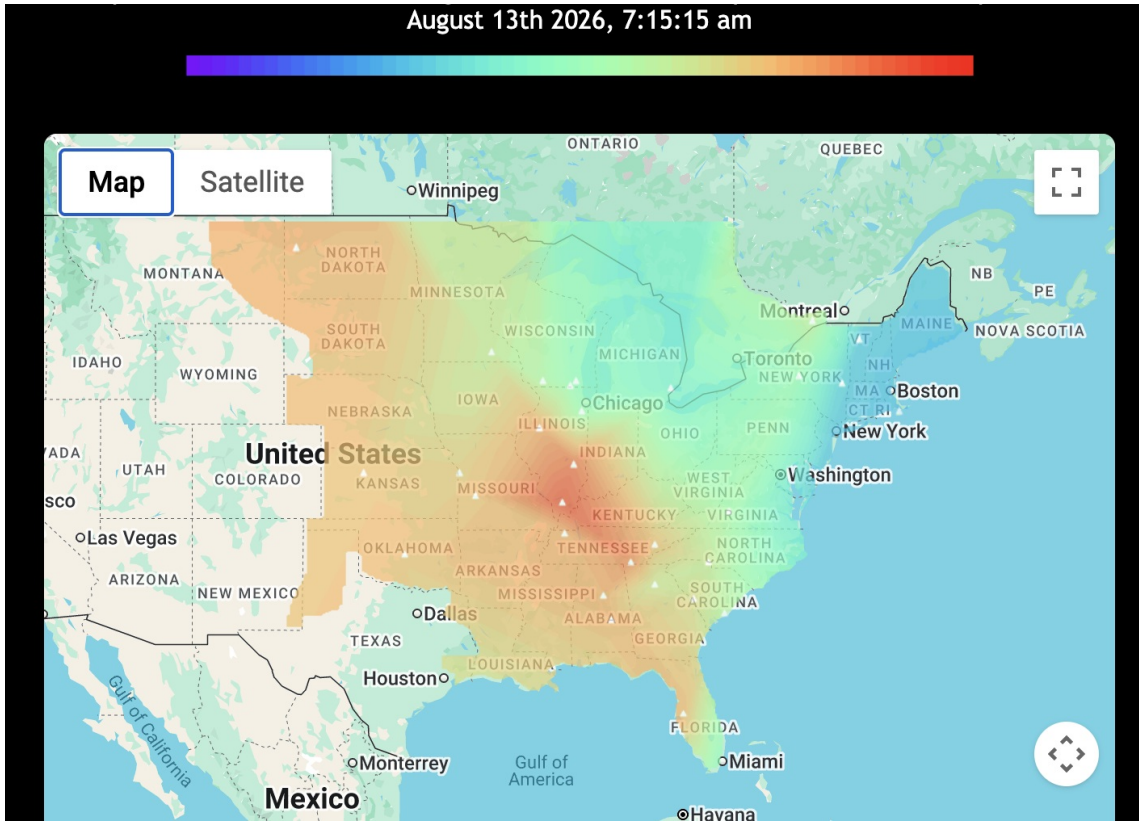
Power Flow Analysis – Part 3

ELEN90060

Keith Moffat



https://fnetpublic.utk.edu/anglecontour.html

Last lecture: can we predict these electricity angles from the power injections?

This lecture: can we *approximately* predict these angles from the power injections using a *linear equation*?

Below is a map of wholesale electricity prices on a winter day in Australia

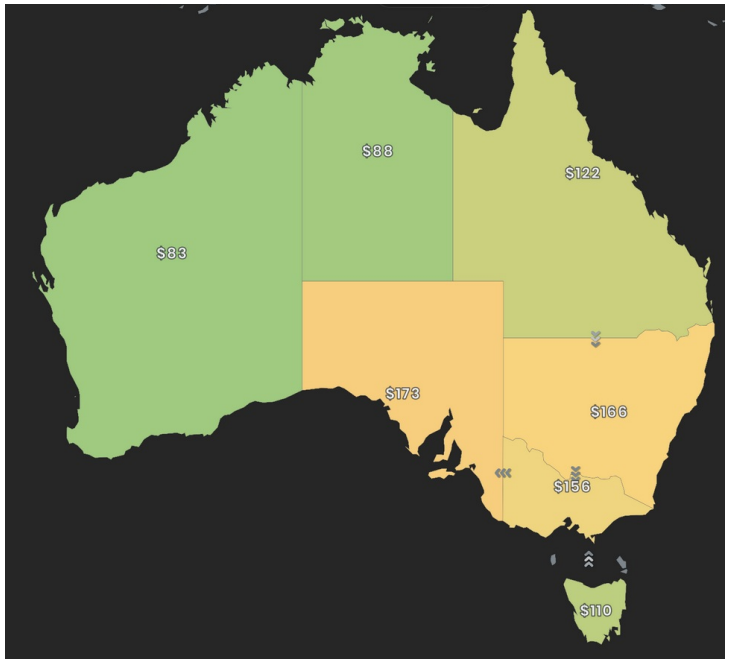
6:00 am



12:00 pm



6:00 pm

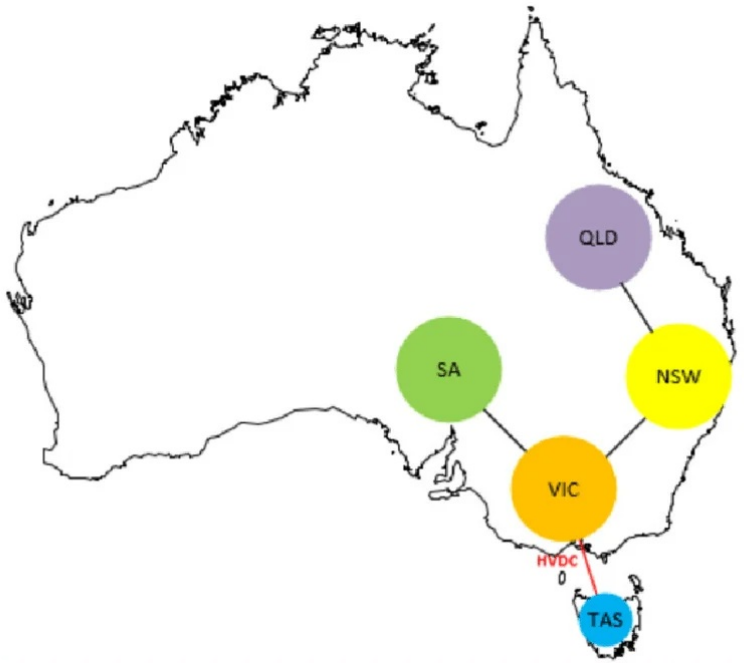


https://app.electricitymaps.com/map/live/fifteen_minutes?signal=electricity-price

The price separation between the states is determined by the flow sensitivities between regions.

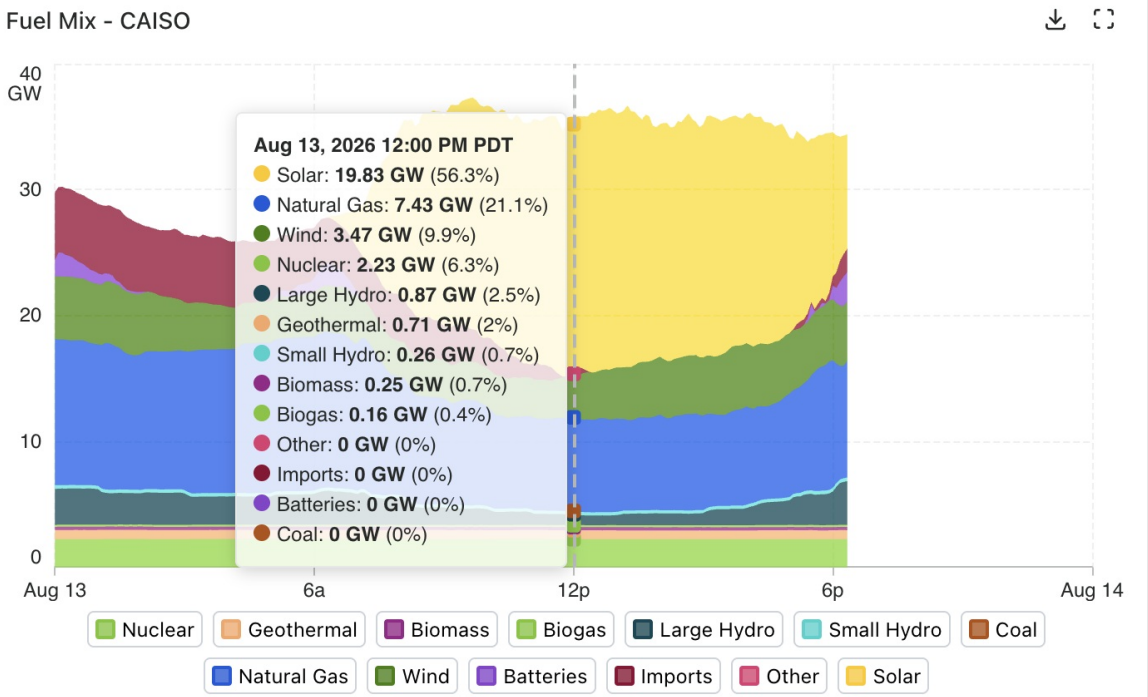
Where do the flow sensitivities come from?

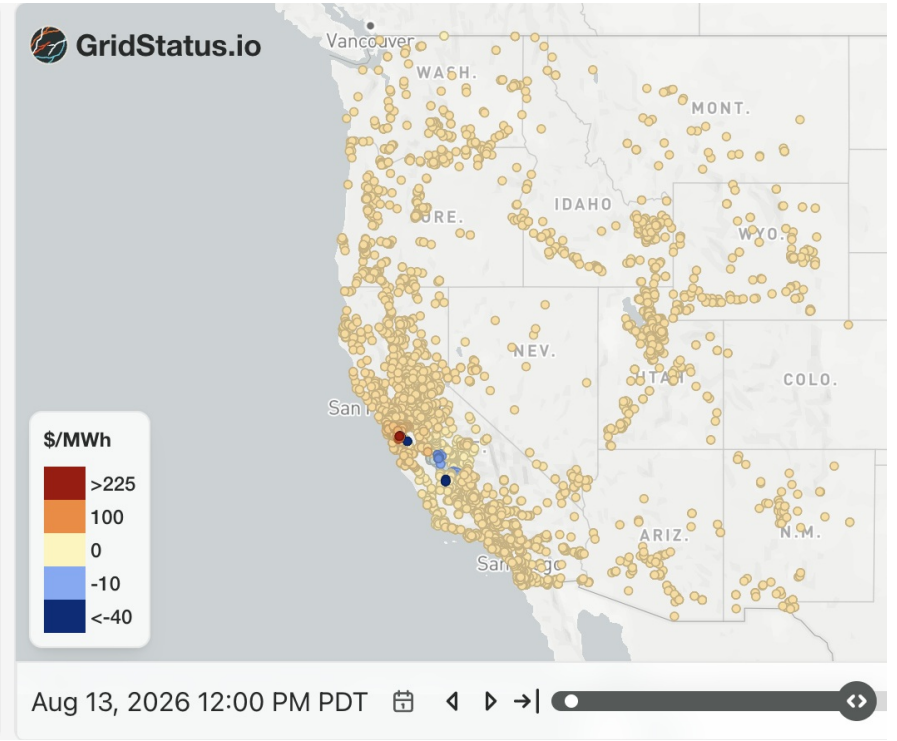
The Eastern-Australian transmission network can be approximated by a 4 node system:

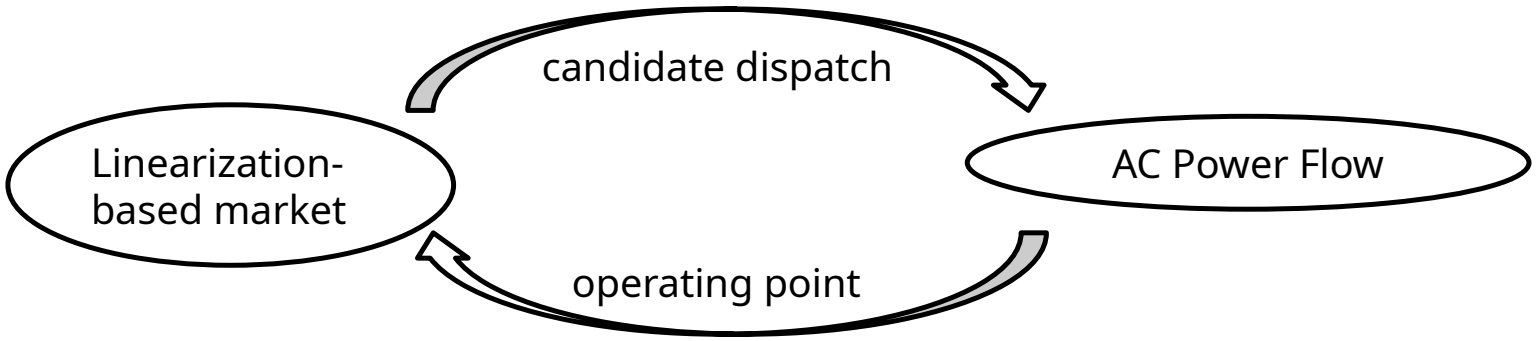


How do we calculate the sensitivities between the 4 nodes?

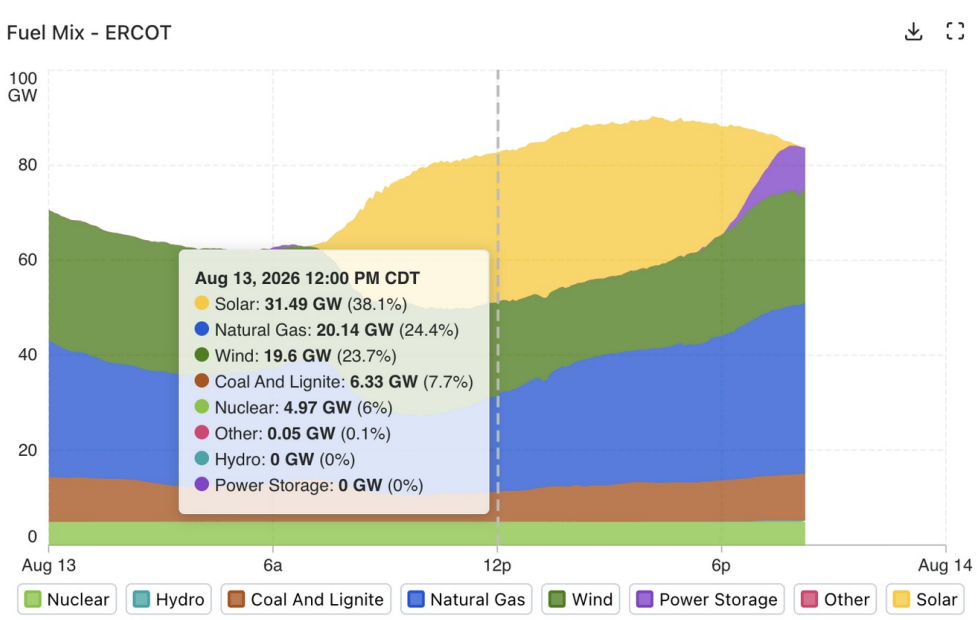
CAISO Locational Marginal Prices

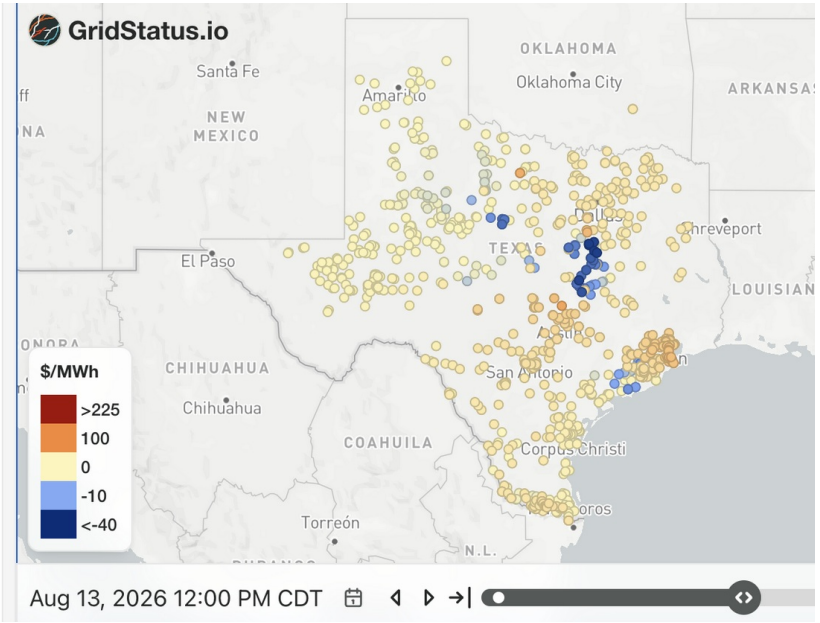


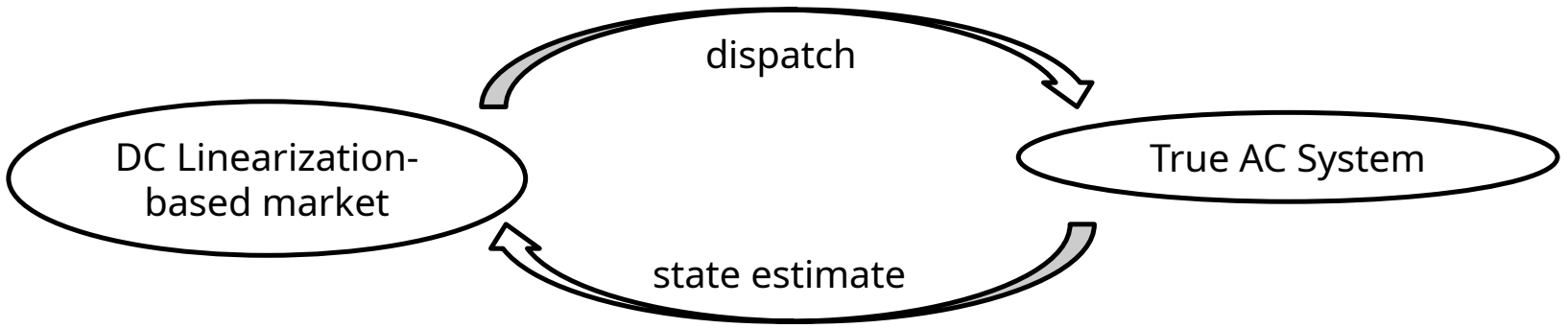




Texas Locational Marginal Prices







Full AC Power Flow		Linearized (e.g., DC) Power Flow
	Attributes	

• System of nonlinear equations
• Provides an “AC feasible” power flow
  • Adheres to system physics
• Iterative numerical solution such as NR
  • Slower, does not always converge
		• System of linear equations
		• Solution is an approximate power flow
		  • Approximates system physics
		• Direct linear solve
		  • Fast, always provides a solution

	Use Cases	
• Verify critical outage cases • Precise connection and network studies • Accurate network losses • AC-feasible generator dispatch		• Quickly screen many outage scenarios • Simulate many planning scenarios • Estimate changes following a small redispatch • Energy markets (always provides a solution!)

Outline

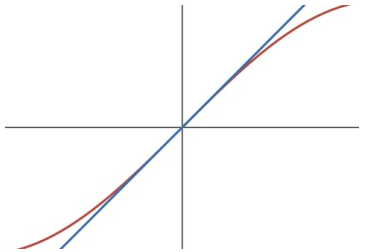
1. DC Power Flow
2. Slack Bus Discussion
3. Decoupled Linearized Power Flow
4. Coupled Linearized Power Flow

DC Power Flow

Interpretation/derivation 1:

Approximately describes the active power/voltage angle relationship, making the following assumptions

- Assumes nominal voltage (1 per unit)
- Assumes the line resistances = 0
- Assumes small angle differences: $sin(\theta_k - \theta_i) \approx \theta_k - \theta_i$
- A single slack bus

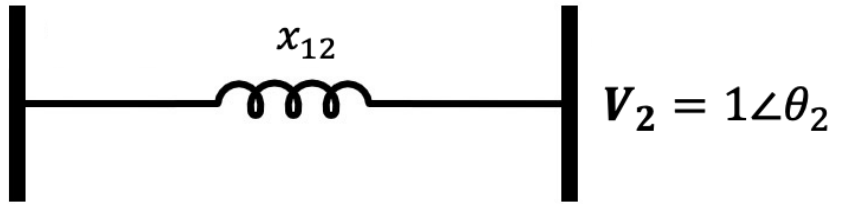


Interpretation/derivation 2:

The active power/voltage angle portion of the power flow linearization at the nominal voltage for a network with no resistance and a single slack bus

(the second interpretation will make sense by the end of this lecture)

Decoupled Power Flow
Two-Bus Transmission System with an Inductive Line



$$P_k^{injected} = \sum_{i \in N_k} V_k V_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}]$$

$$P_2 = V_2 \cdot V_1 \cdot [G_{21} \cos(\theta_2) + B_{21} \sin(\theta_2)]$$
$$= B_{21} V_2 V_1 \sin(\theta_2)$$
$$\approx B_{21}\theta_2 \quad \text{where } \sin(\theta_2 - \theta_1) \approx \theta_2 - \theta_1$$
$$= -b_{12}\theta_2$$
$$= \frac{1}{x_{12}}\theta_2$$

$$z = jx_{12} = \frac{1}{jb_{12}} = \frac{1}{y}$$

$$b_{12} = -\frac{1}{x_{12}}$$

x_{12} is positive, b_{12} is negative!

$$B = \begin{bmatrix} b_{12} & -b_{12} \\ -b_{12} & b_{12} \end{bmatrix}$$

Is there an analogous approximation for a full network, rather than a single line?

DC Power Flow

B_P : the *reduced* susceptance matrix

"reduced": remove the slack bus row and column (just PQ and PV buses)

P_P : the vector of real power injections with the slack bus injection removed

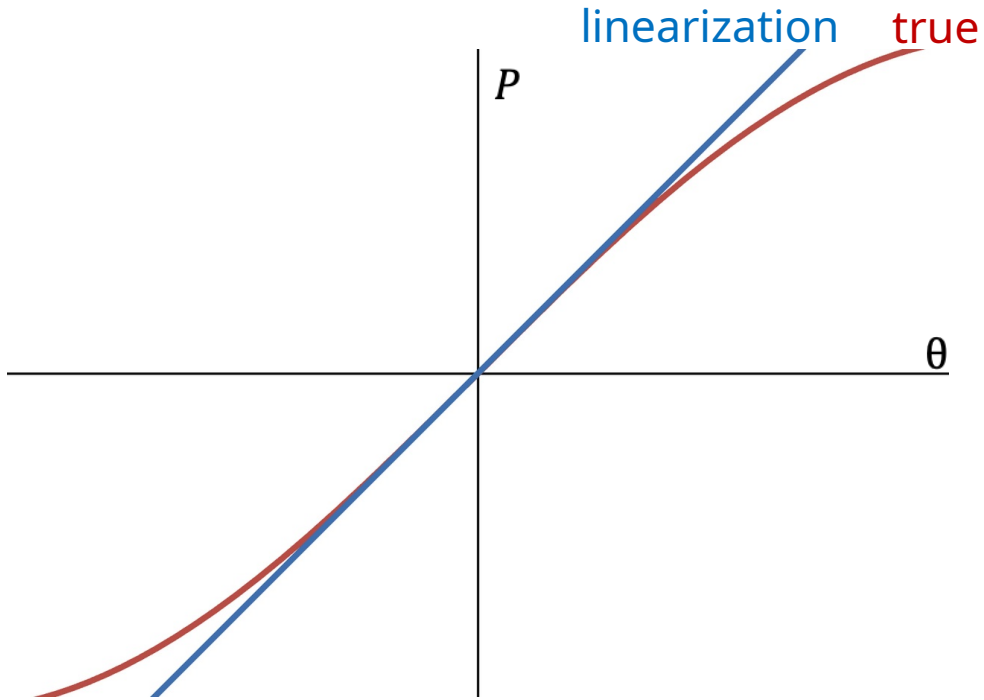
θ_P : bus voltage angle differences to the slack bus, with the slack bus angle = 0 removed

$$P_P \approx -B_P \theta_P$$

High-level Linearization Points

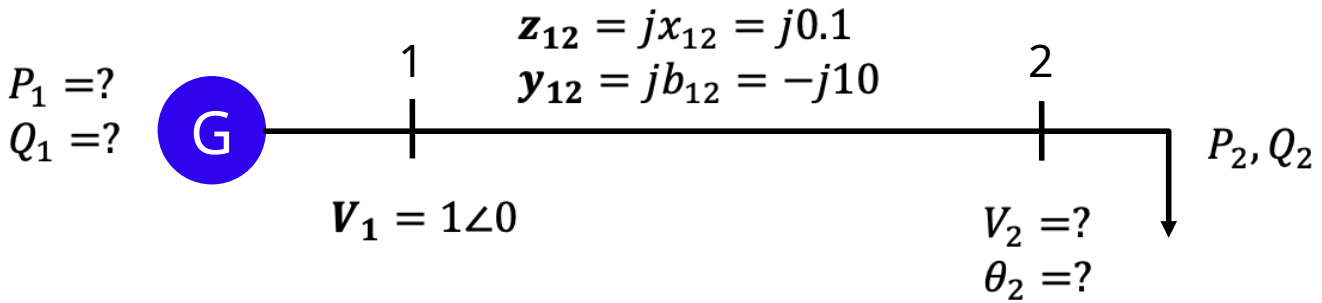
Point 1: Linearizations can be quite accurate close to the linearization point.

Point 2: Linearization accuracy often decreases the further you go from the operating point.



Point 3: Linearizations are approximations and thus will not give a true AC-feasible voltages.

Power Flow Approximation for 2-Bus System

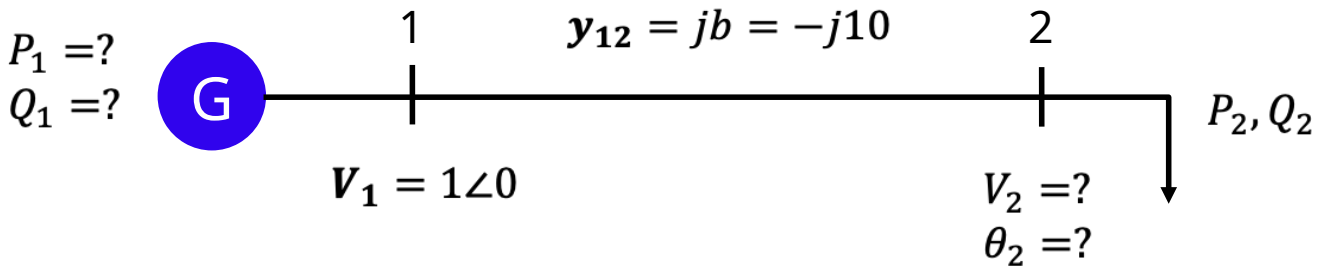


$$Y = G + jB = j\begin{bmatrix} b & -b \\ -b & b \end{bmatrix} = j\begin{bmatrix} -10 & 10 \\ 10 & -10 \end{bmatrix}$$

Class exercise 1: write out DC power flow for this 2_Bus system

What intuition does it give us for the relationship between P_2 and θ_2?

Power Flow Approximation for 2-Bus System

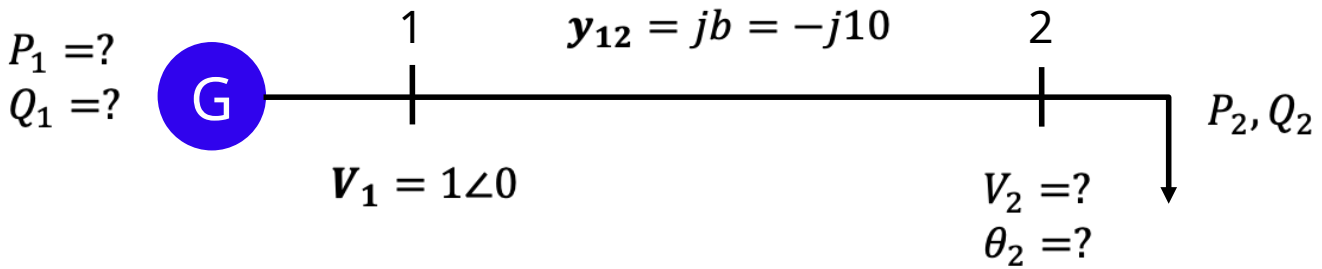


$$B = \begin{bmatrix} -10 & 10 \\ 10 & -10 \end{bmatrix}$$

$$B_R = [-10]$$

Class exercise 2: why do we use B_P and not B?

Power Flow Approximation for 2-Bus System



Class exercise 3: Can you find a linearization from Q_2 to V_2?

$$Q_k^{injected} = \sum_{i \in N_k} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$

$$B = \begin{bmatrix} -10 & 10 \\ 10 & -10 \end{bmatrix}$$

$$\hookrightarrow \quad Q_2 = V_2 V_1 [G_{21} \sin(\theta_2) - B_{21} \cos(\theta_2)] - V_2{}^2 B_{22}$$

Hint: approximations/substitutions:

$$\cos(\theta_2) \approx 1$$

(not as easy as $P \sim \theta$ derivation, next slide gives derivation method)

Two ways of determining sensitivities/linearizations

1) Calculate derivative/Jacobian directly

$$\Delta y = \left(\frac{\partial f}{\partial x}\right)_{x_0} \Delta x$$

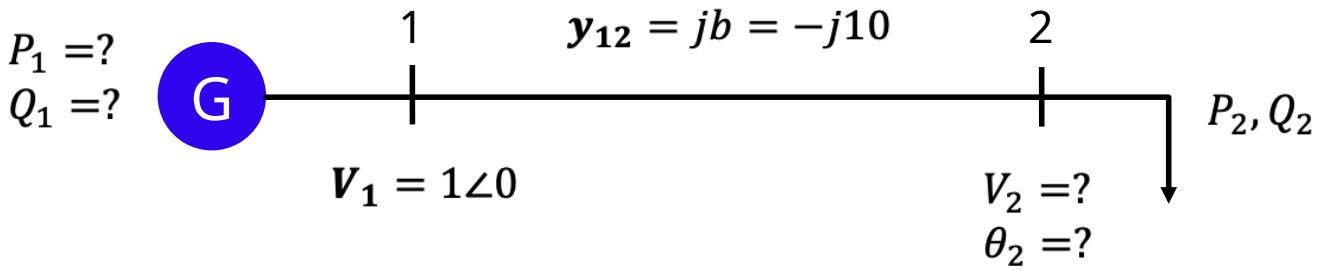
→ done

2) First-order expansion by substituting $x = x^* + \Delta x$

$$y^* + \Delta y = f(x^* + \Delta x)$$

→ discard higher-order terms such as Δx^2

Power Flow Approximation for 2-Bus System



Class exercise 3: Can you find a linearization from Q_2 to V_2?

$$Q_k^{injected} = \sum_{i \in N_k} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$

$$B = \begin{bmatrix} -10 & 10 \\ 10 & -10 \end{bmatrix}$$

$$\begin{aligned} Q_2 &= V_2 V_1 [G_{21} \sin(\theta_2) - B_{21} \cos(\theta_2)] - V_2{}^2 B_{22} \\ &= -10 V_2 \cos\theta_2 + 10 V_2^2 \\ &\approx -10 V_2 + 10 V_2^2 \\ &= -10(1 + \Delta V_2) + 10(1 + \Delta V_2)^2 \\ &= 10\Delta V_2 + 10(\Delta V_2)^2 \\ &\approx 10\Delta V_2 \end{aligned}$$

dropping this term gives the linearization

Hint: approximations/substitutions:

$\cos(\theta_2) \approx 1$

$\Delta V_2^2 \approx 0$

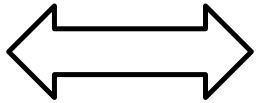
$V_2 = V_2^* + \Delta V_2 = 1 + \Delta V_2$

Two Representations of AC Power Flow

Real-valued bus injection equations

$$P_k = \sum_{i \in N_k} V_k V_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}]$$

$$Q_k = \sum_{i \in N_k} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$



Complex-valued power equations

$$\boldsymbol{S} = \mathrm{diag}(\boldsymbol{V})\bar{\boldsymbol{I}}, \qquad \boldsymbol{I} = \boldsymbol{Y}\boldsymbol{V}$$

$$\boldsymbol{S} = \mathrm{diag}(\boldsymbol{V})\overline{\boldsymbol{Y}\boldsymbol{V}}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \approx \mathrm{J} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \approx \begin{bmatrix} \mathrm{J}_1 & \mathrm{J}_2 \\ \mathrm{J}_3 & \mathrm{J}_4 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$$

Sensitivity Derivation, Method 1

Assumptions: $\boldsymbol{V} = \mathbf{1}$ (angles all are zero), and the network has no shunt admittances

Method 1: Populate $\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}$ element-by-element with partial derivatives of the real-valued PF equations

$$P_k^{injected} = \sum_{i \in N_k} V_k V_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}]$$

$$Q_k^{injected} = \sum_{} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$

Active power / voltage angle

Reactive power / voltage magnitude

for the off-diagonal entries $i \neq k$

$$[J_1]_{ki} = \frac{\partial P_k}{\partial \theta_i} = V_k V_i [G_{ki}\sin(\theta_{ki}) - B_{ki}\cos(\theta_{ki})]$$

$$\approx -V_k V_i B_{ki} \approx -B_{ki}$$

$$[J_4]_{ki} = \frac{\partial Q_k}{\partial V_i} = V_k [G_{ki}\sin(\theta_{ki}) - B_{ki}\cos(\theta_{ki})]$$

$$\approx -V_k B_{ki} \approx -B_{ki}$$

the diagonal entries $i = k$ admit the same approximation

$$[J_1]_{kk} \approx -V_k B_{kk} \approx -B_{kk}$$

$$[J_4]_{kk} \approx -V_k B_{kk} \approx -B_{kk}$$

Sensitivity Derivation, Method 1

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Active power / voltage magnitude

Reactive power / voltage angle

for the off-diagonal entries $i \neq k$

$$[J_2]_{ki} = \frac{\partial P_k}{\partial V_i} = V_k [G_{ki}\cos(\theta_{ki}) + B_{ki}\sin(\theta_{ki})]$$

$$\approx V_k G_{ki} \approx G_{ki}$$

$$[J_3]_{ki} = \frac{\partial Q_k}{\partial \theta_i} = -V_k V_i [G_{ki}\cos(\theta_{ki}) + B_{ki}\sin(\theta_{ki})]$$

$$\approx -V_k V_i G_{ki} \approx -G_{ki}$$

the diagonal entries $i = k$ admit the same approximation

$$[J_2]_{kk} \approx V_k G_{kk} \approx G_{kk}$$

$$[J_3]_{kk} \approx -V_k V_i G_{kk} \approx -G_{kk}$$

1

Sensitivity Derivation, Method 1

Assumptions: $\boldsymbol{V} = \mathbf{1}$ (angles all are zero), and the network has no shunt admittances

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$$P_k^{injected} = \sum_{i \in N_k} V_k V_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}]$$

$$Q_k^{injected} = \sum_{} V_k V_i [G_{ki} \sin\theta_{ki} - B_{ki} \cos\theta_{ki}]$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \approx \begin{bmatrix} -B & G \\ -G & -B \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$$

Two ways of determining sensitivities/linearizations

1) Calculate derivative/Jacobian directly

$$\Delta y = \left(\frac{\partial f}{\partial x}\right)_{x_0} \Delta x$$

→ done

2) First-order expansion by substituting $x = x^* + \Delta x$

$$y^* + \Delta y = f(x^* + \Delta x)$$

→ discard higher-order terms such as Δx^2

Sensitivity Derivation, Method 2

Assumptions: $\boldsymbol{V} = \mathbf{1}$ (angles all are zero), and the network has no shunt admittances

$$\boldsymbol{S} = \text{diag}(\boldsymbol{V})\overline{\boldsymbol{Y}\boldsymbol{V}}$$

$$\begin{aligned}\boldsymbol{S} + \Delta\boldsymbol{S} &= \text{diag}(\boldsymbol{V} + \Delta\boldsymbol{V})\,\overline{(\boldsymbol{I} + \Delta\boldsymbol{I})} \\ &= \text{diag}(\boldsymbol{V})\,\bar{\boldsymbol{I}} + \underbrace{\text{diag}(\bar{\boldsymbol{I}})\,\Delta\boldsymbol{V} + \text{diag}(\boldsymbol{V})\,\Delta\bar{\boldsymbol{I}}}_{\text{linearization}} + \text{diag}(\Delta\boldsymbol{V})\,\Delta\bar{\boldsymbol{I}}\end{aligned}$$

$\boldsymbol{S}$

dropping this term gives the linearization

Sensitivity Derivation, Method 2

Assumptions: $\boldsymbol{V} = \mathbf{1}$ (angles all are zero), and the network has no shunt admittances

Calculate the full Jacobian from the Complex Power Flow Equations:

$$\boldsymbol{S} = \mathrm{diag}(\boldsymbol{V})\overline{\boldsymbol{Y}\boldsymbol{V}}$$

vectors

At the nominal voltage $\boldsymbol{V} = 1$, $\boldsymbol{I} = \boldsymbol{Y}\boldsymbol{V} = 0$

Therefore, $\Delta\boldsymbol{S} \approx \mathrm{diag}(\overline{\boldsymbol{I}})\,\Delta\boldsymbol{V} + \mathrm{diag}(\boldsymbol{V})\,\Delta\overline{\boldsymbol{I}}$ (0 over $\overline{\boldsymbol{I}}$, 1 over $\boldsymbol{V}$)

$$= \Delta\overline{\boldsymbol{I}}$$
$$= \overline{\boldsymbol{Y}\Delta\boldsymbol{V}}$$
$$= \overline{(G + jB)(\Delta V + j\Delta\theta)}$$
$$= G\Delta V - B\Delta\theta - j(B\Delta V + G\Delta\theta)$$

$$\boldsymbol{S} + \Delta\boldsymbol{S} = \mathrm{diag}(\boldsymbol{V} + \Delta\boldsymbol{V})\,\overline{(\boldsymbol{I} + \Delta\boldsymbol{I})}$$

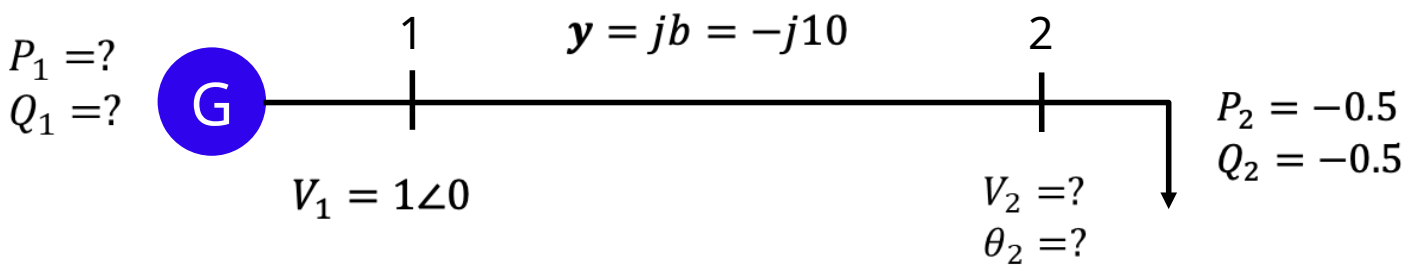
$$\Delta\boldsymbol{S} = \underbrace{\mathrm{diag}(\overline{\boldsymbol{I}})\,\Delta\boldsymbol{V} + \mathrm{diag}(\boldsymbol{V})\,\Delta\overline{\boldsymbol{I}}}_{\text{linear part}} + \mathrm{diag}(\Delta\boldsymbol{V})\,\Delta\overline{\boldsymbol{I}}$$

dropping this term gives the linearization

Equating real and imaginary parts gives, once again,

$$\begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & G \\ -G & -B\end{bmatrix}\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix}$$

Power Flow Approximation for 2-Bus System

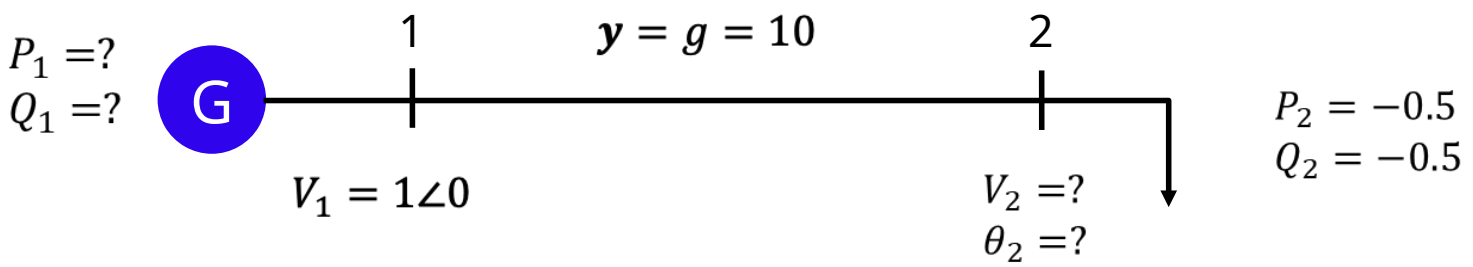


Class exercise 3: Can you find a sensitivity of ΔQ to ΔV?

$$\begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & G \\ -G & -B\end{bmatrix}\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix}$$

Now it's too easy

Power Flow Approximation for 2-Bus System



Class exercise 4: what would the sensitivity of ΔP to $\Delta\theta$ be if the line was resistive?

$$\begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & G \\ -G & -B\end{bmatrix}\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix}$$

Also too easy

Can you derive DC Power Flow from the sensitivity matrix below?

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \approx \begin{bmatrix} -B & G \\ -G & -B \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

What is still left to be done?

2 Bus Example | DC Power Flow

$$\begin{bmatrix}\Delta p_1\\ \Delta p_2\end{bmatrix} \approx -\begin{bmatrix} b_{12} & -b_{12}\\ -b_{12} & b_{12}\end{bmatrix}\begin{bmatrix}0\\ \Delta\theta_2\end{bmatrix}$$

$\Delta\theta_1 = 0$: remove column 1

$\Delta P_1 = -\sum_{i=2}^{n_{\text{bus}}} \Delta P_i$: remove row 1

DC Power Flow: $\Delta p_2 \approx -[b_{12}]\Delta\theta_2$

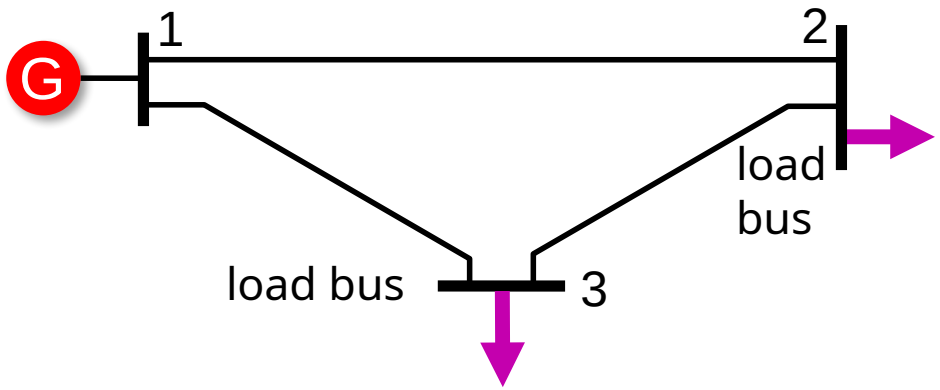
example: $B = \begin{bmatrix}-10 & 10\\ 10 & -10\end{bmatrix}$

example: $B_P = [-10]$

(b_{12} is negative!)

Can you invert B? Can you invert B_P?

3 Bus Example | DC Power Flow



3 Bus Example | DC Power Flow

$$\begin{bmatrix}\Delta P_1\\ \Delta P_2\\ \Delta P_3\end{bmatrix} \approx -\begin{bmatrix}(b_{12}+b_{13}) & b_{12} & b_{13}\\ -b_{12} & (b_{12}+b_{23}) & -b_{23}\\ -b_{13} & -b_{23} & (b_{13}+b_{23})\end{bmatrix}\begin{bmatrix}0\\ \Delta\theta_2\\ \Delta\theta_3\end{bmatrix}$$

$\Delta\theta_1 = 0$: remove column 1

$\Delta P_1 = -\sum_{i=2}^{n_{\text{bus}}} \Delta P_i$: remove row 1

DC Power Flow:

$$\begin{bmatrix}\Delta P_2\\ \Delta P_3\end{bmatrix} \approx -\begin{bmatrix}(b_{12}+b_{23}) & -b_{23}\\ -b_{23} & (b_{13}+b_{23})\end{bmatrix}\begin{bmatrix}\Delta\theta_2\\ \Delta\theta_3\end{bmatrix}$$

Can you invert B? Can you invert B_P?

Why would we want to invert the linearization?

This allows us to determine the power angles as a function of the power injections, which is often what we want to do.

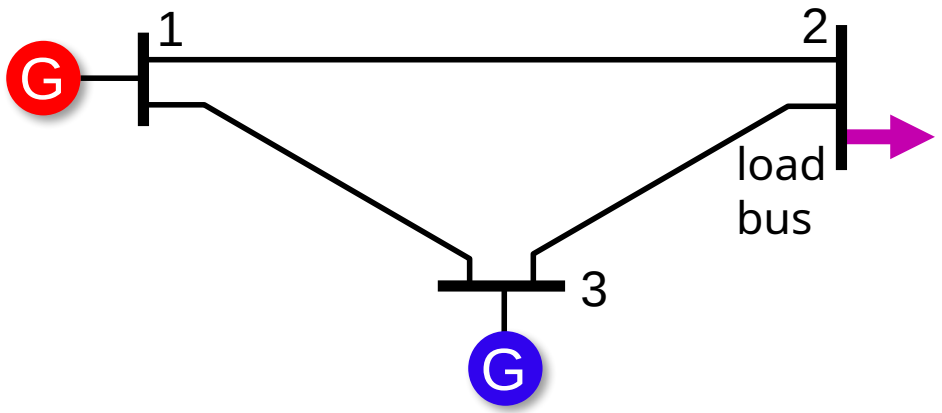
$$\Delta\theta_P = -B_P^{-1}\Delta P_P$$

The “Slack Bus”

1. Fixed voltage angle: removes the arbitrary angle-reference ambiguity
2. Adjustable power injection: maintains network power balance

3-Bus Example | DC Power Flow

Physical reality:



Modeling: either Bus 1 or Bus 3 could be chosen as the slack bus

DC Power Flow determines the network angles as a function of the power injections

$$\Delta\theta_P = -B_P^{-1}\Delta P_P$$

What impact does our selection of the slack bus have on the "roles" of G_1 and G_3?

The “Slack Bus”

1. Fixed voltage angle: removes the arbitrary angle-reference ambiguity
2. Adjustable power injection: maintains network power balance
3. Fixed voltage magnitude: represents a regulated generator or a stiff substation

The standard slack-bus model assigns all three functions to one bus for simplicity.

1. Any bus can provide the angle reference
2. Power balancing (both P and Q) can be assigned to any bus or shared among several buses
3. Often, many generators are controlled to be constant voltage

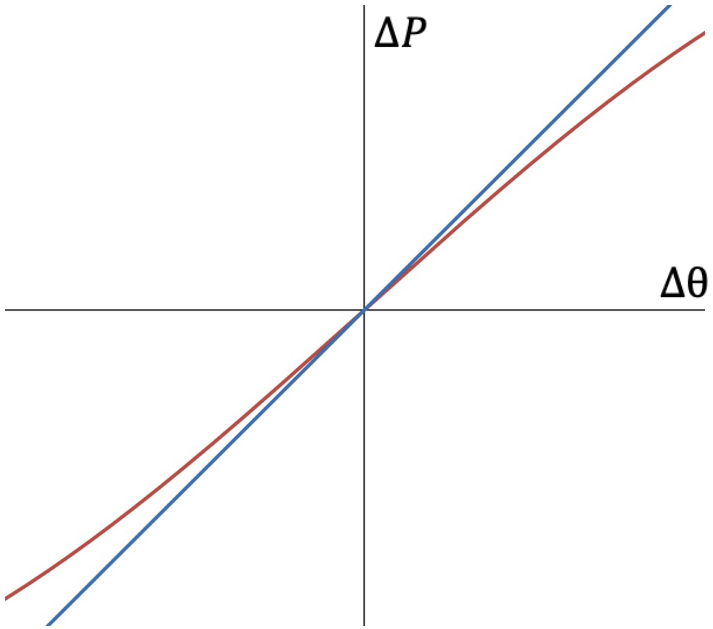
Class Exercise

For each task, choose DC, AC, or both. Justify your decision.

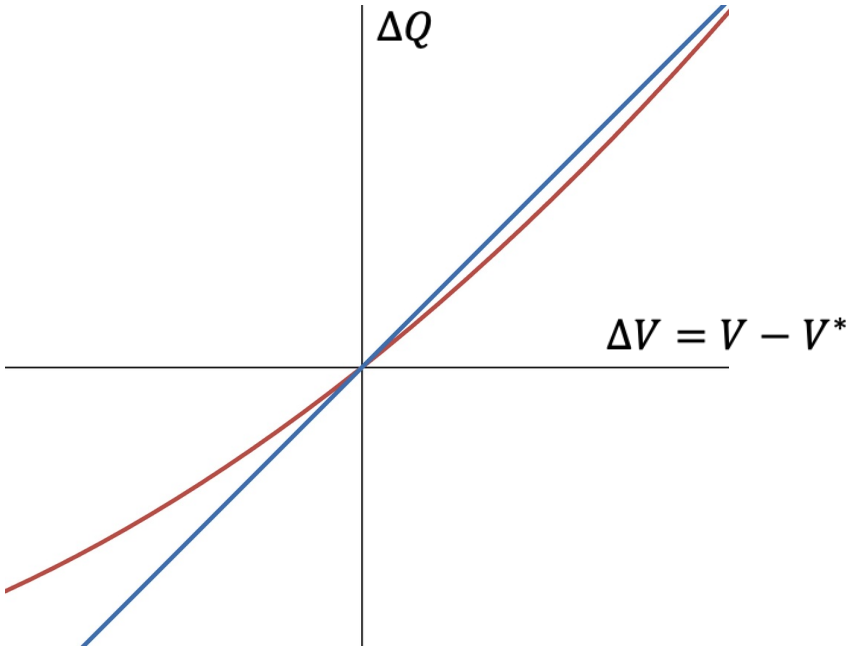
Grid-operator task	Your choice
Screen 100,000 possible line outages for overloaded lines	
Investigate low voltages after one severe outage	
Recalculate the least-cost generator dispatch every five minutes	
Verify that the chosen dispatch satisfies voltage and reactive-power limits	
Estimate how flows change following a small generator redispatch	

What matters in each task: speed and scale, or voltage and reactive-power detail?

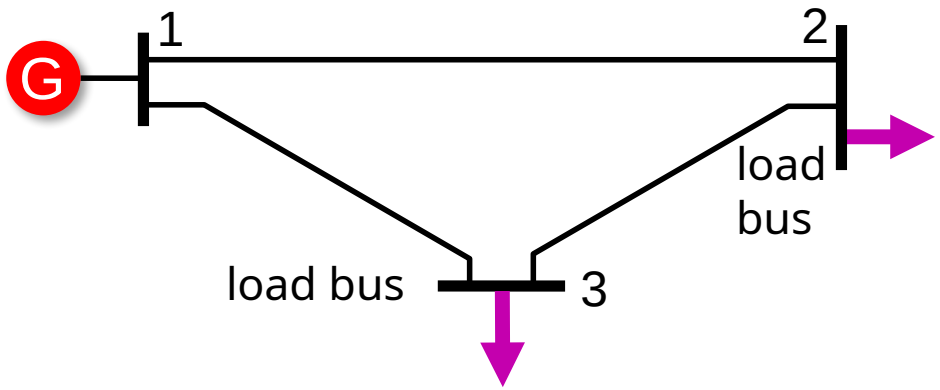
We have DC Power Flow for real power and voltage angle.



Can we accompany it with a linearization for voltage magnitude and reactive power?



3 Bus Example | DC Power Flow



3-Bus Example | Decoupled Linearized Power Flow

Now, we will stop ignoring voltage magnitudes

Can we add an additional sensitivity between voltage magnitude and reactive power?

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & 0\\ 0 & -B\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix}$$

Consider the case of a single, constant voltage bus, chosen to be the slack bus.

$$\begin{bmatrix}\Delta Q_1\\ \Delta Q_2\\ \Delta Q_3\end{bmatrix} \approx -\begin{bmatrix}(b_{12}+b_{13}) & b_{12} & b_{13}\\ -b_{12} & (b_{12}+b_{23}) & -b_{23}\\ -b_{13} & -b_{23} & (b_{13}+b_{23})\end{bmatrix}\begin{bmatrix}0\\ \Delta V_2\\ \Delta V_3\end{bmatrix}$$

What assumptions are made by removing the slack bus row/column?

$\Delta V_1 = 0$: remove column 1

$\Delta Q_1 = -\sum_{i=2}^{n_{\text{bus}}} \Delta Q_i$: remove row 1

What does the invertible B_P allow us to answer that the non-invertible B does not?

3-Bus Example | Decoupled Linearized Power Flow

Now, we will stop ignoring voltage magnitudes

Can we add an additional sensitivity between voltage magnitude and reactive power?

$$\begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & 0 \\ 0 & -B\end{bmatrix}\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix}$$

Consider the case of a single, constant voltage bus, chosen to be the slack bus.

$$\Delta\theta_P = -B_P^{-1}\Delta P_P$$

$$\begin{bmatrix}\Delta Q_2 \\ \Delta Q_3\end{bmatrix} \approx -\begin{bmatrix}b_{12}+b_{23} & -b_{23} \\ -b_{23} & b_{13}+b_{23}\end{bmatrix}\begin{bmatrix}\Delta V_2 \\ \Delta V_3\end{bmatrix}$$

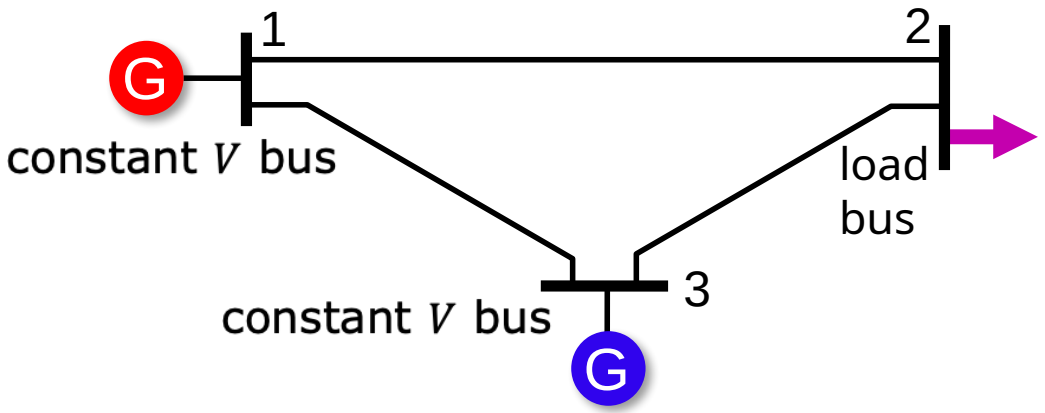
$$\Delta\theta_1 = 0, \qquad \Delta P_1 = -\sum_{i=2}^{n_{\text{bus}}} \Delta P_i$$

$$\Delta V_1 = 0, \qquad \Delta Q_1 = -\sum_{i=2}^{n_{\text{bus}}} \Delta Q_i$$

But what if there are many constant voltage buses?

3-Bus Example | Decoupled Linearized Power Flow

Physical reality:



$$\Delta\theta_P \approx -B_P^{-1}\Delta P_P$$

$$\Delta V_? \approx -B_?^{-1}\Delta Q_?$$

What voltages and reactive power injections should be included?

Step 1: Determine the voltage of the PQ buses

$$\Delta V_Q \approx -B_Q^{-1}\Delta Q_Q$$

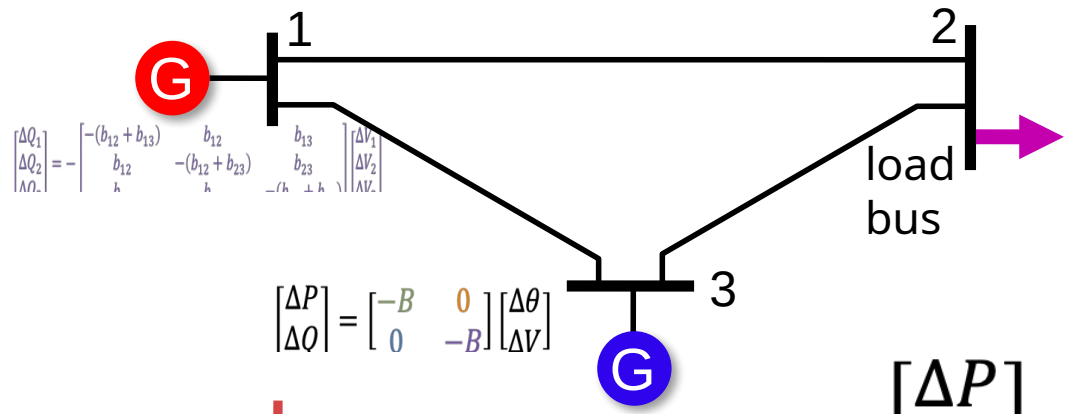
B with only PQ columns and rows remaining

Step 2: Determine the remaining Q injections

$$\Delta Q_V \approx -B_V\Delta V_Q$$

B with
- columns: PQ
- rows: slack bus and PV

3 Bus Linearized Power Flow



$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}-B & 0\\ 0 & -B\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix}$$

Step 1:

$$\begin{bmatrix}\Delta Q_1\\ \Delta Q_2\\ \Delta Q_3\end{bmatrix} \approx -\begin{bmatrix}(b_{12}+b_{13}) & b_{12} & b_{13}\\ -b_{12} & (b_{12}+b_{23}) & -b_{23}\\ -b_{13} & -b_{23} & (b_{13}+b_{23})\end{bmatrix}\begin{bmatrix}0\\ \Delta V_2\\ 0\end{bmatrix}$$

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & 0\\ 0 & -B\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix}$$

$$\Delta V_2 \approx -B_Q^{-1}\Delta Q_2$$
$$\Delta V_2 \approx -(b_{12}+b_{23})^{-1}\Delta Q_2$$

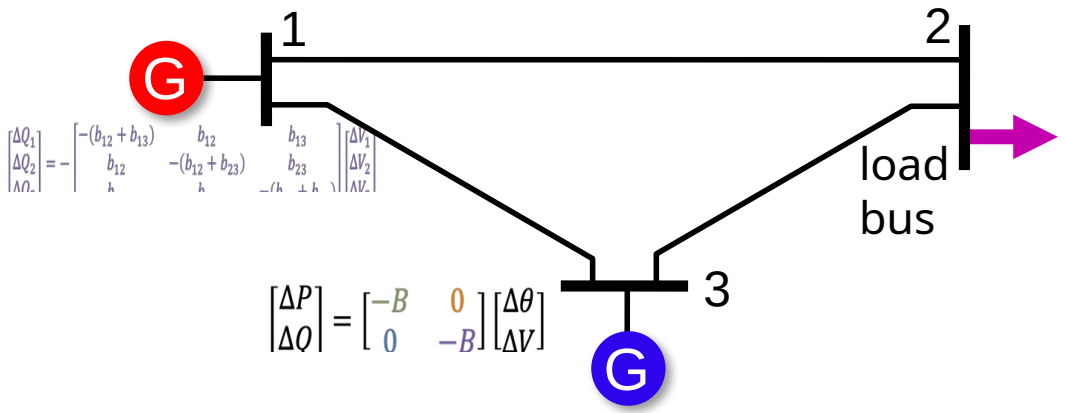
B_Q is B with only the rows and columns for the PQ buses remaining

What is the sensitivity of the voltage magnitude at Bus 2 to the voltage reactive power injection at Bus 2?

assume $b_{13} = b_{23} = -1$, which corresponds to an inductive transmission line

$$-(b_{12}+b_{23})^{-1} = -0.5$$

3 Bus Linearized Power Flow



Step 2:

once you know ΔV_2, how do you determine ΔQ_1 and ΔQ_3?

$$\begin{bmatrix}\Delta Q_1\\ \Delta Q_2\\ \Delta Q_3\end{bmatrix} \approx -\begin{bmatrix}(b_{12}+b_{13}) & -b_{12} & -b_{13}\\ -b_{12} & (b_{12}+b_{23}) & -b_{23}\\ -b_{13} & -b_{23} & (b_{13}+b_{23})\end{bmatrix}\begin{bmatrix}0\\ \Delta V_2\\ 0\end{bmatrix}$$

B_V is B with

- the columns for the slack bus and PV buses removed
- the rows for the PQ buses removed

$$\Delta Q_Q \approx B_V \Delta V_V$$

$$\begin{bmatrix}\Delta Q_1\\ \Delta Q_3\end{bmatrix} \approx \begin{bmatrix}b_{12}\\ b_{23}\end{bmatrix}\Delta V_2$$

Decoupled Linearized Power Flow

$$\begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & 0 \\ 0 & -B\end{bmatrix}\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix}$$

$$\Delta\theta_P \approx -B_P^{-1}\Delta P_P \qquad \Delta V_Q \approx -B_Q^{-1}\Delta Q_Q$$

B_Q is B with the rows and columns of the constant-V buses removed

ΔV_Q is the vector of non-constant voltages (PQ buses)

ΔQ_Q is the vector of reactive power injections for the PQ buses

3 bus network with two constant-voltage buses example:

$$\begin{bmatrix}\Delta P_2 \\ \Delta P_3\end{bmatrix} \approx -\begin{bmatrix}(b_{12}+b_{23}) & -b_{23} \\ -b_{23} & (b_{13}+b_{23})\end{bmatrix}\begin{bmatrix}\Delta\theta_2 \\ \Delta\theta_3\end{bmatrix}$$

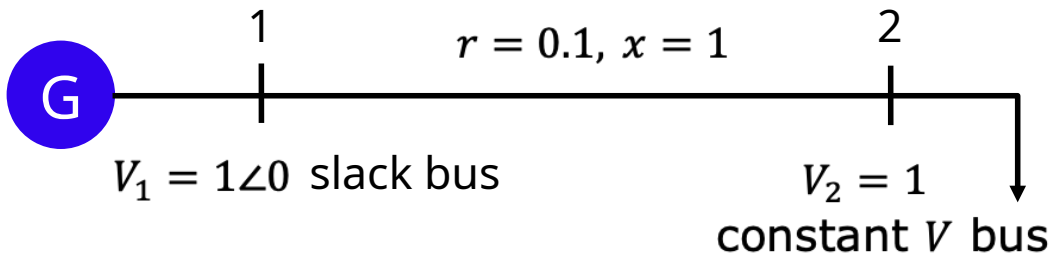
$$\Delta V_2 \approx -(b_{12}+b_{23})^{-1}\Delta Q_2$$

Coupled Linearized Power Flow

Now, we will stop ignoring network resistance

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \approx \begin{bmatrix} -B & G \\ -G & -B \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

Power Flow Approximation for 2-Bus System, small resistance

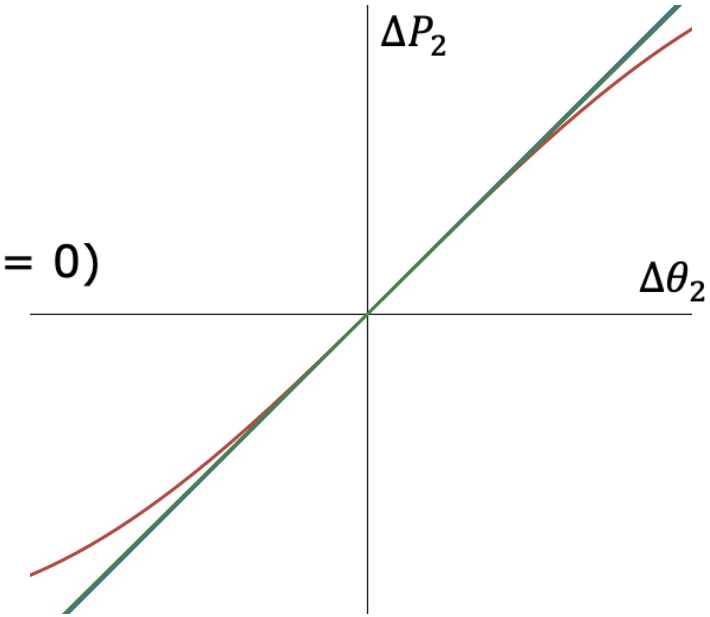


True Nonlinear Power Flow: $\Delta P_2 = 0.099 - 0.099\cos(\Delta\theta_2) + 0.99\sin(\Delta\theta_2)$

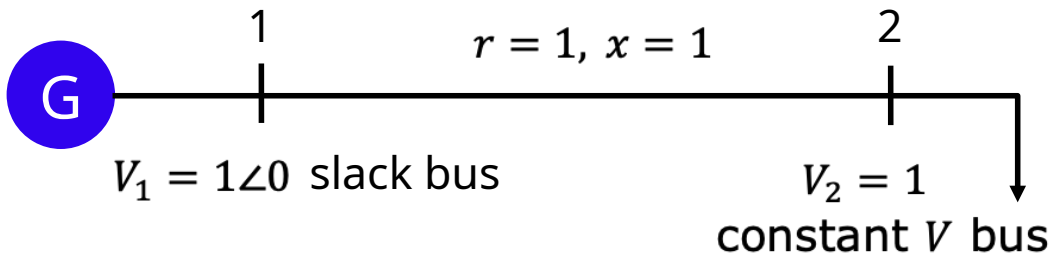
True linearization around $\theta_2^* = 0$: $\Delta P_2 \approx 0.99\Delta\theta_2$

DC Power Flow for $x = 1$ (assumes $r = 0$): $\Delta P_2 \approx \Delta\theta_2$

With small resistance, DC Power Flow is accurate (around $\theta_2^* = 0$)



Power Flow Approximation for 2-Bus System, big resistance

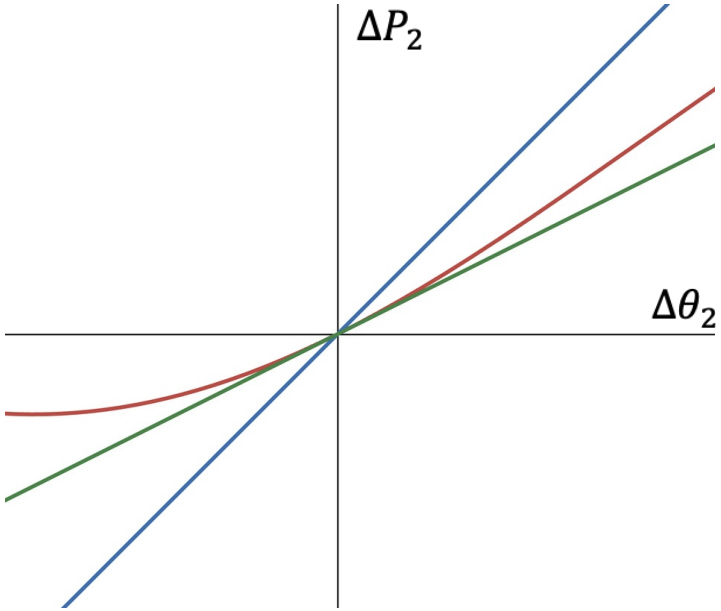


True Nonlinear Power Flow: $\Delta P_2 = 0.5 - 0.5\cos(\Delta\theta_2) + 0.5\sin(\Delta\theta_2)$

True linearization around $\theta_2^* = 0$: $\Delta P_2 \approx 0.5\Delta\theta_2$

DC Power Flow for $x = 1$ (assumes $r = 0$): $\Delta P_2 \approx \Delta\theta_2$

With large resistance, DC Power Flow is not accurate



Coupled Linearized Power Flow

Now, we will stop ignoring network resistance

$$\begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & G \\ -G & -B\end{bmatrix}\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix}$$

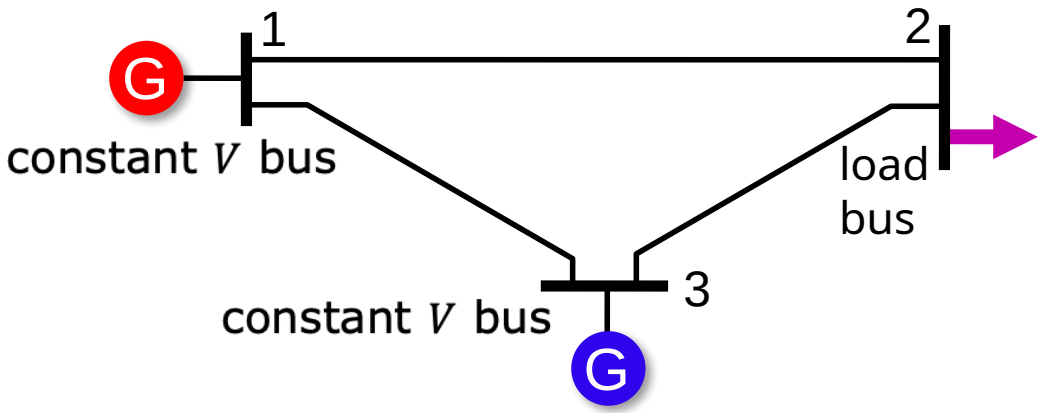
Removing the slack bus and constant voltage buses,

$$\begin{bmatrix}\Delta P_P \\ \Delta Q_Q\end{bmatrix} \approx \begin{bmatrix}-B_{PP} & G_{PQ} \\ -G_{QP} & -B_{QQ}\end{bmatrix}\begin{bmatrix}\Delta\theta_P \\ \Delta V_Q\end{bmatrix}$$

G_{PQ} : G with the slack bus rows and constant-voltage columns removed

G_{QP} : G with the constant-voltage rows and slack bus columns removed

Coupled Linearized Power Flow, 3 Bus Example



Removing the slack bus and constant voltage buses,

denotes matrix dimension

$$\begin{bmatrix}\Delta P_P \\ \Delta Q_Q\end{bmatrix} \approx \begin{bmatrix}-B_{PP} & G_{PQ} \\ -G_{QP} & -B_{QQ}\end{bmatrix}\begin{bmatrix}\Delta\theta_P \\ \Delta V_Q\end{bmatrix}$$

$\mathbb{R}^{2\times 2}$ $\mathbb{R}^{2\times 1}$ $\mathbb{R}^{1\times 2}$ $\mathbb{R}^{1\times 1}$

$$\begin{bmatrix}\Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3\end{bmatrix} \approx -\begin{bmatrix}g_{12}+g_{13} & -g_{12} & -g_{13} \\ -g_{12} & g_{12}+g_{23} & -g_{23} \\ -g_{13} & -g_{23} & g_{13}+g_{23}\end{bmatrix}\begin{bmatrix}0 \\ \Delta\theta_2 \\ \Delta\theta_3\end{bmatrix}$$

$$\begin{bmatrix}\Delta P_1 \\ \Delta P_2 \\ \Delta P_3\end{bmatrix} \approx \begin{bmatrix}g_{12}+g_{13} & -g_{12} & -g_{13} \\ -g_{12} & g_{12}+g_{23} & -g_{23} \\ -g_{13} & -g_{23} & g_{13}+g_{23}\end{bmatrix}\begin{bmatrix}0 \\ \Delta V_2 \\ 0\end{bmatrix}$$

Coupled Linearized Power Flow

Now, we will stop ignoring network resistance

$$\begin{bmatrix}\Delta P \\ \Delta Q\end{bmatrix} \approx \begin{bmatrix}-B & G \\ -G & -B\end{bmatrix}\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix}$$

Removing the slack bus and constant voltage buses,

$$\begin{bmatrix}\Delta P_P \\ \Delta Q_Q\end{bmatrix} \approx \begin{bmatrix}-B_{PP} & G_{PQ} \\ -G_{QP} & -B_{QQ}\end{bmatrix}\begin{bmatrix}\Delta\theta_P \\ \Delta V_Q\end{bmatrix}$$

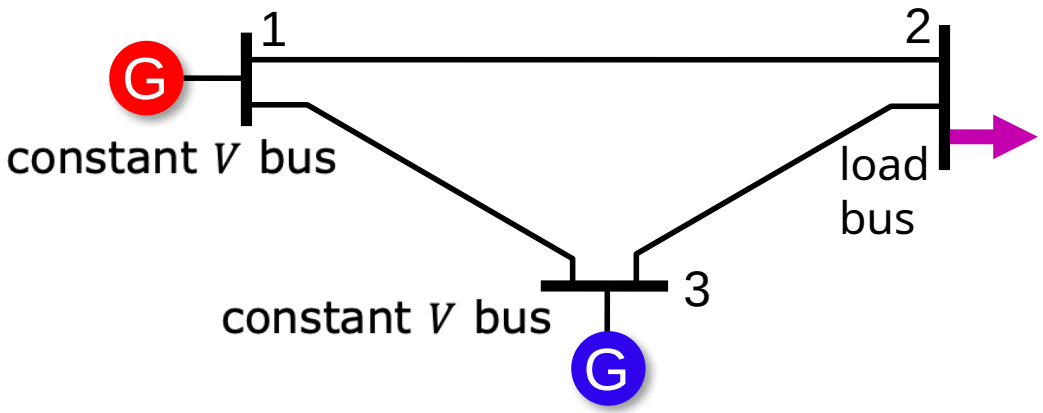
Step 1: Determine the voltages

$$\begin{bmatrix}\Delta\theta_P \\ \Delta V_Q\end{bmatrix} \approx \begin{bmatrix}-B_{PP} & G_{PQ} \\ -G_{QP} & -B_{QQ}\end{bmatrix}^{-1}\begin{bmatrix}\Delta P_P \\ \Delta Q_Q\end{bmatrix}$$

Step 2: Determine the remaining injections

$$\begin{bmatrix}\Delta P_0 \\ \Delta Q_V\end{bmatrix} \approx \begin{bmatrix}-B_{0P} & G_{0Q} \\ -G_{VP} & -B_{VP}\end{bmatrix}\begin{bmatrix}\Delta\theta_P \\ \Delta V_Q\end{bmatrix}$$

Three Bus Example | Coupled Linearized Power Flow

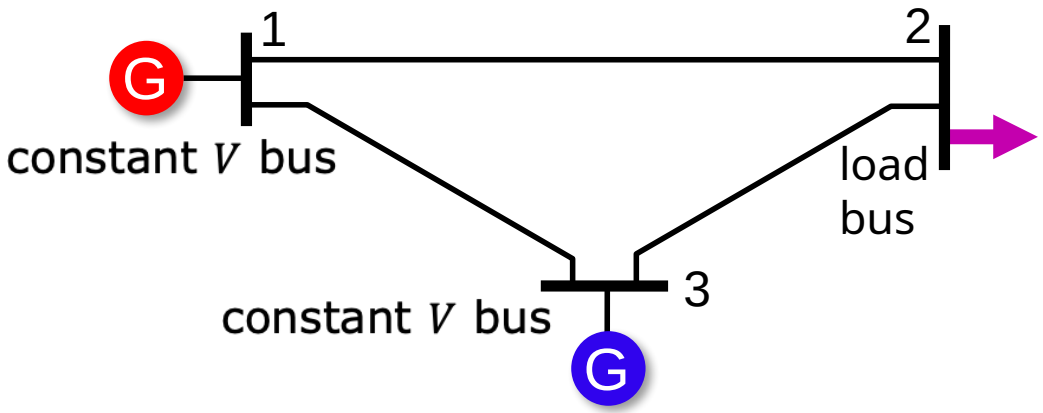


Step 1: Determine the voltages

$$\begin{bmatrix}\Delta\theta_P \\ \Delta V_Q\end{bmatrix} \approx \begin{bmatrix}-B_{PP} & G_{PQ} \\ -G_{QP} & -B_{QQ}\end{bmatrix}^{-1} \begin{bmatrix}\Delta P_P \\ \Delta Q_Q\end{bmatrix}$$

$$\begin{bmatrix}\Delta\theta_2 \\ \Delta\theta_3 \\ \Delta V_2\end{bmatrix} \approx \begin{bmatrix}-(b_{12}+b_{23}) & b_{23} & g_{12}+g_{23} \\ b_{23} & -(b_{13}+b_{23}) & -g_{23} \\ -(g_{12}+g_{23}) & g_{23} & -(b_{12}+b_{23})\end{bmatrix}^{-1} \begin{bmatrix}\Delta P_2 \\ \Delta P_3 \\ \Delta Q_2\end{bmatrix}$$

Three Bus Example | Coupled Linearized Power Flow



Step 2: Determine the remaining injections

$$\begin{bmatrix}\Delta P_0 \\ \Delta Q_V\end{bmatrix} \approx \begin{bmatrix}-B_{0P} & G_{0Q} \\ -G_{VP} & -B_{VP}\end{bmatrix}\begin{bmatrix}\Delta\theta_P \\ \Delta V_Q\end{bmatrix}$$

$$\begin{bmatrix}\Delta P_1 \\ \Delta Q_1 \\ \Delta Q_3\end{bmatrix} \approx \begin{bmatrix}b_{12} & b_{13} & -g_{12} \\ g_{12} & g_{13} & b_{12} \\ g_{23} & -(g_{13}+g_{23}) & b_{23}\end{bmatrix}\begin{bmatrix}\Delta\theta_2 \\ \Delta\theta_3 \\ \Delta V_2\end{bmatrix}$$

Below is a map of wholesale electricity prices on a winter day in Australia

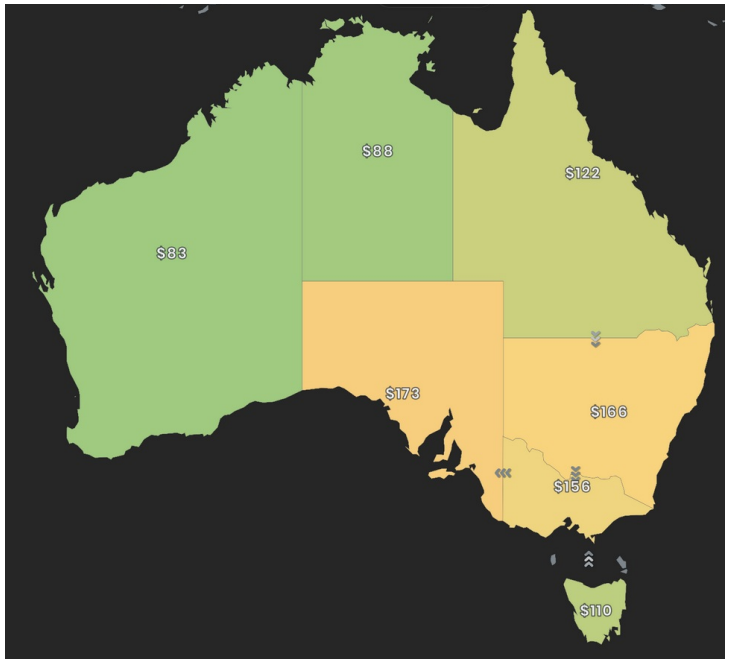
6:00 am



12:00 pm



6:00 pm



https://app.electricitymaps.com/map/live/fifteen_minutes?signal=electricity-price

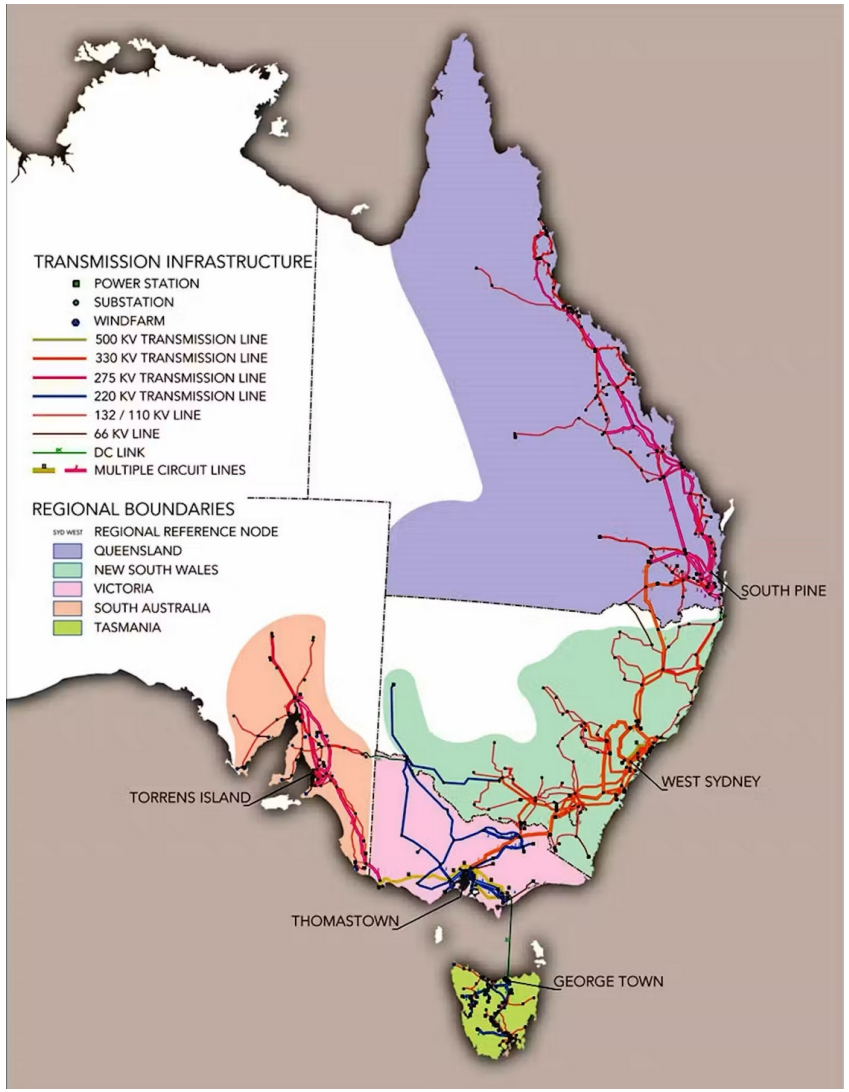
The price separation between the states is determined by the flow sensitivities between regions.

Where do the flow sensitivities come from?

The Eastern-Australian transmission network can be approximated by a 4 node system:



Any of the linearizations can be used to determine the sensitivities between the 4 nodes.

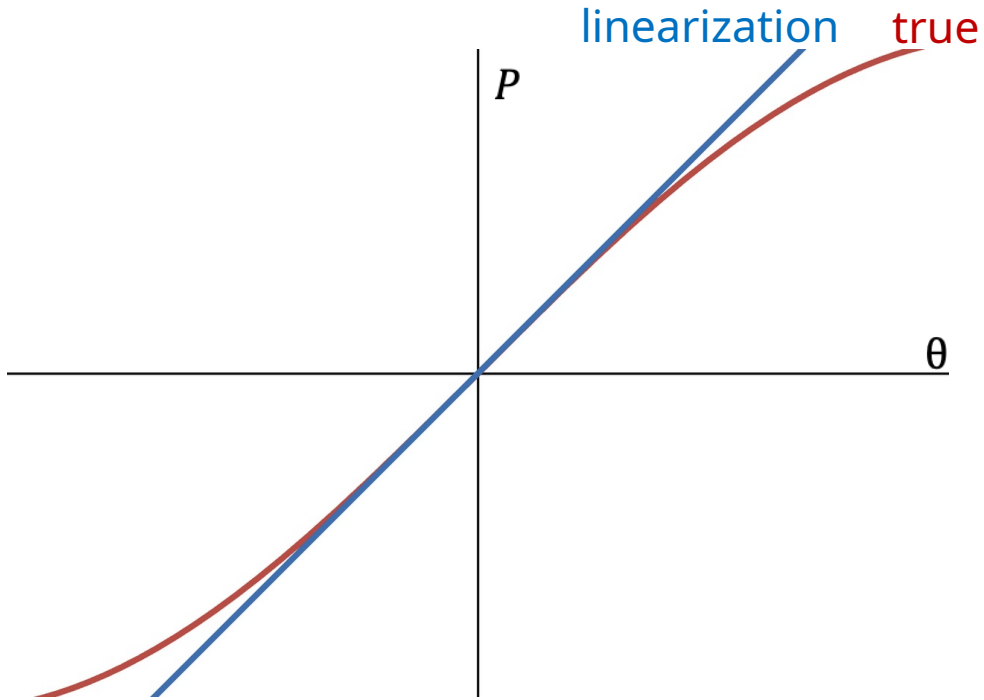


Or, any of the linearizations can be used to determine the sensitivities for the true system model.

High-level Linearization Points

Point 1: Linearizations can be quite accurate close to the linearization point.

Point 2: Linearization accuracy often decreases the further you go from the operating point.



Point 3: Linearizations are approximations and thus will not give a true AC-feasible voltages.

Choosing a Linearized Power Flow Model

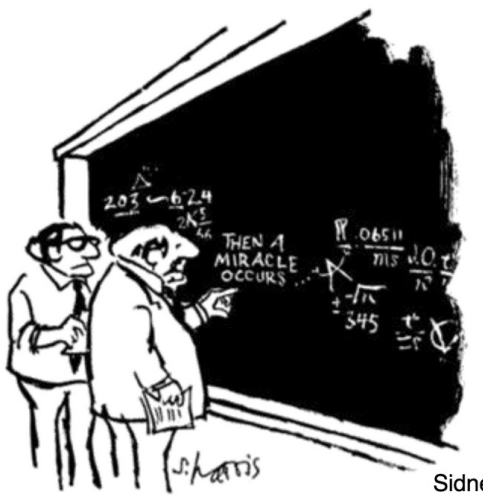
Simpler

DC Power Flow	+ Simple - Assumes zero resistance - Ignores voltage/reactive-power sensitivity
Decoupled Linearized Power Flow	0 Simple, except for constant-voltage buses - Assumes zero resistance + Describes nominal voltage/reactive-power sensitivity
Coupled Linearized Power Flow	0 Simple, except for constant-voltage buses 0 Considers network resistance but does not capture losses + Describes nominal voltage/reactive-power sensitivity
Full-Linearized AC Power Flow	- Not simple + Considers network resistance, describes marginal losses + Most accurate, at the given setpoint

More accurate

Current Industry Practice

Clear markets using the DC power flow approximation → **Engineering intuition and heuristics** → Feasible operating point for the AC power flow equations



Sidney Harris,
Science Cartoons Plus

"I think you should be more explicit here in step two."

Power Flow

Credit: Dan Molzahn
https://www.energy-markets-school.dk/documents/2017/Molzahn/Molzahn%20-%20DTU%20Tutorial.pdf?utm_source=chatgpt.com

Power Flow Analysis – Part 3

ELEN90060

Keith Moffat