Fault Analysis

ELEN90060

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2016 South Australian Blackout
Source: ABC News (Tom Fedorowytsch)

What

Power system faults are unexpected current flows caused by insulation breakdown.

This breakdown can be due to a variety of different factors:

- lightning
- wires blowing together in the wind
- animals or plants coming in contact with the wires
- salt spray or pollution on insulators

The faults can cause explosions and ignite fires, putting people and infrastructure at risk.



http://i.imgur.com/bIIGRU2.png



https://patch.com/california/losalamitos/mylar-balloon-power-outage-affects-2500-north-oc-homes

Why

The goal of fault analysis is to determine the magnitudes of the currents present during the fault.

- We need to determine the current to design the network protection.
- We also need to ensure the network infrastructure can survive the fault.

Fault Types

There are two main types of faults:

- **Symmetrical Faults** (balanced 3-phase). System remains balanced; these faults are relatively rare but are the easiest to analyze so we'll consider them first.

- **Asymmetrical Faults**. System is no longer balanced. More common, more difficult to analyze.

The **most common type** of fault on a three-phase system is the **single line-to-ground**, followed by the line-to-line faults, double line-to-ground faults, and balanced three-phase faults.

Network Simplifications

To simplify analysis of fault currents in networks we'll make several simplifications:

- **Transmission lines** are represented by their **series reactance**
- **Transformers** are represented by their **leakage reactances**
- **Synchronous machines** are modeled as a **constant voltage** behind direct-axis sub-transient reactance
- **Induction motors** are ignored or treated as **synchronous machines**
- Other **(non-spinning) loads** are **ignored**
- **Power electronics generator/loads** are... ***we are still figuring this out***

Solution Techniques

Circuit models used during the fault allow the network to be represented as a linear circuit.

There are two main methods for solving for fault currents:

1. **Direct Method**: Solve directly for the fault current, which is modelled as a short to ground. This requires first solving for the internal machine voltages using pre-fault conditions; then apply fault and solve directly.

2. **Superposition**: Fault is represented by two opposing voltage sources; solve system by superposition of two systems. (Power circuits are ~linear from voltage to current!)
   1) The first system is just the pre-fault operating point.
   2) The second system only has a single voltage source counteracting the pre-fault voltage at the fault location.

Example 1 using Direct Method

Assume a network with one 100MVA, 13.8kV, X”=0.15pu generator supplying power to one 100MVA, 13.8kV, X”=0.2pu motor. They are connected through a 138kV transmission line (j19.5Ω). The generator has a step-up Δ-Y transformer (j0.1pu) and the motor a step-down Y-Δ transformer (j0.1pu).

The generator has 1.05pu terminal voltage and supplies 100MVA with 0.95 lag pf. Assume a fault on its terminal. Calculate the fault current.

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A:

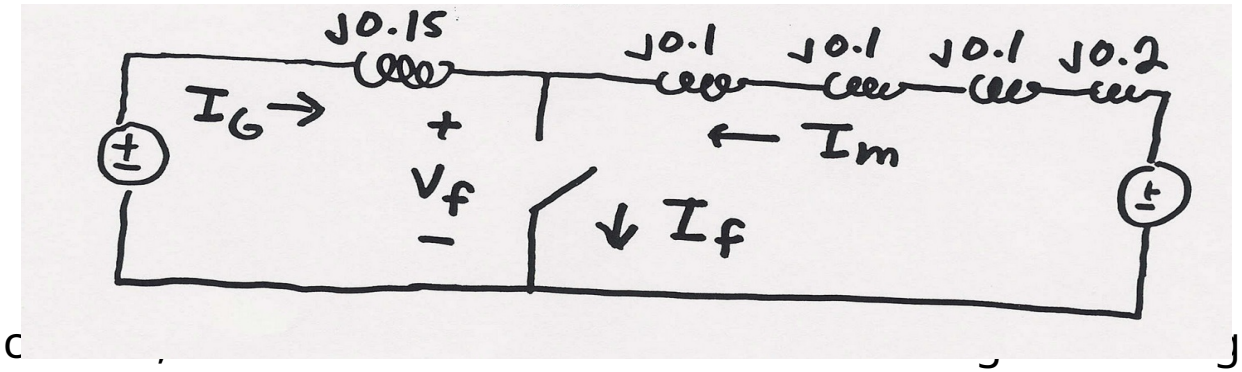
$$X_{line} = \frac{19.5}{138^2/_{100}} = 0.1\ pu$$

Base impedance so we can do calculations in pu.

Example 1 using Direct Method

A:

Produce the faulted network per unit diagram.



To determine the fault c[illegible] generator and motor.

$$\boldsymbol{I}_{load} = \left(\frac{\boldsymbol{S}_g}{\boldsymbol{V}_f}\right)^* = \left(\frac{1.0\angle 18.2°}{1.05\angle 0°}\right)^* = 0.952\angle -18.2°\, pu$$

$$\boldsymbol{E}_g = \boldsymbol{V}_f + \boldsymbol{I}_{Load}X_g = 1.10\angle 7.1°\, pu$$

Pre-fault conditions

Example 1 using Direct Method

A:

The motor's terminal voltage is then

$$\boldsymbol{V}_m = \boldsymbol{V}_f - \boldsymbol{I}_{Load} \times \sum X$$

Pre-fault conditions

$$\boldsymbol{V}_m = 1.05\angle 0° - (0.952\angle -18.2°) \times j0.3 = 1.0\angle -15.8°\, pu$$

$$\boldsymbol{E}_m = \boldsymbol{V}_m - \boldsymbol{I}_{Load} \times X_m = 1.01\angle -26.6°\, pu$$

For the fault, we can then solve as a linear circuit:

$$\boldsymbol{I}_f = \frac{\boldsymbol{E}_g}{X_g} + \frac{\boldsymbol{E}_m}{\sum X} = \frac{1.10\angle 7.1°}{j0.15} + \frac{1.01\angle -26.6°}{j0.5}$$

$$\boldsymbol{I}_f = 9.1\angle -90° = -j9.1\, pu$$

The Direct Method is intuitive but it can be time-consuming for a large network.

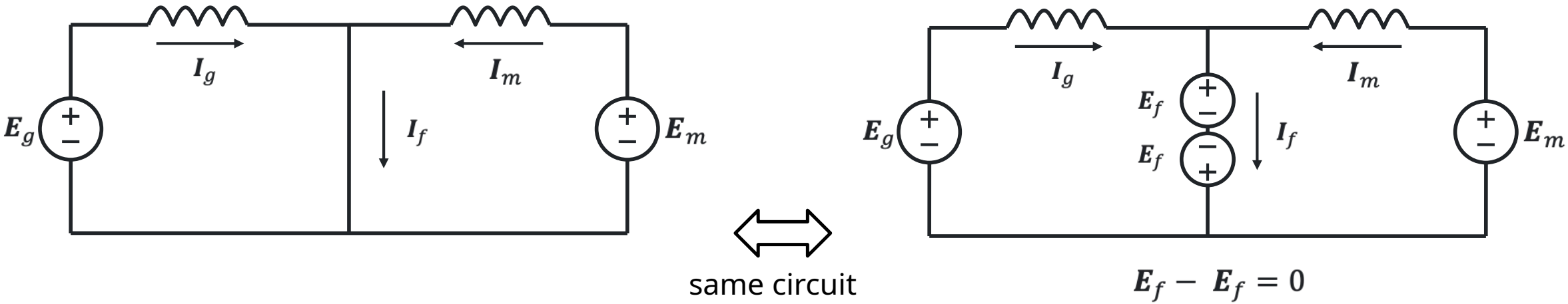
Superposition

The fault is represented *by two equal and opposite voltage sources*,
each equal to the pre-fault voltage.

Since this is now a linear network, the faulted voltages and currents are just the sum of:

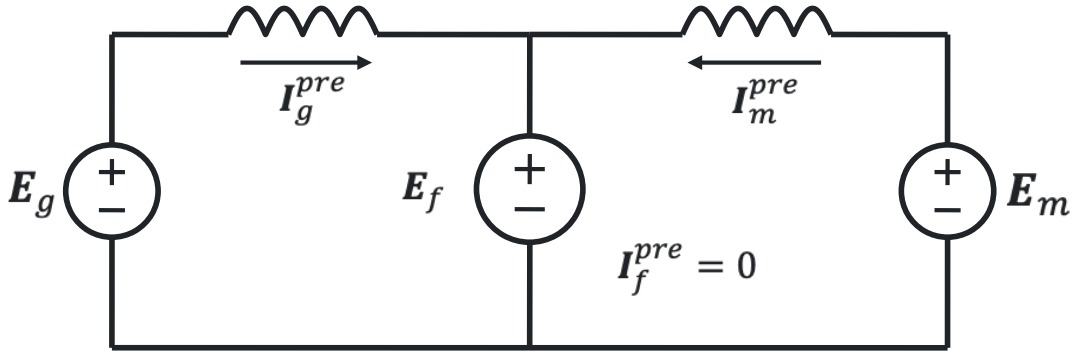
- the pre-fault conditions and
- the conditions with just a single voltage source at the fault location

Superposition Example

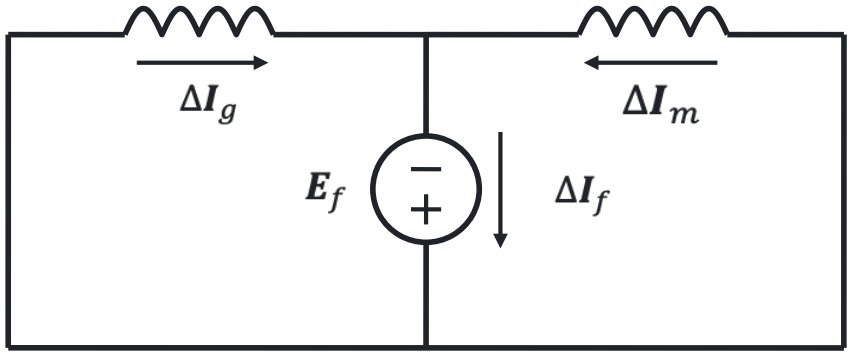


Superposition Example

Pre-fault circuit:



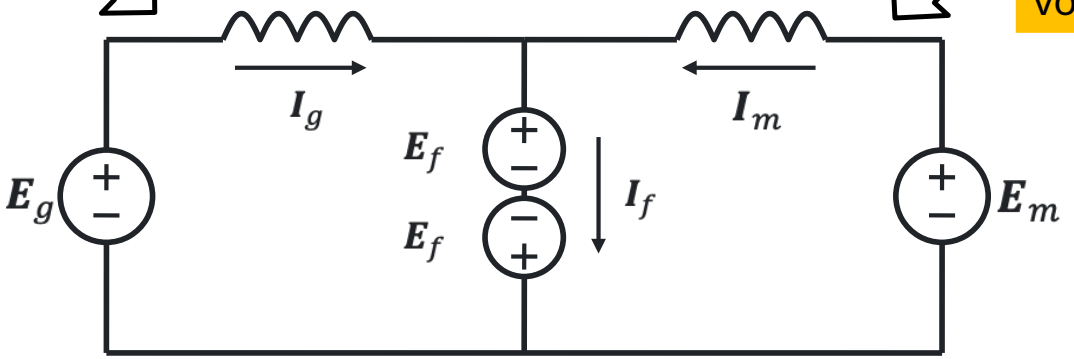
Fault-increment circuit:



the pre-fault "fault current" is zero!

This is the superposition

It's an increment, so the other voltage sources are set to 0!



$$I_g = I_g^{pre} + \Delta I_g \qquad I_m = I_m^{pre} + \Delta I_m$$

$$I_f = I_f^{pre} + \Delta I_f = 0 + \Delta I_f = \Delta I_f$$

Example 1 using Superposition

Assume a network with one 100MVA, 13.8kV, X”=0.15pu generator supplying power to one 100MVA, 13.8kV, X”=0.2pu motor. They are connected through a 138kV transmission line (j19.5Ω). The generator has a step-up Δ-Y transformer (j0.1pu) and the motor a step-down Y-Δ transformer (j0.1pu).

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A:

We now can consider $E_f = 1.05\angle 0° \, pu$

Also, $\boldsymbol{I}_g^{pre} = 0.952\angle -18.2°$ and $\boldsymbol{I}_m^{pre} = -0.952\angle -18.2° \, pu$

Example 1 using Superposition

A:

Solving for the fault increment circuit, we get

$$\Delta \boldsymbol{I}_g = \frac{\boldsymbol{E}_f}{j0.15} = \frac{1.05\angle 0°}{j0.15} = -j7\ pu$$

$$\Delta \boldsymbol{I}_m = \frac{\boldsymbol{E}_f}{j0.5} = \frac{1.05\angle 0°}{j0.5} = -j2.1\ pu$$

$$\boldsymbol{I}_f = -j7 - j2.1 = -j9.1\ pu$$

This matches what we calculated earlier

Superposition | Extension to Larger Systems

The superposition approach can be easily extended to larger systems.

Using the $\boldsymbol{Y}_{bus}^{fault}$, we have $\boldsymbol{Y}_{bus}^{fault}\Delta\boldsymbol{V} = \Delta\boldsymbol{I}$

For the fault-increment system, $\Delta\boldsymbol{I}$ is all zeros except at the fault location:

$$\Delta\boldsymbol{I} = \begin{bmatrix} 0 \\ \vdots \\ -\boldsymbol{I}_f \\ \vdots \\ 0 \end{bmatrix}$$

Superposition | Extension to Larger Systems

Define the fault-defining bus impedance matrix $\boldsymbol{Z}_{bus}^{fault}$ as

$$\boldsymbol{Z}_{bus}^{fault} = \left(\boldsymbol{Y}_{bus}^{fault}\right)^{-1} \qquad \Delta\boldsymbol{V} = \boldsymbol{Z}_{bus}^{fault}\Delta\boldsymbol{I}$$

Then, for the fault-increment system:

$$\begin{bmatrix} \boldsymbol{Z}_{11}^{fault} & \cdots & \boldsymbol{Z}_{1n}^{fault} \\ \vdots & \ddots & \vdots \\ \boldsymbol{Z}_{n1}^{fault} & \cdots & \boldsymbol{Z}_{nn}^{fault} \end{bmatrix} \begin{bmatrix} \vdots \\ 0 \\ -\boldsymbol{I}_f \\ 0 \\ \vdots \end{bmatrix} = \Delta\boldsymbol{V}$$

For a fault at bus i, we get:

$$-\boldsymbol{I}_f\boldsymbol{Z}_{ii}^{fault} = \Delta\boldsymbol{V}_i = -\boldsymbol{V}_i^{pre}$$

Thus:

$$\boldsymbol{I}_f = \frac{\boldsymbol{V}_i^{pre}}{\boldsymbol{Z}_{ii}^{fault}}$$

Superposition

1. Find the pre-fault bus voltage vector: $\boldsymbol{V}^{pre}$
2. Find $\boldsymbol{Z}_{bus}^{fault} = \left(\boldsymbol{Y}_{bus}^{fault}\right)^{-1}$
3. Calculate the fault current:

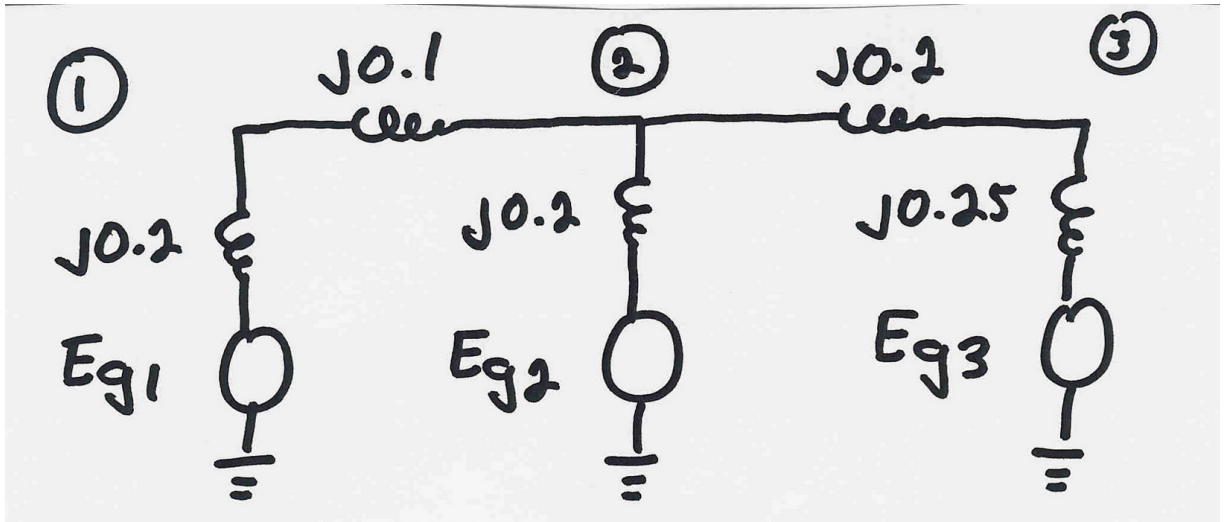
$$\boldsymbol{I}_f = \frac{\boldsymbol{V}_k^{pre}}{\boldsymbol{Z}_{kk}^{fault}}$$

4. If needed, calculate voltages: $\boldsymbol{V} = \boldsymbol{V}^{pre} - \boldsymbol{Z}_{bus}^{fault} e_k \boldsymbol{I}_f$

e_k : k^{th} standard basis vector: 1 at bus k, 0 elsewhere

$$e_k = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

Example 2 | Three-Generator System using Superposition



For simplicity, assume the system is unloaded before the fault with

$$\boldsymbol{E}_{g1} = \boldsymbol{E}_{g2} = \boldsymbol{E}_{g3} = 1.05\angle 0°\ pu$$

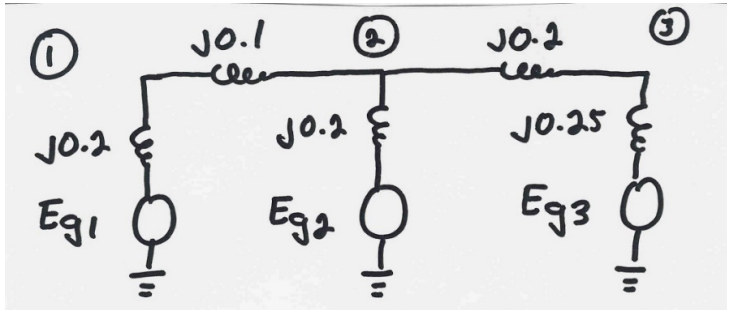
For a fault at bus 1, calculate the fault current and the voltages during the fault.

Example 2 | Three-Generator System using Superposition

A:

First, using the $\boldsymbol{Y}_{bus}^{fault}$, we calculate the $\boldsymbol{Z}_{bus}^{fault}$ for the faulted network

$$\boldsymbol{Y}_{bus}^{fault} = j\begin{bmatrix} -15 & 10 & 0 \\ 10 & -20 & 5 \\ 0 & 5 & -9 \end{bmatrix} pu$$



$$\boldsymbol{Z}_{bus}^{fault} = \left(\boldsymbol{Y}_{bus}^{fault}\right)^{-1} = j\begin{bmatrix} 0.1088 & 0.0632 & 0.0351 \\ 0.0632 & 0.0947 & 0.0526 \\ 0.0351 & 0.0526 & 0.1404 \end{bmatrix} pu$$

Example 2 | Three-Generator System using Superposition

A:

Then, we calculate the fault current at bus 1

$$\boldsymbol{I}_f = \boldsymbol{I}_1 = \frac{\boldsymbol{V}_1^{pre}}{\boldsymbol{Z}_{11}^{fault}} = \frac{1.05}{j0.1088} = -j9.6\ pu$$

Finally, we can compute the voltages during the fault

$$\begin{bmatrix}\Delta\boldsymbol{V}_1\\ \Delta\boldsymbol{V}_2\\ \Delta\boldsymbol{V}_3\end{bmatrix} = j\begin{bmatrix}0.1088 & 0.0632 & 0.0351\\ 0.0632 & 0.0947 & 0.0526\\ 0.0351 & 0.0526 & 0.1404\end{bmatrix}\begin{bmatrix}j9.6\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}-1.05\angle 0°\\ -0.606\angle 0°\\ -0.337\angle 0°\end{bmatrix} pu$$

$$\begin{bmatrix}\boldsymbol{V}_1\\ \boldsymbol{V}_2\\ \boldsymbol{V}_3\end{bmatrix} = \begin{bmatrix}1.05\angle 0°\\ 1.05\angle 0°\\ 1.05\angle 0°\end{bmatrix} + \begin{bmatrix}-1.05\angle 0°\\ -0.606\angle 0°\\ -0.337\angle 0°\end{bmatrix} = \begin{bmatrix}0\angle 0°\\ 0.444\angle 0°\\ 0.713\angle 0°\end{bmatrix} pu$$

Some faults are balanced 3-phase...



2016 South Australian Blackout
Source: News.com.au ; News Corp Australia

but most are not.

How do we deal with unbalanced faults?

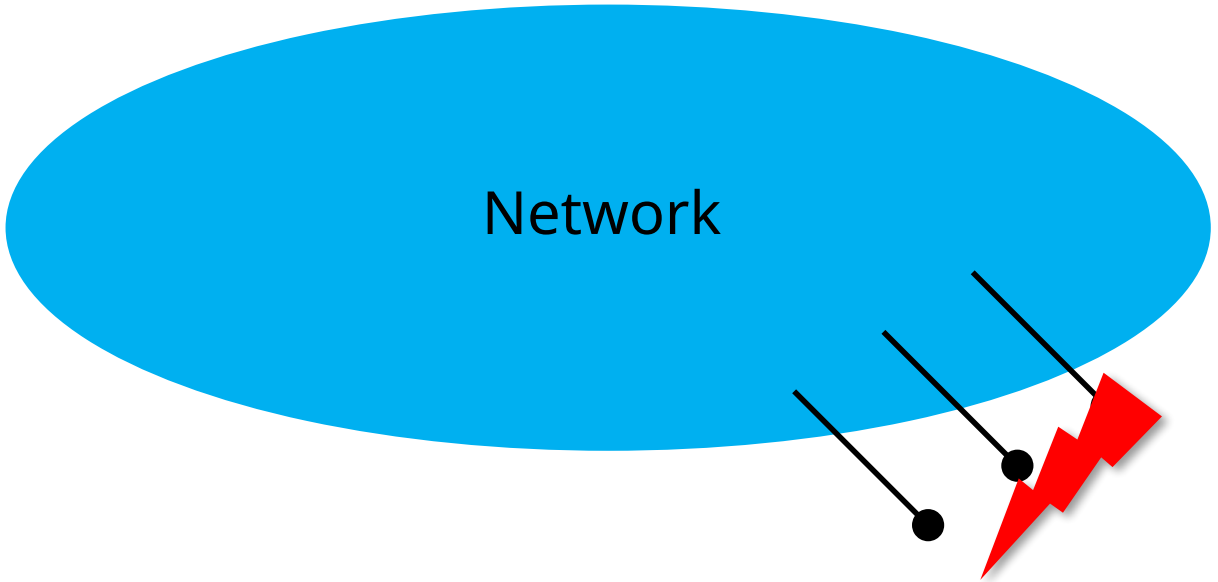
Outline: Dealing With Unbalanced Faults

1. Balanced and Unbalanced Currents
2. Symmetrical Components
3. Currents and Voltages in Symmetrical Components
4. Sequence Networks
5. Sequence Circuits of Transformers and Generators
6. Modelling Faults

Note:

- Special thanks to Prof. Ochoa and Dr. Robin Preece (Univ. of Manchester)
- “Power System Analysis & Design” by Glover, Sarma and Overbye

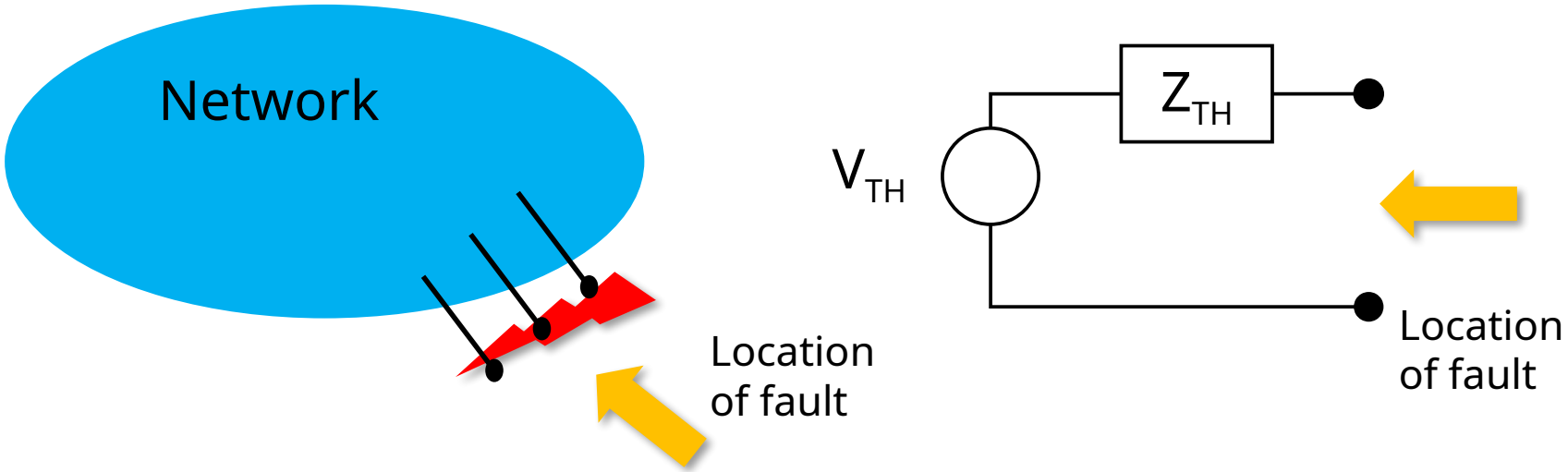
Abstraction:



Pre fault: Current flows around intact network.

During fault: Current flows from the network into the ground (earth) or into another phase of the network.

To calculate fault currents, we need a model of the network from the POV of the fault.



For balanced faults, the entire network is replaced by its Thevenin equivalent.

Fault does not destroy the symmetry of the system, so a single phase-equivalent representation is sufficient.

That single-phase equivalent is the positive sequence model, which we will learn about next.

The most common type of fault on a three-phase system is single line-to-ground,
followed by the line-to-line faults, double line-to-ground faults, and balanced three-phase faults.

Not a good idea

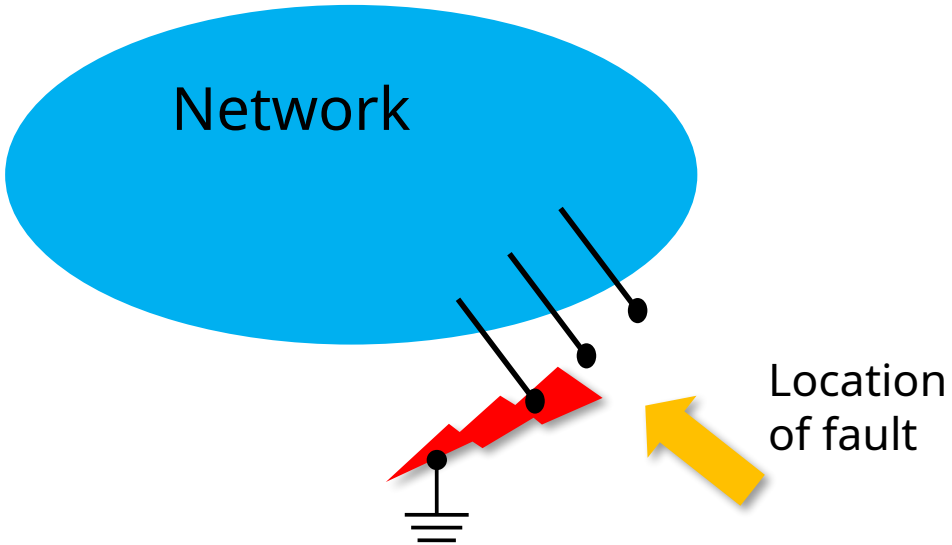




Meanwhile, on Reddit







For unbalanced faults, we no longer have a symmetrical system.

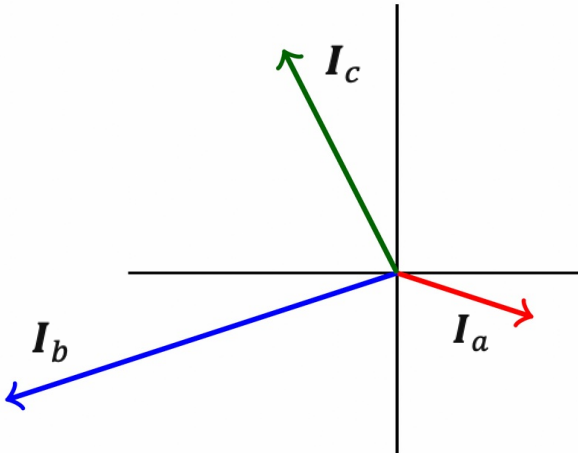
Thus, a single positive-sequence equivalent model no longer works.

We have to look at what happens in each phase.

Unbalanced Currents

Unbalanced three-phase systems are difficult to study in phase currents/voltages because what happens on each phase is affected by what happens in the other phases due to the mutual coupling of the power lines.

Solution: Transform the currents or voltages in **symmetrical components**



Symmetrical Components

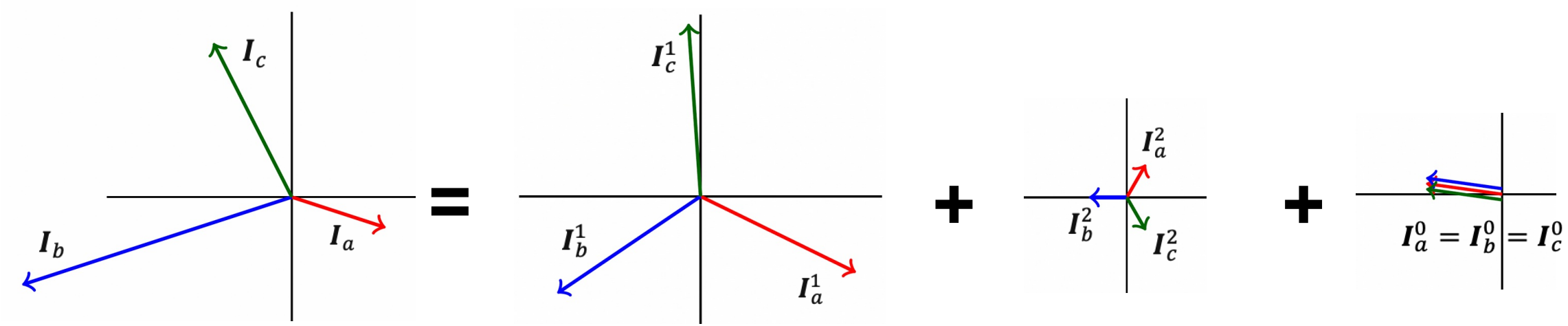
Any set of three-phase currents (or voltages) can be decomposed into the sum of three symmetrical-component sets, each consisting of three phasors with magnitude and phase:

1. **A positive sequence component**
   - Set of three current or voltage phasors of equal magnitude, separated by 120° in the *positive* phase sequence
2. **A negative sequence component**
   - Set of three current or voltage phasors of equal magnitude, separated by 120° in the *negative* phase sequence
3. **A zero sequence component**
   - Set of three current or voltage phasors of equal magnitude and phase

We analyse the network as a set of decoupled symmetrical component systems.

To recover the phase currents (voltages), we sum the symmetrical component currents (voltages).

Symmetrical Components



Three Phase | Positive Sequence | Negative Sequence | Zero Sequence

Can we express this as a matrix-vector product?

$$\begin{aligned} \mathbf{I}_a &= \mathbf{I}_a^0 + \mathbf{I}_a^1 + \mathbf{I}_a^2 \\ \mathbf{I}_b &= \mathbf{I}_b^0 + \mathbf{I}_b^1 + \mathbf{I}_b^2 \\ \mathbf{I}_c &= \mathbf{I}_c^0 + \mathbf{I}_c^1 + \mathbf{I}_c^2 \end{aligned}$$

The Rotation Operator $\boldsymbol{\alpha}$

The operator $\boldsymbol{\alpha}$ rotates a phasor by 120°

In polar notation: $\boldsymbol{\alpha} = 1\angle 120°$

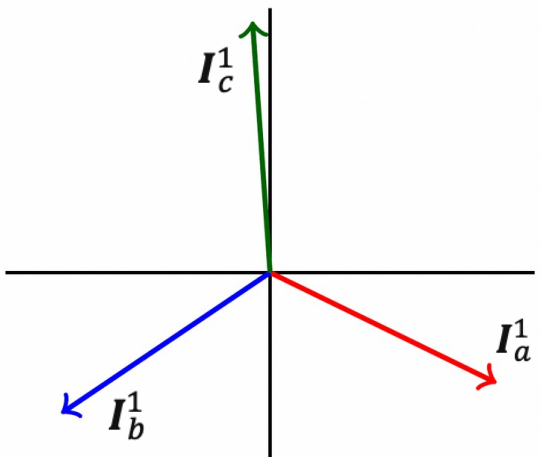
Therefore we have:

$$\boldsymbol{\alpha}^2 = 1\angle 240° \quad \Rightarrow \quad \text{Rotation of } 240^\circ$$

$$\boldsymbol{\alpha}^3 = 1\angle 360° = 1\angle 0° \quad \Rightarrow \quad \text{No rotation}$$

and: $1 + \boldsymbol{\alpha} + \boldsymbol{\alpha}^2 = 0$

Positive Sequence



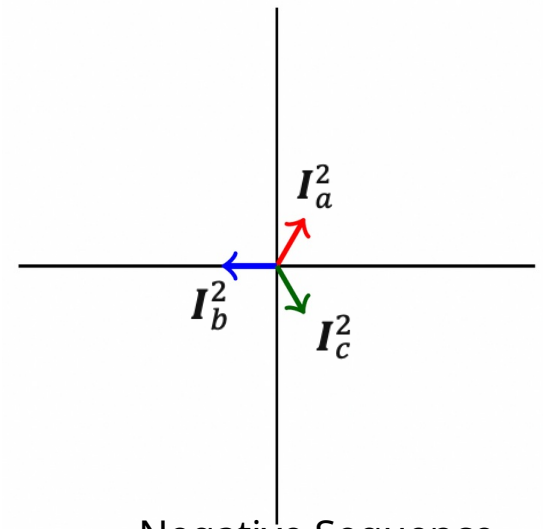
$$I_a^1 = I^1$$

$$I_b^1 = \alpha^2 I_a^1$$

$$I_c^1 = \alpha I_a^1$$

When rotating the phasors counterclockwise, you see $a \rightarrow b \rightarrow c$

Negative Sequence



Negative Sequence

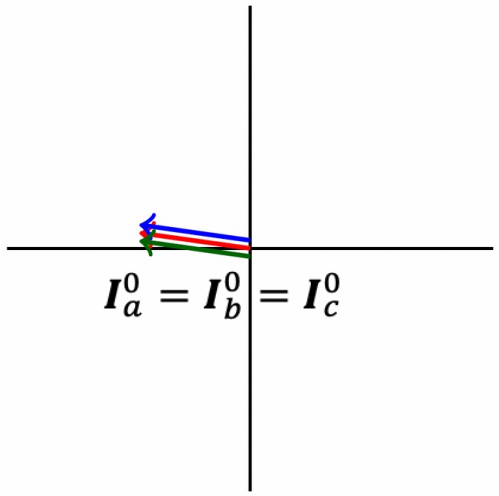
$$I_a^2 = I^2$$

$$I_b^2 = \alpha I_a^2$$

$$I_c^2 = \alpha^2 I_a^2$$

When rotating the phasors counterclockwise, you see $a \rightarrow c \rightarrow b$

Zero Sequence

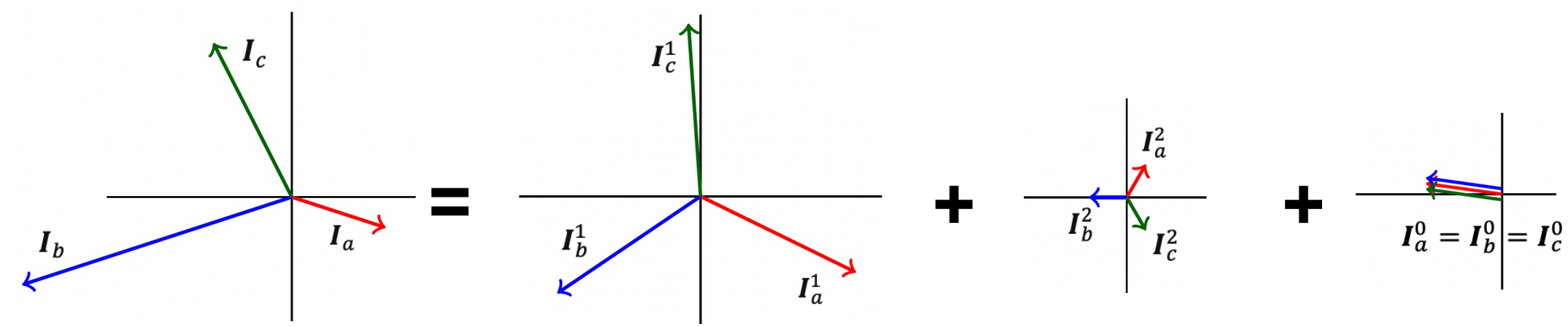


Zero Sequence

$$I_a^0 = I_b^0 = I_c^0 = I^0$$

When rotating the phasors counterclockwise, you see a, b, c at once

Symmetrical Components



Three Phase | Positive Sequence | Negative Sequence | Zero Sequence

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix} \Leftrightarrow \begin{aligned} \mathbf{I}_a &= \mathbf{I}^0 + \mathbf{I}^1 + \mathbf{I}^2 \\ \mathbf{I}_b &= \mathbf{I}^0 + \alpha^2\mathbf{I}^1 + \alpha\mathbf{I}^2 \\ \mathbf{I}_c &= \mathbf{I}^0 + \alpha\mathbf{I}^1 + \alpha^2\mathbf{I}^2 \end{aligned} \Leftrightarrow \begin{aligned} \mathbf{I}_a &= \mathbf{I}_a^0 + \mathbf{I}_a^1 + \mathbf{I}_a^2 \\ \mathbf{I}_b &= \mathbf{I}_b^0 + \mathbf{I}_b^1 + \mathbf{I}_b^2 \\ \mathbf{I}_c &= \mathbf{I}_c^0 + \mathbf{I}_c^1 + \mathbf{I}_c^2 \end{aligned}$$

Symmetrical components Phase components

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}}_{A} \begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$AI_s = I_p$$

$$I_s = A^{-1} I_p$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Check this!

$$\begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix} = \underbrace{\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}}_{A^{-1}} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Currents and Voltages in Symmetrical Components

The transformation applies to voltages as well as currents

$$\boldsymbol{A I_s = I_p} \quad \Longleftrightarrow \quad \boldsymbol{I_s = A^{-1}\, I_p}$$

$$\boldsymbol{A V_s = V_p} \quad \Longleftrightarrow \quad \boldsymbol{V_s = A^{-1} V_p}$$

Symmetrical Components Example 1

Determine the symmetrical components of the following line currents:

$$I_a = 5 \angle 90° \text{ pu}$$
$$I_b = 5 \angle 340° \text{ pu}$$
$$I_c = 5 \angle 200° \text{ pu}$$

$$\boldsymbol{I_s = A^{-1} I_p}$$

$$\begin{bmatrix} \boldsymbol{I^0} \\ \boldsymbol{I^1} \\ \boldsymbol{I^2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{\alpha} & \boldsymbol{\alpha^2} \\ 1 & \boldsymbol{\alpha^2} & \boldsymbol{\alpha} \end{bmatrix} \begin{bmatrix} 5\angle 90° \\ 5\angle 340° \\ 5\angle 200° \end{bmatrix}$$

$$= \frac{5}{3} \begin{bmatrix} 1\angle 90° + 1\angle 340° + 1\angle 200° \\ 1\angle 90° + 1\angle 100° + 1\angle 80° \\ 1\angle 90° + 1\angle 220° + 1\angle 320° \end{bmatrix}$$

$$= \frac{5}{3} \begin{bmatrix} 0 + j0.3 \\ 0 + j3.0 \\ 0 - j0.3 \end{bmatrix} = \begin{bmatrix} 0.5\angle +90° \\ 4.9\angle +90° \\ 0.5\angle -90° \end{bmatrix}$$

Symmetrical Components Example 2

Determine the phase voltage corresponding to the symmetrical components:

$$V^0 = 20 \angle 80° \text{ pu}$$
$$V^1 = 100 \angle 0° \text{ pu}$$
$$V^2 = 30 \angle 180° \text{ pu}$$

$$\boldsymbol{V}_p = \boldsymbol{A}\boldsymbol{V}_s$$

$$\begin{bmatrix} \boldsymbol{V}_a \\ \boldsymbol{V}_b \\ \boldsymbol{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{\alpha}^2 & \boldsymbol{\alpha} \\ 1 & \boldsymbol{\alpha} & \boldsymbol{\alpha}^2 \end{bmatrix} \begin{bmatrix} 20\angle 80° \\ 100\angle 0° \\ 30\angle 180° \end{bmatrix}$$

$$= \begin{bmatrix} 20\angle 80° + 100\angle 0° + 30\angle 180° \\ 20\angle 80° + 100\angle 240° + 30\angle 300° \\ 20\angle 80° + 100\angle 120° + 30\angle 60° \end{bmatrix}$$

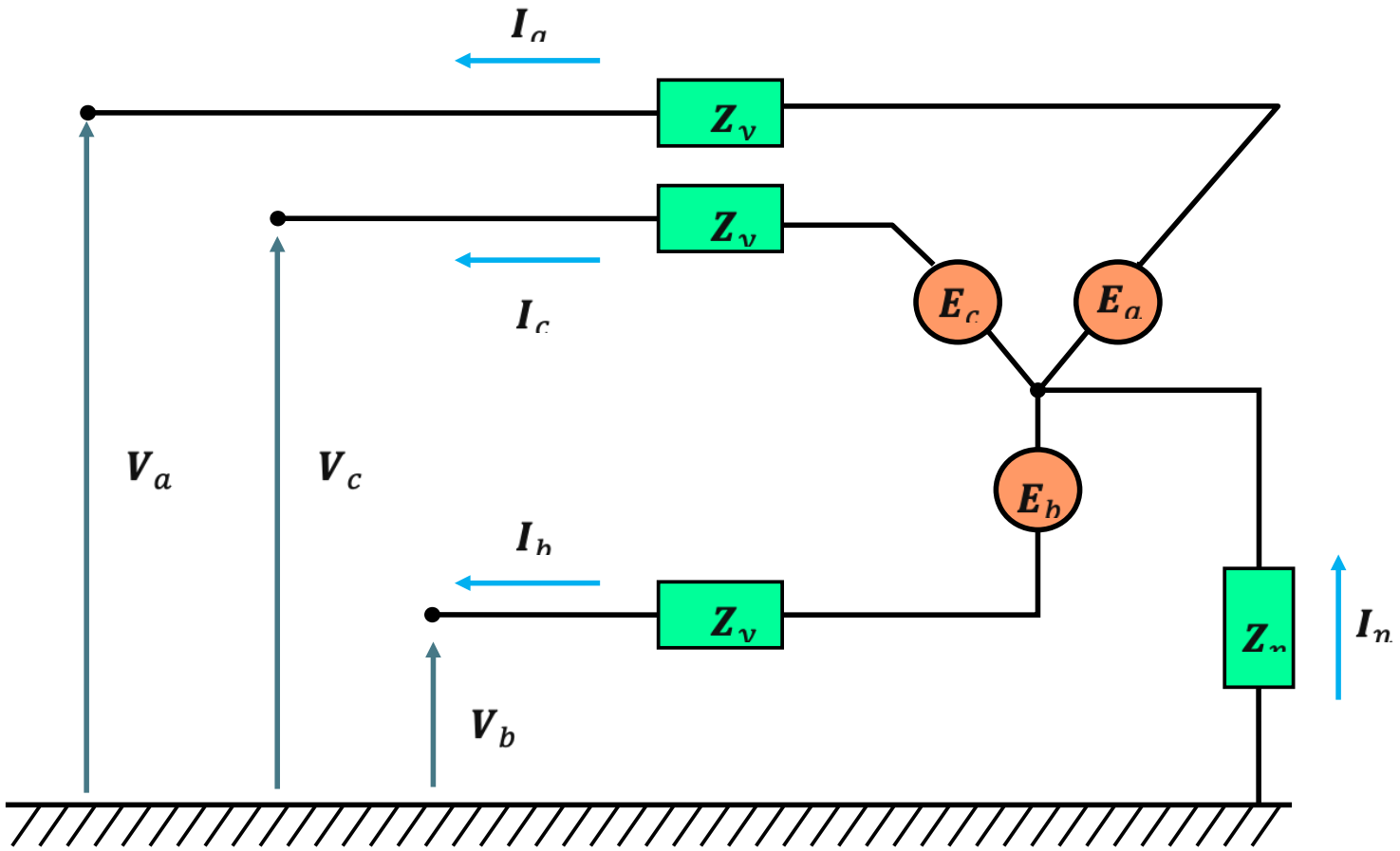
$$= \begin{bmatrix} 74 + j20 \\ -32 - j93 \\ -32 + j132 \end{bmatrix} = \begin{bmatrix} 76\angle 15.01° \\ 98\angle 251.3° \\ 136\angle 103.4° \end{bmatrix}$$

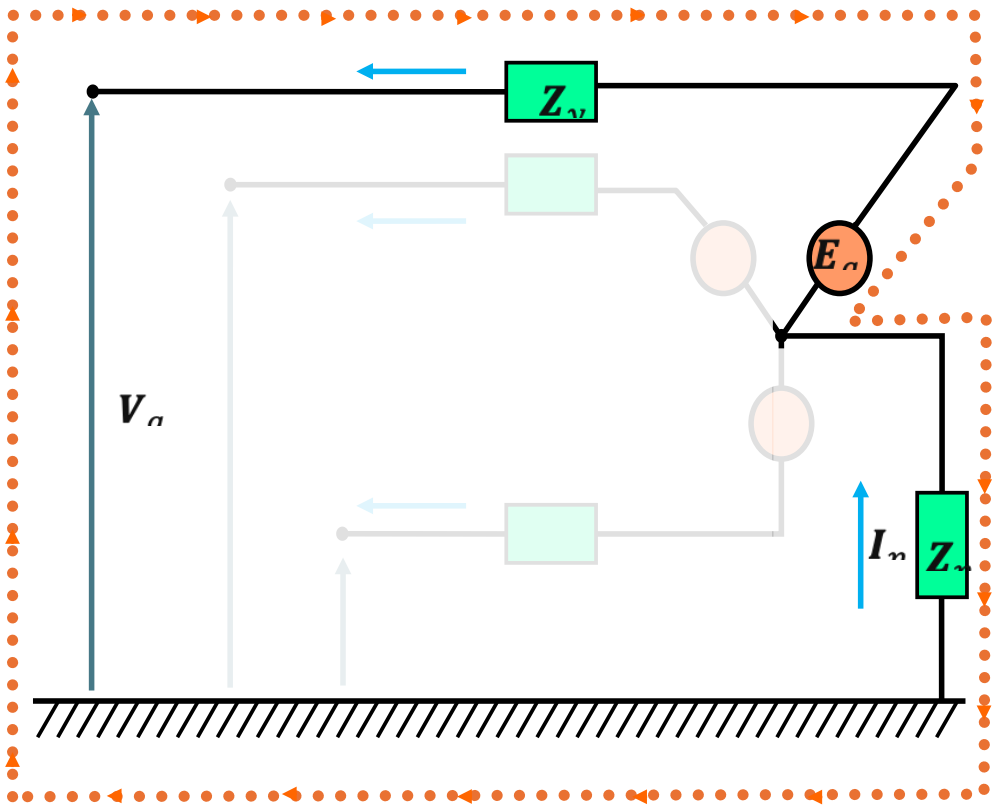
Now, how do we integrate sequence with fault analysis?

We need the three-phase coupling impedance matrix $\boldsymbol{Z}_p$.

For Y-connections with a neutral/ground connection, $\boldsymbol{Z}_p$ includes the neutral/ground impedance.

Phase representation of a three-phase active network:





$$V_a = E_a - Z_y I_a - Z_n I_n$$

$$= E_a - Z_y I_a - Z_n (I_a + I_b + I_c)$$

$$= E_a - (Z_y + Z_n) I_a - Z_n I_b - Z_n I_c$$

V_a includes I_b and I_c

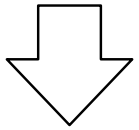
This is called (interphase) coupling and it makes modelling harder!

Three-Phase Ohm's Law

$$V_a = E_a - (Z_y + Z_n) I_a - Z_n I_b - Z_n I_c$$

$$V_b = E_b - Z_n I_a - (Z_y + Z_n) I_b - Z_n I_c$$

$$V_c = E_c - Z_n I_a - Z_n I_b - (Z_y + Z_n) I_c$$



Full matrix Coupling!

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} - \underbrace{\begin{bmatrix} Z_y + Z_n & Z_n & Z_n \\ Z_n & Z_y + Z_n & Z_n \\ Z_n & Z_n & Z_y + Z_n \end{bmatrix}}_{Z_p} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$V_p = E_p - Z_p I_p$$

Three-Phase Ohm's Law

Ohm's Law in phase coordinates:

$$\boldsymbol{V}_p = \boldsymbol{E}_p - \boldsymbol{Z}_p \boldsymbol{I}_p$$

$$\boldsymbol{V}_p = \boldsymbol{A}\boldsymbol{V}_s \qquad \boldsymbol{I}_p = \boldsymbol{A}\boldsymbol{I}_s$$

$$\boldsymbol{A}\boldsymbol{V}_s = \boldsymbol{E}_p - \boldsymbol{Z}_p \boldsymbol{A}\boldsymbol{I}_s$$

Ohm's Law in sequence coordinates:

$$\boldsymbol{V}_s = \boldsymbol{A}^{-1}\boldsymbol{E}_p - \boldsymbol{A}^{-1}\boldsymbol{Z}_p \boldsymbol{A}\boldsymbol{I}_s$$

The Symmetrical Component Equation

$$\boldsymbol{V_s = A^{-1} E_p - A^{-1} Z_p A I_s}$$

$$\boldsymbol{A^{-1} E_p = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha\end{bmatrix}\begin{bmatrix}E_a\\ E_b\\ E_c\end{bmatrix} = \begin{bmatrix}0\\ E^1\\ 0\end{bmatrix} = E_s}$$

A balanced source has only positive sequence voltage

$$\boldsymbol{A^{-1} Z_p A = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha\end{bmatrix}\begin{bmatrix}Z_y+Z_n & Z_n & Z_n\\ Z_n & Z_y+Z_n & Z_n\\ Z_n & Z_n & Z_y+Z_n\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha^2 & \alpha\\ 1 & \alpha & \alpha^2\end{bmatrix} = \begin{bmatrix}Z_y+3Z_n & 0 & 0\\ 0 & Z_y & 0\\ 0 & 0 & Z_y\end{bmatrix} = Z_s}$$

The sequence matrix is diagonal

→ no coupling between sequence components!

The Symmetrical Component Equation

$$\boldsymbol{V}_s = \boldsymbol{E}_s - \boldsymbol{Z}_s \boldsymbol{I}_s$$

$$\begin{bmatrix} \boldsymbol{V}^0 \\ \boldsymbol{V}^1 \\ \boldsymbol{V}^2 \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{E}^1 \\ 0 \end{bmatrix} - \begin{bmatrix} \boldsymbol{Z}_y + 3\boldsymbol{Z}_n & 0 & 0 \\ 0 & \boldsymbol{Z}_y & 0 \\ 0 & 0 & \boldsymbol{Z}_y \end{bmatrix} \begin{bmatrix} \boldsymbol{I}^0 \\ \boldsymbol{I}^1 \\ \boldsymbol{I}^2 \end{bmatrix}$$

$$\begin{aligned} \mathbf{V}^0 &= -\mathbf{Z}_0 \mathbf{I}^0 \\ \mathbf{V}^1 &= \mathbf{E}^1 - \mathbf{Z}_1 \mathbf{I}^1 \\ \mathbf{V}^2 &= -\mathbf{Z}_2 \mathbf{I}^2 \end{aligned} \quad \text{with} \quad \begin{aligned} \mathbf{Z}_0 &= \mathbf{Z}_y + 3\mathbf{Z}_n \\ \mathbf{Z}_1 &= \mathbf{Z}_y \\ \mathbf{Z}_2 &= \mathbf{Z}_y \end{aligned}$$

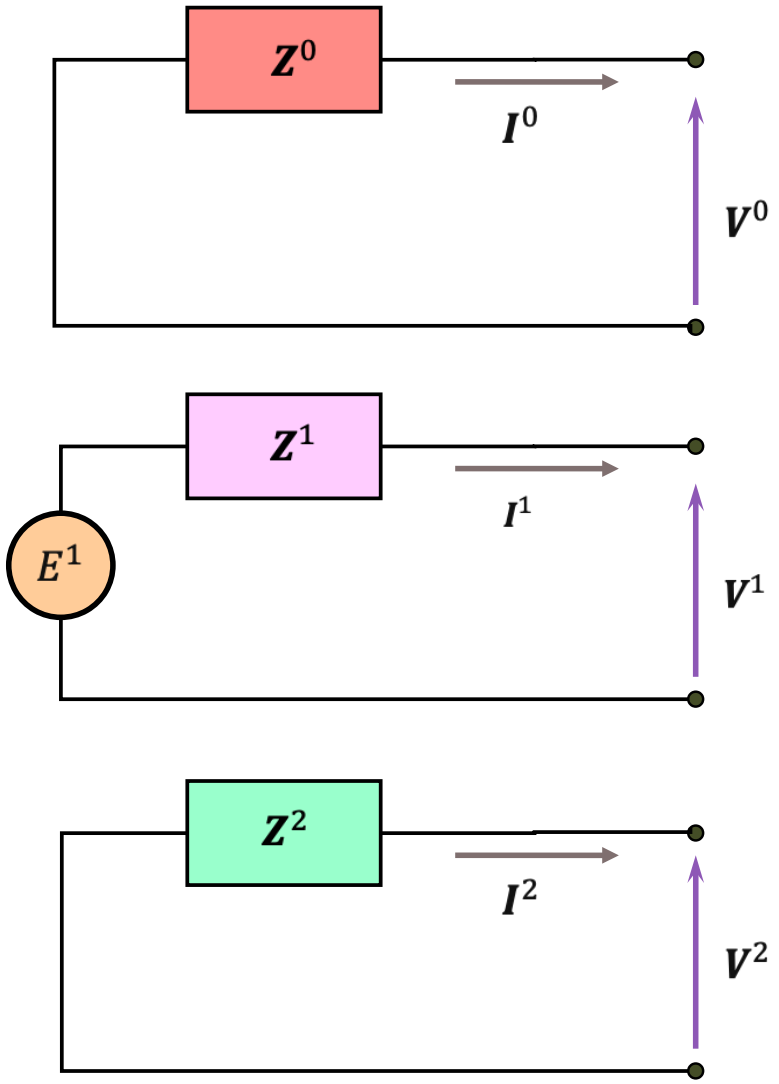
complex-valued scalars (not matrices)

Decoupled Sequence Networks

$$V^0 = -Z^0 I^0$$

$$V^1 = E^1 - Z^1 I^1$$

$$V^2 = -Z^2 I^2$$



Zero sequence circuit

Positive sequence circuit

Negative sequence circuit

Coupling

- Phase circuits are never decoupled (even if there is no neutral current path, there is mutual inductance)
- Sequence circuits are decoupled if the impedances are balanced (this is a common approximation)

Neutral impedance appears only **in zero sequence**

- Positive and negative sequence currents do not flow in the neutral circuit as they are balanced
- No neutral connection is equivalent to $\boldsymbol{Z}_n = \infty \rightarrow \boldsymbol{I}^0 = 0$

Sequence can decompose phase currents and voltages into decoupled sequence components

For faults that just affect one sequence, we can now analyse just that sequence

→ this is what we did for the balanced 3-phase faults, which is a purely positive-sequence fault

What do we do when a fault affects more than one sequence?

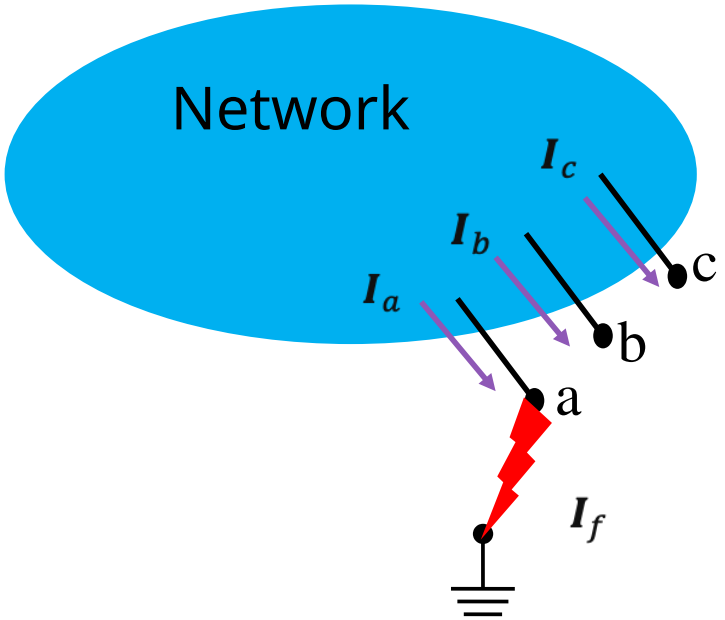
→ need to combine the effect of each sequence

Case 1: Single-Line-to-Ground Fault

Not a good idea



Case 1: Single-Line-to-Ground Fault



$$I_a = I_f \neq 0$$

$$I_b = 0$$

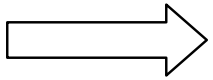
$$I_c = 0$$

Case 1: Single-Line-to-Ground Fault

$$I_a = I_f \neq 0$$

$$I_b = 0$$

$$I_c = 0$$



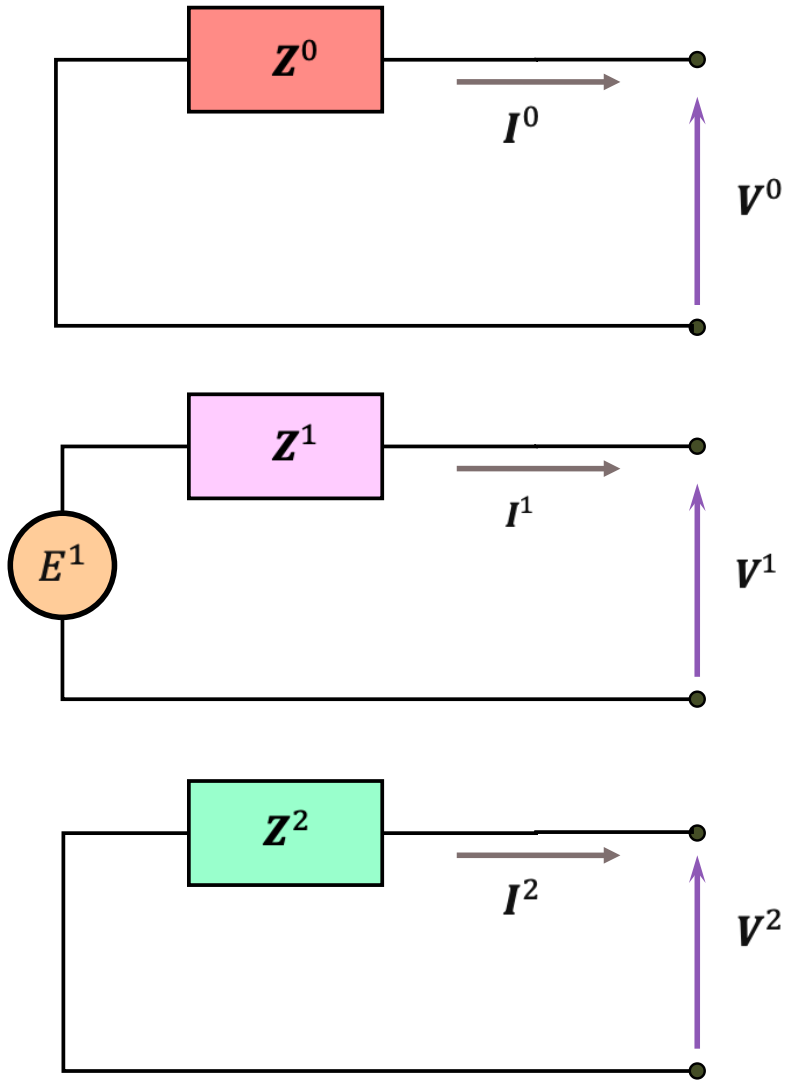
$$\begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} I_a \\ I_a \\ I_a \end{bmatrix}$$

$$I^0 = I^1 = I^2 = \frac{1}{3} I_a$$

Case 1: Single-Line-to-Ground Fault

How do we implement a single-line-to-ground fault with the sequence circuit models?

$$I^0 = I^1 = I^2$$



Case 1: Single-Line-to-Ground Fault

How do we implement a single-line-to-ground fault with the sequence circuit models?

$$I^0 = I^1 = I^2$$

Single line to ground faults are equivalent to connecting the three sequence circuits in series:

The intuition comes from the equation above.

Connecting the circuit this way implies Ohm's Law:

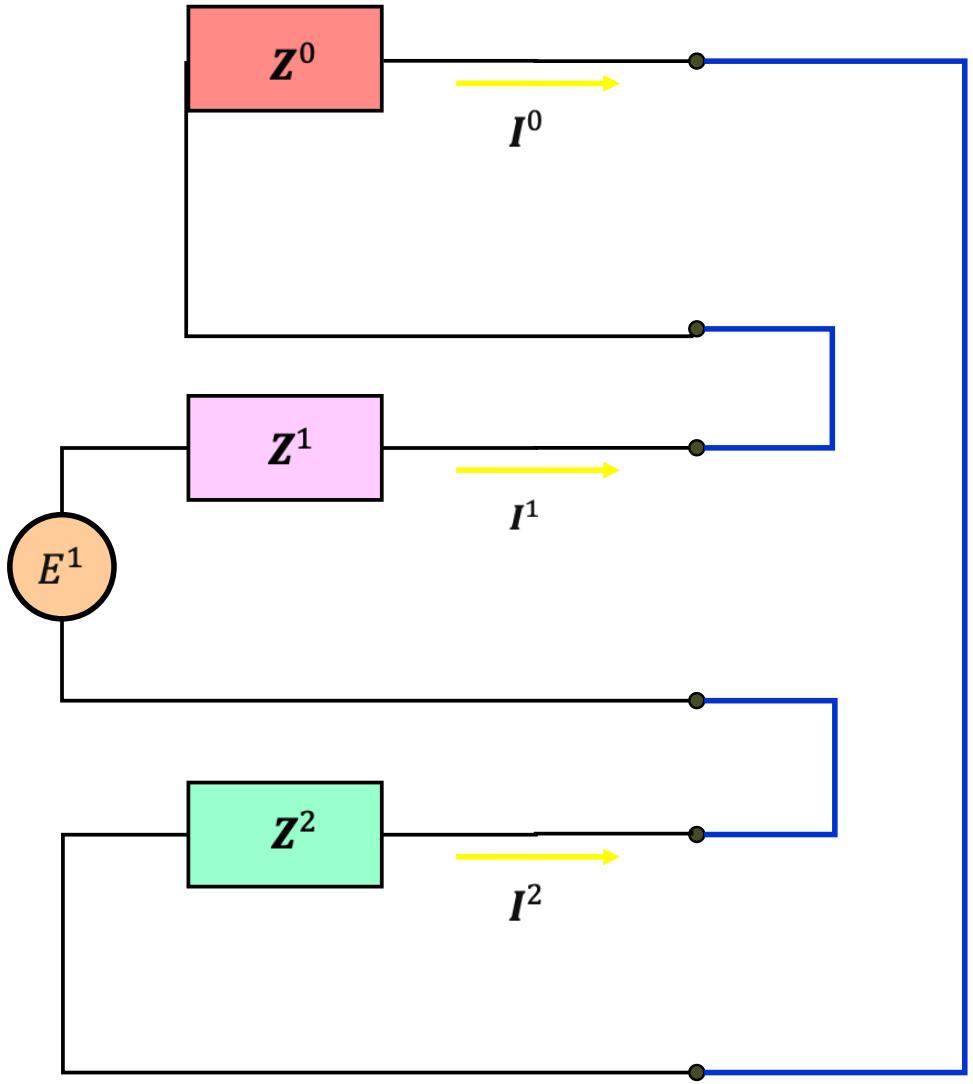
$$I^1 = \frac{E^1}{Z^0 + Z^1 + Z^2}$$

$$\mathbf{Z}_0 = \mathbf{Z}_y + 3\mathbf{Z}_n$$

Where $\mathbf{Z}_1 = \mathbf{Z}_y$ are scalars (not matrices)

$$\mathbf{Z}_2 = \mathbf{Z}_y$$

How do we know that we can do this?



Why do we connect the components in series?

$$\begin{bmatrix}\mathbf{V}_a\\ \mathbf{V}_b\\ \mathbf{V}_c\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\\ 1 & \boldsymbol{\alpha}^2 & \boldsymbol{\alpha}\\ 1 & \boldsymbol{\alpha} & \boldsymbol{\alpha}^2\end{bmatrix}\begin{bmatrix}\mathbf{V}^0\\ \mathbf{V}^1\\ \mathbf{V}^2\end{bmatrix}$$

There is no voltage drop from phase a to ground: $\boldsymbol{V}_a=\boldsymbol{V}^0+\boldsymbol{V}^1+\boldsymbol{V}^2=0$

The sequence-circuit voltages are: $\boldsymbol{V}^1=\boldsymbol{E}^1-\boldsymbol{Z}^1\boldsymbol{I}^1,\qquad \boldsymbol{V}^2=-\boldsymbol{Z}^2\boldsymbol{I}^2,\qquad \boldsymbol{V}^0=-\boldsymbol{Z}^0\boldsymbol{I}^0$

Summing, $0=\boldsymbol{E}^1-(\boldsymbol{Z}^0+\boldsymbol{Z}^1+\boldsymbol{Z}^2)I^1$ because $\boldsymbol{I}^0=\boldsymbol{I}^1=\boldsymbol{I}^2$ and $\boldsymbol{V}^0+\boldsymbol{V}^1+\boldsymbol{V}^2=0$

Thus, $\boldsymbol{I}^1=\frac{\boldsymbol{E}^1}{\boldsymbol{Z}^0+\boldsymbol{Z}^1+\boldsymbol{Z}^2}$

This is the equation you get by connecting the component circuits in series!

Case 1: Single-Line-to-Ground Fault

How do we implement a single-line-to-ground fault with the sequence circuit models?

$$I^0 = I^1 = I^2 = \frac{1}{3} I_a$$

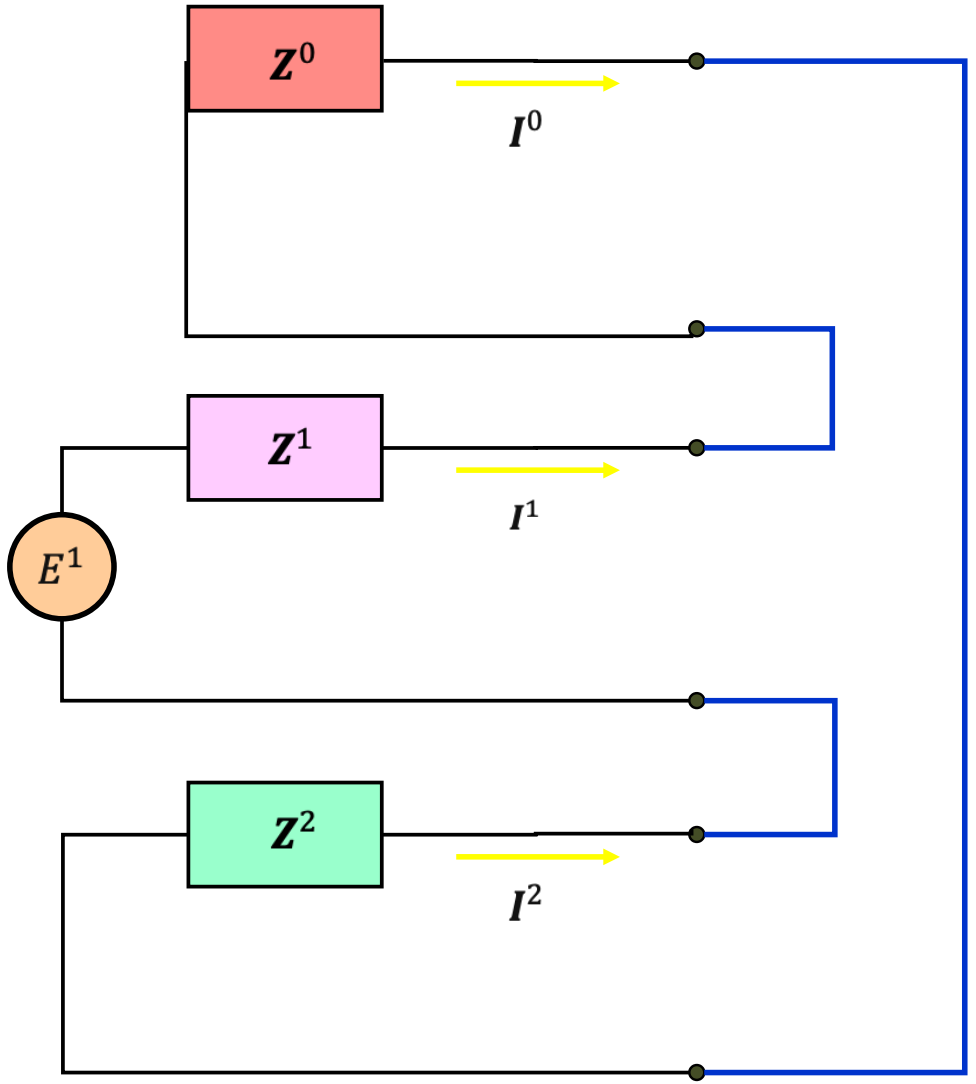
Single line to ground faults are equivalent to connecting the three sequence circuits in series:

And what current flows through the fault?

Phases b and c are not connected: $I_b = I_c = 0$

$$I^0 = I^1 = I^2 = \frac{E^1}{Z^0 + Z^1 + Z^2}$$

$$I_a = \frac{3E^1}{Z^0 + Z^1 + Z^2}$$



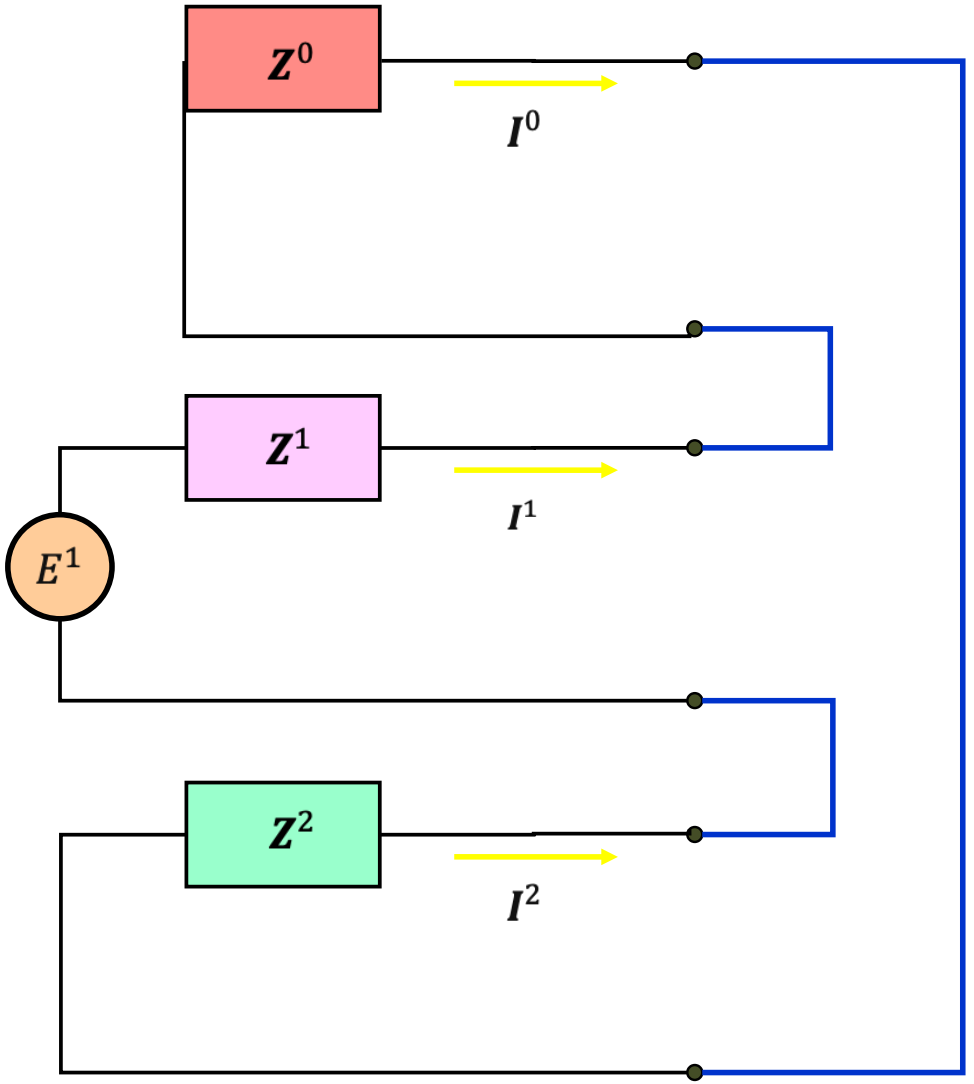
Case 1: Single-Line-to-Ground Fault

How do we implement a single-line-to-ground fault with the sequence circuit models?

$$I^0 = I^1 = I^2 = \frac{1}{3} I_a = \frac{E^1}{Z^0 + Z^1 + Z^2}$$

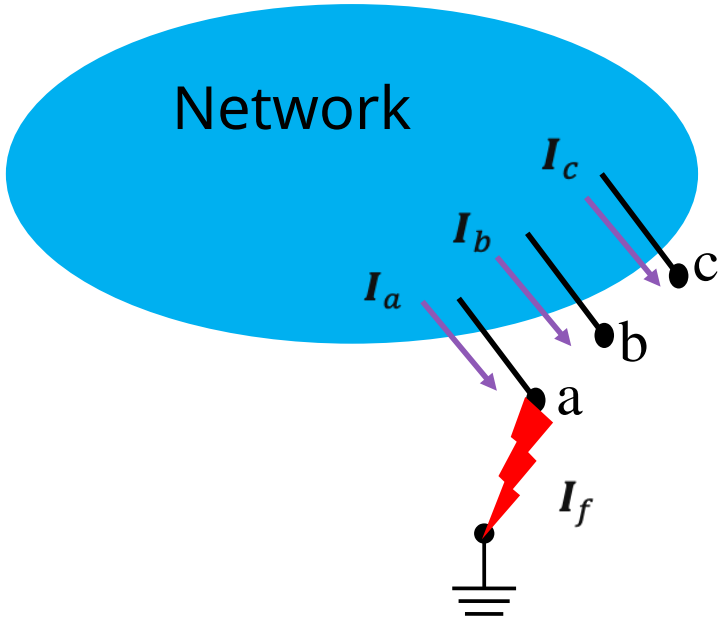
So, *why do we use symmetrical components for single-phase faults?*

We can model the single-phase fault using a scalar equation!



Single-Line-to-Ground Fault: What if $z_0 = \infty$?

What happens if there is no path to ground for the neutral current in the circuit?



$$Z^0 = \infty$$

$$\Rightarrow \quad I_a = \frac{3E^1}{Z^0 + Z^1 + Z^2} = 0$$

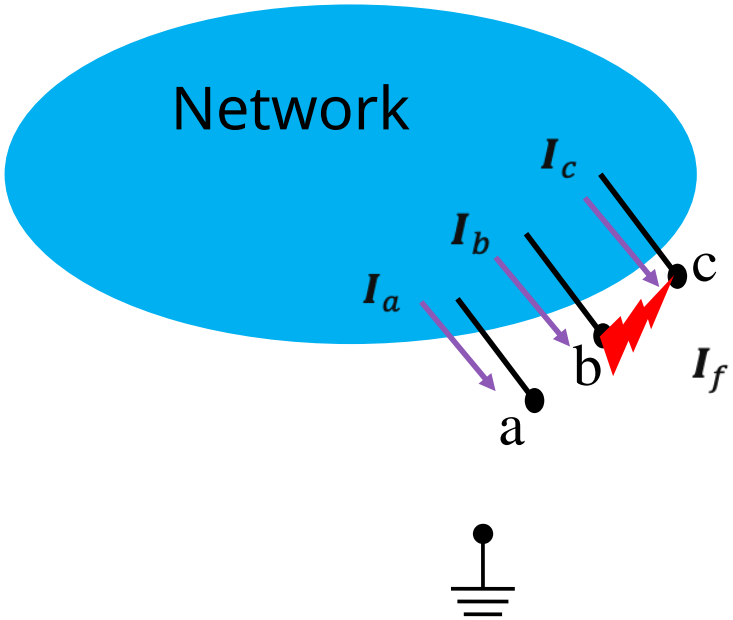
→ Fault current is zero!

Is having zero fault current a good thing or a bad thing?

Case 2: Line-to-Line Fault

Case 2: Line-to-Line Fault





$$I_a = 0$$

$$I_b = I_f$$

$$I_c = -I_f$$

Case 2: Line-to-Line Fault

$$I_a = 0$$

$$I_b = I_f$$

$$I_c = -I_f$$

$$\Rightarrow \quad \begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 0 \\ I_f \\ -I_f \end{bmatrix}$$

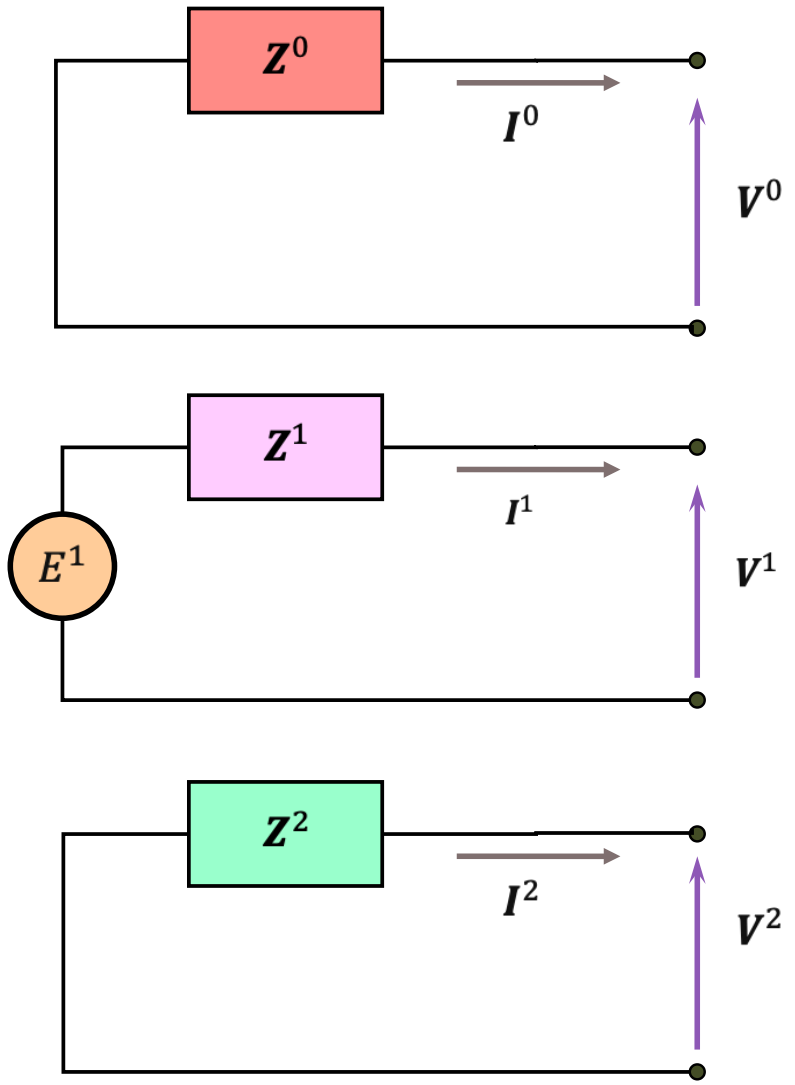
$$\begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 \\ \alpha I_f - \alpha^2 I_f \\ \alpha^2 I_f - \alpha I_f \end{bmatrix}$$

$$I^0 = 0 \qquad I^1 = -I^2$$

Case 2: Line-to-Line Fault

How do we implement a line-to-line fault with the sequence circuit models?

$$I^0 = 0 \qquad I^1 = -I^2$$



Case 2: Line-to-Line Fault

How do we implement a line-to-line fault with the sequence circuit models?

$$I^0 = 0 \qquad I^1 = -I^2$$

Phase-to-phase faults are equivalent to connecting the pos/neg sequence circuits back-to-back:

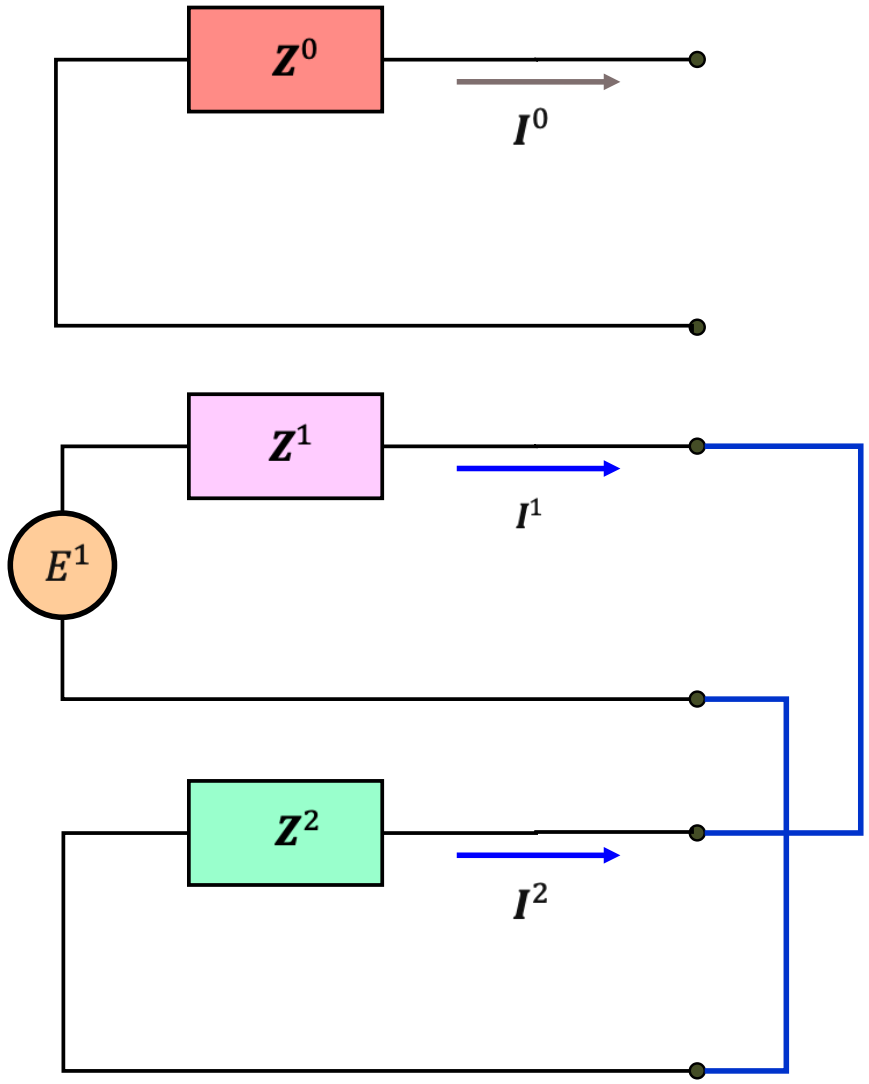
The intuition comes from the equation above.

Connecting the circuit this way implies Ohm's Law:

$$I^1 = \frac{E^1}{Z^1 + Z^2}$$

Where Z^1 and Z^2 are scalars (not matrices)

How do we know that we can do this?



Why do we connect the pos/neg components back-to-back?

For a line-to-line fault, $\mathbf{V}_b = \mathbf{V}_c$

$$\begin{bmatrix}\mathbf{V}_a\\ \mathbf{V}_b\\ \mathbf{V}_c\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & \boldsymbol{\alpha}^2 & \boldsymbol{\alpha}\\ 1 & \boldsymbol{\alpha} & \boldsymbol{\alpha}^2\end{bmatrix}\begin{bmatrix}\mathbf{V}^0\\ \mathbf{V}^1\\ \mathbf{V}^2\end{bmatrix}$$

Also, $\mathbf{V}_b - \mathbf{V}_c = (\boldsymbol{\alpha}^2 - \boldsymbol{\alpha})(\mathbf{V}^1 - \mathbf{V}^2) = 0 \quad \Rightarrow \quad \mathbf{V}^1 = \mathbf{V}^2$

From the circuit equations, $\boldsymbol{V}^1 = \boldsymbol{E}^1 - \boldsymbol{Z}^1\boldsymbol{I}^1, \qquad \boldsymbol{V}^2 = -\boldsymbol{Z}^2\boldsymbol{I}^2$

Setting the voltages equal, $\boldsymbol{E}^1 - \boldsymbol{Z}^1\boldsymbol{I}^1 = -\boldsymbol{Z}^2\boldsymbol{I}^2$

Using $\boldsymbol{I}^2 = -\boldsymbol{I}^1, \quad \boldsymbol{I}^1 = \frac{\boldsymbol{E}^1}{\boldsymbol{Z}^1+\boldsymbol{Z}^2}$

This is the equation you get by connecting the component circuits back-to-back!

Case 2: Line-to-Line Fault

How do we implement a line-to-line fault with the sequence circuit models?

$$I^0 = 0 \qquad I^1 = -I^2 = \frac{E^1}{Z^1 + Z^2}$$

And what current flows through the fault?

Phase a is not connected to the fault: . $I_a = 0$

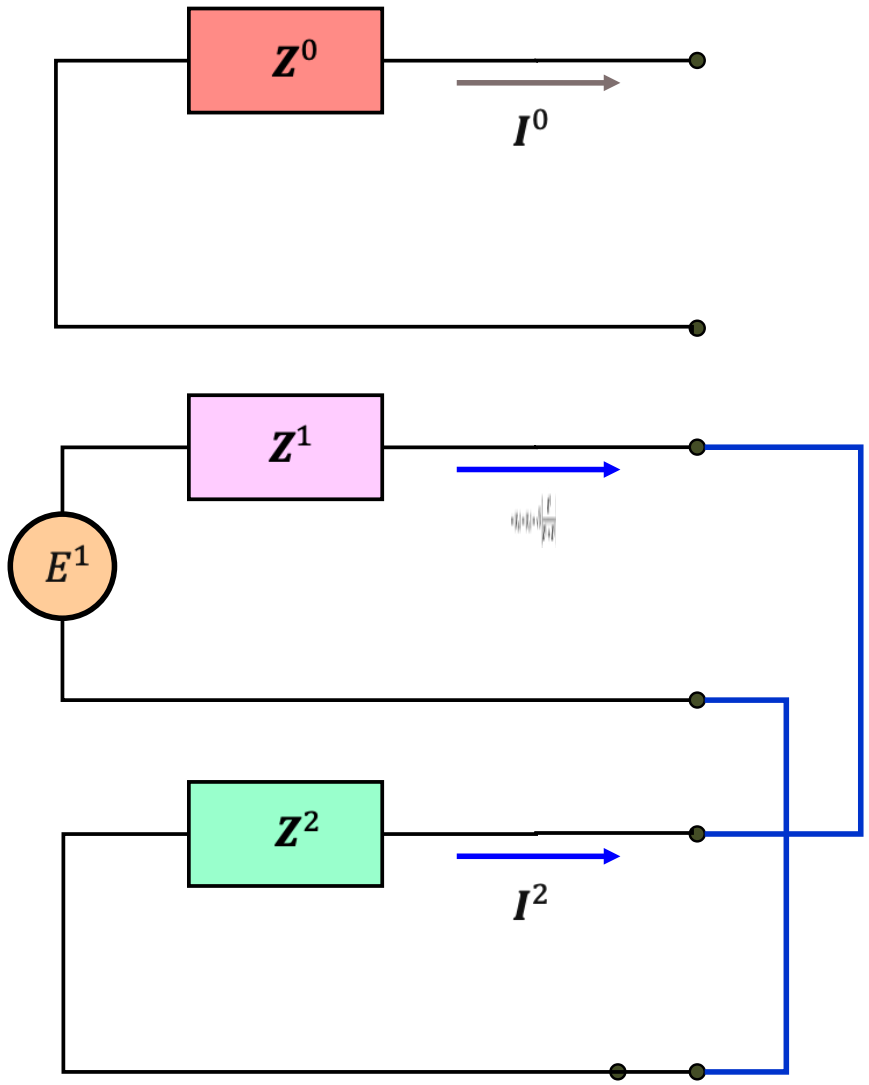
The fault current enters through phase b: $I_b = I_f$

The fault current returns through phase c: $I_c = -I_f$

$$I^1 = \frac{1}{3}\left(\alpha I_f - \alpha^2 I_f\right) = \frac{\alpha - \alpha^2}{3} I_f = \frac{j\sqrt{3}}{3} I_f = \frac{j}{\sqrt{3}} I_f$$

$$|I_f| = \sqrt{3}|I^1|$$

$$_f| = |I_b| = |I_c| = \sqrt{3}\left|\frac{E^1}{Z^1 + Z^2}\right|$$



Case 2: Line-to-Line Fault

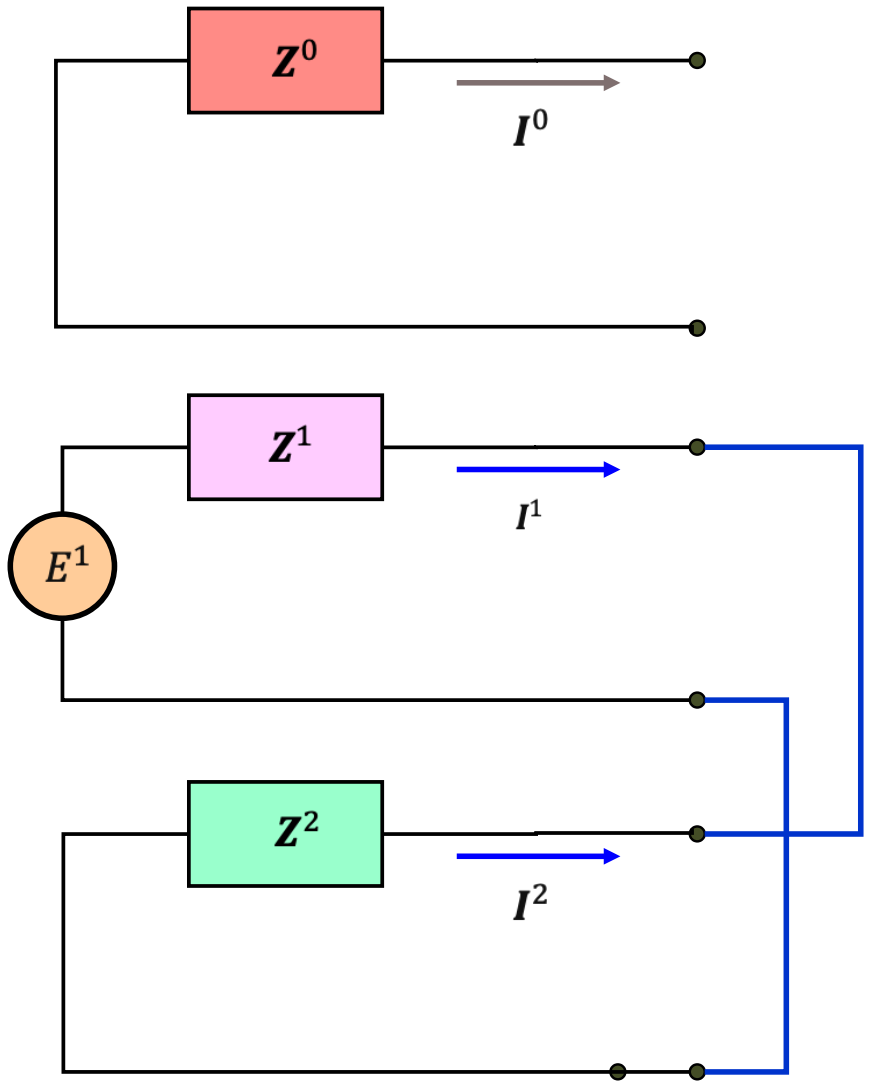
How do we implement a line-to-line fault with the sequence circuit models?

$$I^0 = 0 \qquad I^1 = -I^2 = \frac{E^1}{Z^1 + Z^2}$$

$$|_f| = |I_b| = |I_c| = \sqrt{3}\left|\frac{E^1}{Z^1 + Z^2}\right|$$

So, *why do we use symmetrical components for line-to-line faults?*

We can model the line-to-line fault using a scalar equation!



Sequence can decompose phase currents and voltages into decoupled sequence components

For faults that just affect one sequence, we can now analyse just that sequence

→ this is what we did for the balanced 3-phase faults, which is a purely positive-sequence fault

When a fault affects more than one sequence, we combine the sequences accordingly

Where do we get the sequence impedance models?

→ manufacturers provide them.

Fault Analysis

ELEN90060

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