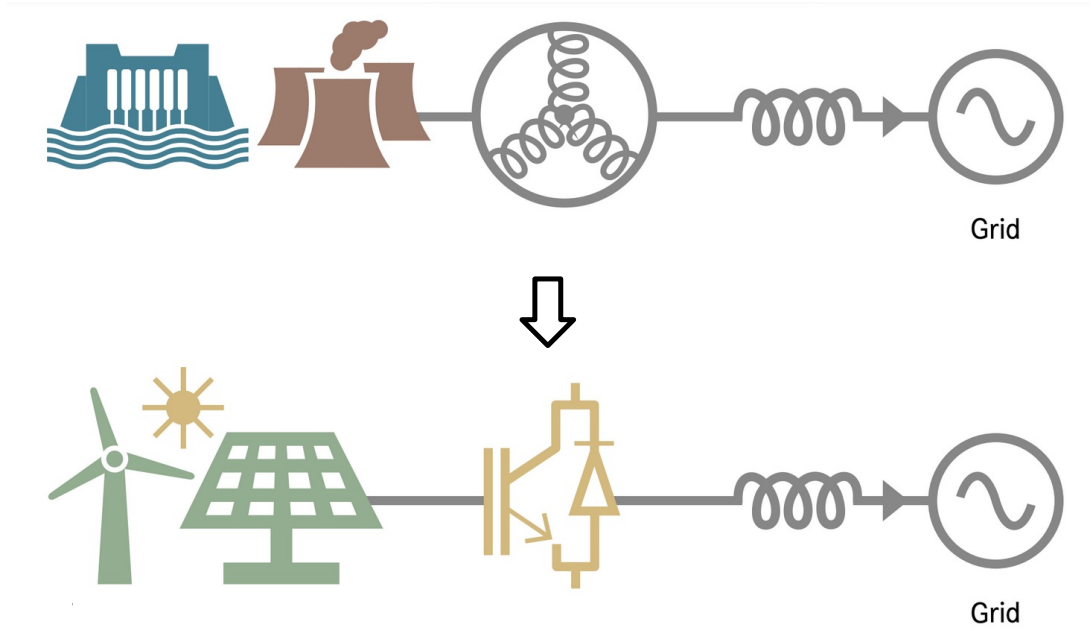


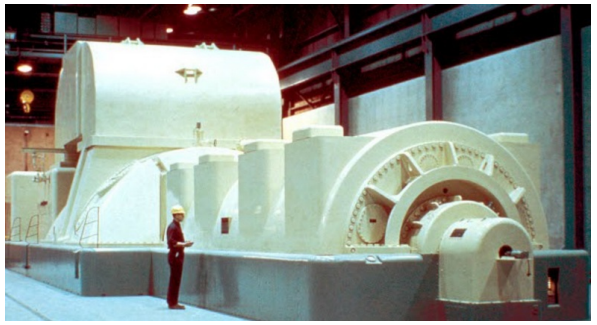
Source: Gomez-Exposito et al.





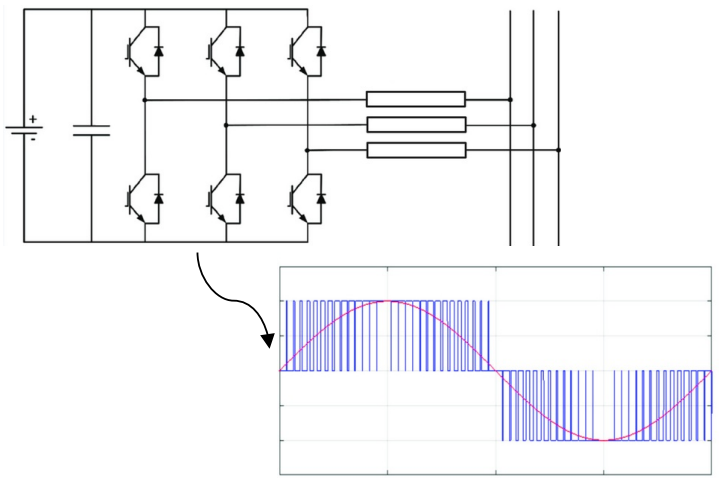
Energy Transition





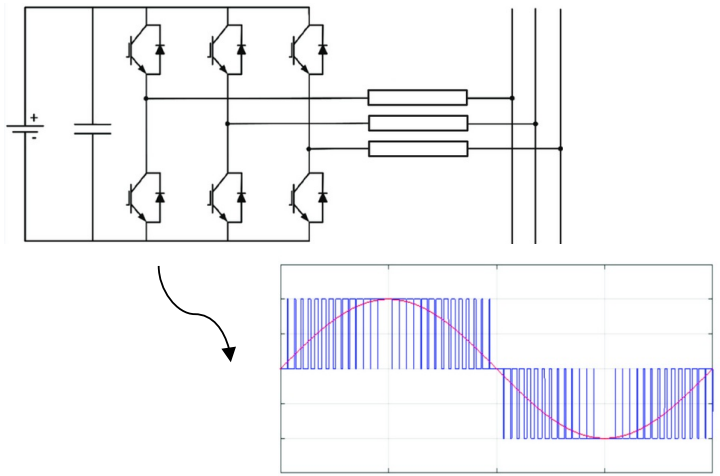






Inverter-Connected Generators

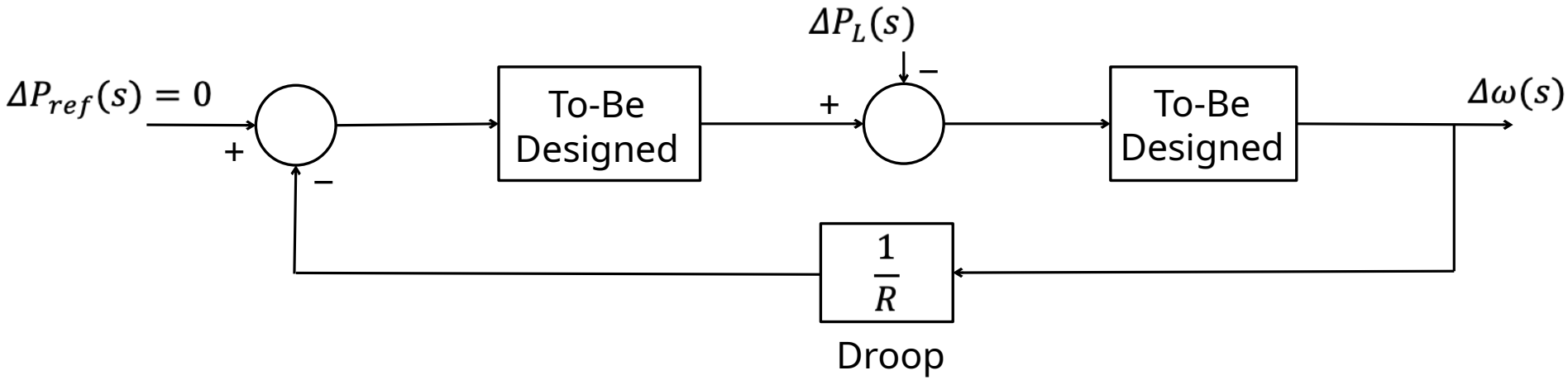




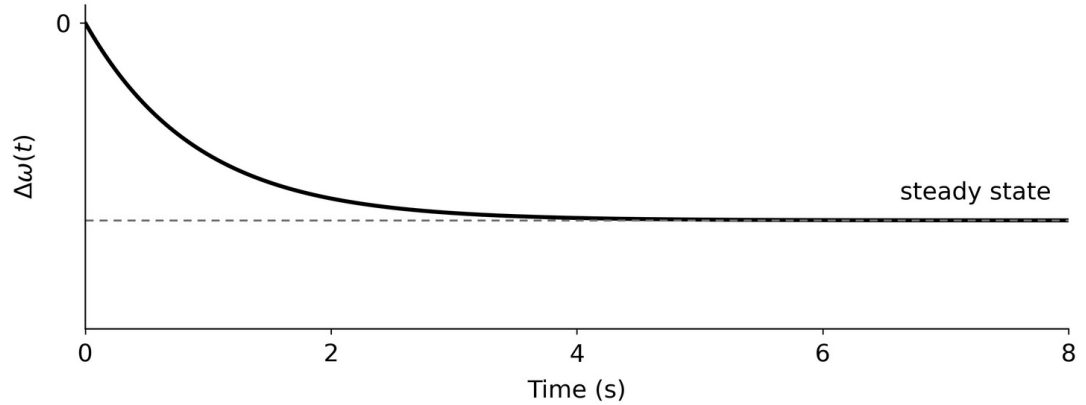


https://www.pv-magazine-australia.com/2022/07/27/hornsdale-big-battery-begins-providing-inertia-grid-services-at-scale-in-world-first/

Inverter-Connected Generators



Will we still have frequency-overshoot with inverters, or can we achieve first-order decay behavior?



Power System Dynamics Fundamentals

ELEN90060 Power System Analysis

Keith Moffat